Top-Quark Mass Predictions from W, Z Masses and Z Partial Widths

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We use recent measurements of the W- and Z-boson masses and the leptonic, hadronic, and total Z widths to constrain the top-quark mass in the standard model, including full radiative corrections. From a maximum-likelihood analysis we find the most likely value of m_t to be 151 GeV and we obtain the bound $m_t \leq 200$ GeV at 95% C.L., based on the central measured value of the Z mass assuming a Higgs-boson mass of 100 GeV and $\alpha_s(M_Z^2)=0.12$.

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With the advent of the SLAC Linear Collider (SLC) and the CERN e^+e^- collider LEP, the detailed structure of the radiative corrections in the standard model (SM) is being tested for the first time. Comparisons between data and SM predictions are currently hampered by the fact that two elements of the SM, the top quark and Higgs boson, are yet to be discovered. However, consistency between SM predictions and existing data may restrict the possible ranges of the masses of these yet undiscovered particles.[1]

In this paper we seek to constrain the value of the top-quark mass m_t by using the most up-to-date experimental data on the properties of the W and Z gauge bosons without any reference to neutral-current measurements at lower energies. In doing so we avoid several possible ambiguities present in neutral-current analyses, including, an imprecise knowledge of structure functions, the values of various phenomenological parameters (such as the charm-quark mass[2] deduced from neutrino-induced dimuon production data), the effects of slow rescaling,[3] and the contributions to neutrino-nucleon scattering arising from the higher-twist operators.[4] Using the well-determined value of the Z mass (M_Z) as input, we perform a fit to the following pieces of data: the weighted average of the CDF (Ref. 5) and UA2 (Ref. 6) measurements of the W mass (M_W) and the mass ratio M_W/M_Z, as well as a weighted average of the most recent LEP data[7] on the Z total width and leptonic and hadronic partial widths. The experimental data are summarized in Table I. In our analysis, we restrict the Higgs-boson mass m_H to lie in the range $41.6 \leq m_H \leq 1000$ GeV where the lower limit arises from direct Higgs-boson searches[7] at LEP and the upper limit is the usual bound[8] from requiring that perturbative calculations remain valid in the SM. We also include the CDF lower limit[5] of $m_t \geq 89$ GeV in our analysis.

Our analysis of the radiative corrections proceeds as follows. Using the Marciano and Sirlin[9] renormalization scheme, the Z and W masses are given by

$$M_Z^2 = \frac{M_W^2}{1-x_W} = \frac{A}{x_W(1-x_W)}\,\frac{1}{1-\Delta r}\,, \tag{1}$$

where $A \equiv \pi\alpha(m_e)(\sqrt{2}G_F)^{-1} \simeq (37.2802\ \text{GeV})^2$ (Ref. 10). Here, Δr summarizes the effects of the radiative corrections to the tree-level gauge-boson masses. For given top-quark and Higgs-boson masses we take the value of M_Z as determined by LEP data and calculate the values of Δr and x_W precisely to one-loop order using the program of Morris[11] which is based on the work of

TABLE I. Experimental data from LEP, CDF, and UA2 and the weighted-average values of the data used in our analysis. Also shown for comparison are the results from our combined fit which correspond to the most probable value of $m_t=151$ GeV, assuming $M_Z=91.177$ GeV, $m_H=100$ GeV, $c_3=30$, and $\alpha_s=0.12$.

	ALEPH	DELPHI	L3	OPAL	Average	Fit
M_Z (GeV)	91.186 ± 0.013[a]	91.188 ± 0.013[a]	91.161 ± 0.013[a]	91.174 ± 0.011[a]	91.173 ± 0.031	Input
Γ_Z (GeV)	2.506 ± 0.026	2.476 ± 0.026	2.492 ± 0.025	2.505 ± 0.020	2.496 ± 0.016	2.498
Γ_l (MeV)	84.9 ± 1.1	82.0 ± 1.7	84.3 ± 1.4	82.7 ± 1.0	83.7 ± 0.7	84.0
Γ_h (MeV)	1764 ± 23	1756 ± 31	1748 ± 35	1778 ± 24	1764 ± 16	1745
σ_{had} (nb)	41.78 ± 0.55	42.38 ± 0.96	41.38 ± 0.65	41.88 ± 0.62	41.78 ± 0.52	41.52
Γ_{inv} (MeV)	489 ± 20	469 ± 27	494 ± 30	476 ± 23	482 ± 16	501
Γ_h/Γ_l	20.95 ± 0.30	21.02 ± 0.47	21.02 ± 0.62	21.26 ± 0.31	21.08 ± 0.20	20.76
	CDF	UA2			Average	Fit
M_W (GeV)	79.91 ± 0.47	80.79 ± 0.89			80.10 ± 0.42	80.25
M_W/M_Z	0.8775 ± 0.0051	0.8831 ± 0.0055			0.8801 ± 0.0037	0.8802

[a] ± 0.030 systematic error.

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Hollik.[12] We then include by an iterative procedure the leading two-loop contributions to Δr which arise from (i) QCD corrections[13] to the one-loop gauge-boson self-energies from heavy fermions [which are of the order $\alpha\alpha_s(m_t^2/M_W^2)$] and (ii) heavy-fermion irreducible contributions to the ρ parameter[14] [which are of order $\alpha^2(m_t^2/M_W^2)^2$]. The incorporation of these two-loop contributions to Δr leads to an improvement in the predicted value of x_W obtained from Eq. (1); the details of this procedure will be discussed elsewhere.[15] Using this value of x_W, the measured M_Z, and the relation $M_W = M_Z(1-x_W)^{1/2}$ from Eq. (1), M_W is determined to better than 0.8 MeV (for a given set of values for m_t and m_H). The value $\alpha_s(M_Z^2) = 0.12 \pm 0.02$ is assumed[16] in our numerical calculations.

In Fig. 1(a) we show the resulting predicted value of M_W vs m_t for $m_H = 42$, 100, and 1000 GeV and $M_Z = 91.177 \pm 0.031$ GeV. Also presented in the figure are the $\pm 1\sigma$ ranges of M_W determined by CDF and UA2. Similarly, in Fig. 1(b) we display the corresponding predictions for M_W/M_Z as a function of m_t, and the $\pm 1\sigma$ ranges from CDF and UA2. As can be seen from the figures, the CDF data prefer smaller values of m_t (~ 100 GeV), while the UA2 results favor larger values of m_t (~ 220 GeV). From the measurements of M_W and M_W/M_Z in these two experiments, we find that the most likely value of m_t is $m_t = 141$ GeV with the 95%-C.L. upper limit of $m_t \leq 210$ GeV (for $m_H = 100$ GeV, $M_Z = 91.177$ GeV, and $\alpha_s = 0.12$).

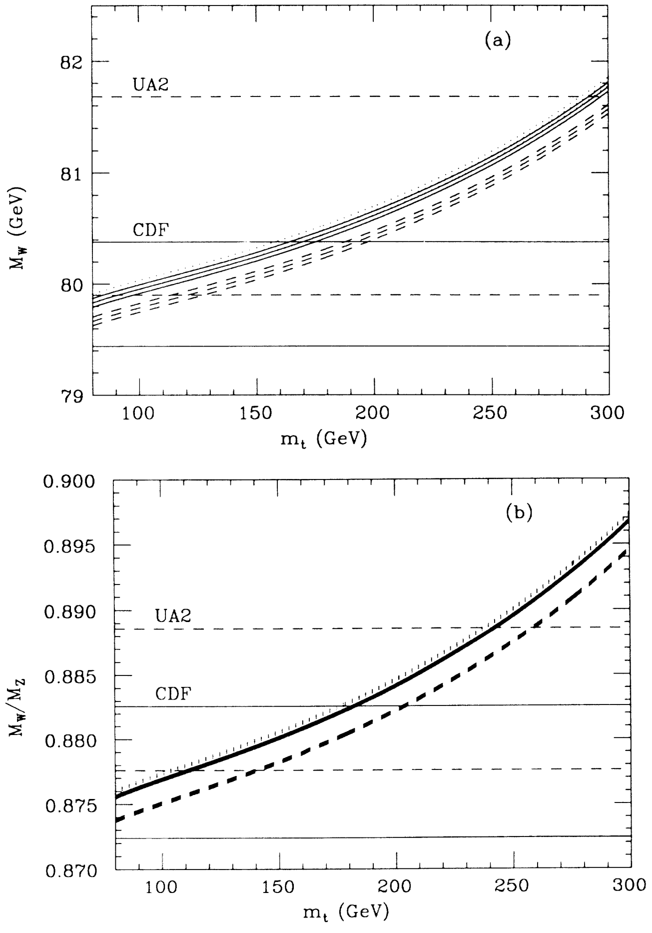

FIG. 1. (a) Predicted value of M_W for $M_Z = 91.177 \pm 0.031$ GeV, $\alpha_s = 0.12$, and $m_H = 42$ (dotted curve), 100 (solid curve), or 1000 (dashed curve) GeV as a function of m_t. The $\pm 1\sigma$ ranges for M_W from CDF (solid) and UA2 (dashed) are also shown as horizontal lines. (b) Same as in (a) but for the predicted value of M_W/M_Z as a function of m_t.

To compare SM predictions with the LEP data we use the improved Born approximation[17] to include radiative corrections to the Z partial and total widths. The following changes must be made in the usual tree-level expressions for the Z partial widths as expressed in terms of G_F: (a) x_W, as it appears in the fermion vector coupling constants, is replaced by the "running" parameter $\tilde{x}_W \equiv \sin^2\theta_W(M_Z)$ given by[17,18]

$$\tilde{x}_W = x_W + (1-x_W)\delta\rho_t + \frac{\alpha}{4\pi}\left[\ln\left(\frac{m_H}{17.3\text{ GeV}}+1\right)-2\right]+\delta_t\,, \tag{2}$$

where $\delta\rho_t$ contains the full irreducible one-loop[19] and leading two-loop contributions[13,14] to the ρ parameter from QCD corrections and heavy top quarks. δ_t summarizes all of the nonleading contributions to the shift in x_W. (b) The overall effective coupling strength G_F is rescaled, i.e., $G_F \to \tilde{G}_F = G_F(1-\delta\rho_t)^{-1}$. (c) The vector and axial-vector couplings of the b quark are modified by corrections of order $G_F m_t^2/\sqrt{2}\pi^2$ in order to account for the rather large vertex corrections arising from top quark loops.[12,17] (d) The partial widths into charged final states are rescaled by a factor of $1+(3\alpha/4\pi)Q_f^2$, where Q_f is the fermion charge, to account for pure QED corrections at the one-loop level. (e) QCD corrections are included for $q\bar{q}$ final states through order α_s^3 (Ref. 20) including the m_t-dependent correction to the axial-vector couplings in order α_s^2.[20] The numerical value of the coefficient (c_3) of the $(\alpha_s/\pi)^3$ QCD correction is presently uncertain. The authors of Ref. 21 have found errors in their original calculation which yielded $c_3 = 64$, and are now repeating the calculation. Maxwell[22] has recently estimated that $c_3 \sim 30$. In order to account for the uncertainties in c_3, we consider three possible values for this coefficient, $c_3 = 0$, 30, and 64. In the case of $b\bar{b}$ final states the finite m_b^2/M_Z^2 terms of order α_s (Ref. 23) are retained. (f) Phase-space corrections to the Z widths from finite τ, b, c, and s masses are included. Combining (a)-(f), the Z partial widths can be written as

$$\Gamma_f = N_c\frac{\tilde{G}_F M_Z^3}{6\sqrt{2}\pi}\left(1+\frac{3\alpha}{4\pi}Q_f^2\right) \times\left[\tfrac{1}{2}\beta_f(3-\beta_f^2)\tilde{v}_f^2+\beta_f^3\tilde{a}_f^2\right], \tag{3}$$

where N_c is a color factor, $\beta_f \equiv (1-4m_f^2/M_Z^2)^{1/2}$, and

$\tilde{v}_f, \tilde{a}_f$ are effective vector and axial-vector couplings which include the above QCD and electroweak corrections. Using Eq. (3) to compute Γ_l, Γ_h, and Γ_Z, we perform our fits using the statistically weighted average of the most recent data obtained from LEP as presented in Table I. Since the LEP experiments[24] take into account correlations between the systematic uncertainties of the decay widths when quoting their results, we treat the errors in Γ_Z, Γ_l, and Γ_h as approximately uncorrelated in our analysis.

We stress that the peak cross section for $e^+e^- \to Z \to$ hadrons ($\sigma_{\rm had}$), the partial width $\Gamma(Z \to b\bar{b})$, and Γ_h/Γ_l are very insensitive to the value of m_t due to the approximate cancellation of the dominant t-quark radiative corrections to these quantities.

The results of a likelihood analysis from a fit to the LEP values of Γ_l, and Γ_h, and Γ_Z are displayed in Fig. 2(a). This figure shows the logarithm of the likelihood function versus m_t for $m_H = 42$, 100 and 1000 GeV and $M_Z = 91.177 \pm 0.031$ GeV with $c_3 = 30$ and $\alpha_s = 0.12$. The 90%- and 95%-C.L. bounds on m_t marked in the figures are based on the central value of M_Z and $m_H = 100$ GeV. Note that as m_H increases, the maximum likelihood favors larger values of m_t. This figure shows that the LEP data alone yield a most likely value for m_t of $m_t = 161$ GeV (for $M_Z = 91.177$ GeV, $c_3 = 30$, $\alpha_s = 0.12$, and $m_H = 100$ GeV) with a 95%-C.L. upper limit of $m_t \leq 221$ GeV.

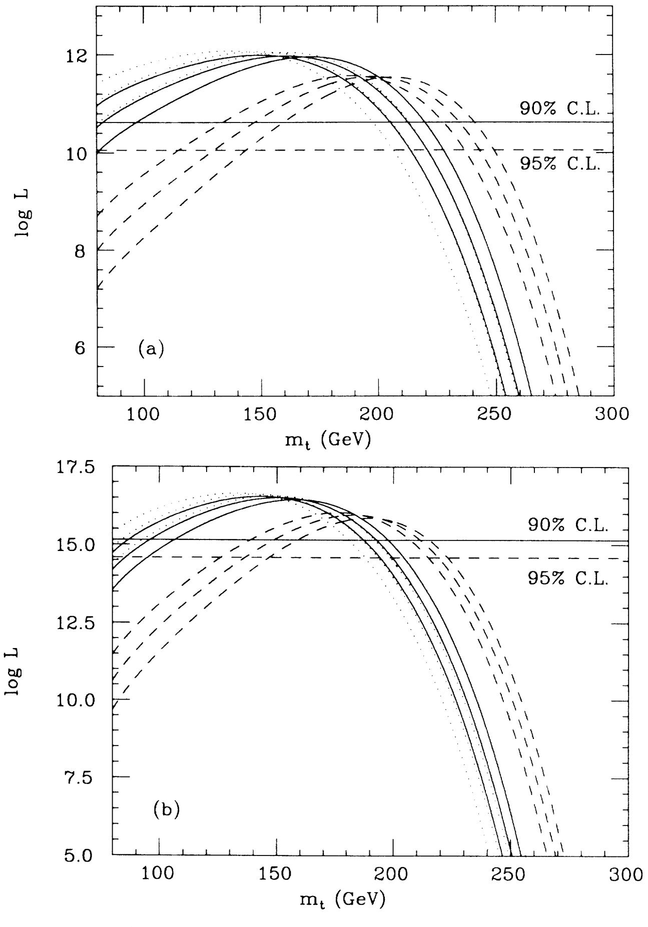

FIG. 2. (a) The logarithm of the likelihood function from a fit to the LEP data as a function of m_t for the same parameters as in Fig. 1 with $c_3 = 30$. (b) Same as in (a), but for the full data set (CDF, UA2, and LEP). In both cases the 90%- (solid) and 95%- (dashed) C.L. limits for $M_Z = 91.177$ GeV, $m_H = 100$ GeV, $c_3 = 30$, and $\alpha_s = 0.12$ are shown as horizontal lines.

If we now combine the LEP, CDF, and UA2 data in a single fit, we obtain the results shown in Fig. 2(b). From this combined analysis we find the most likely value $m_t = 151$ GeV and obtain the bound $m_t \leq 193$ ($m_t \leq 200$) GeV at the 90% (95%) C.L. These bounds are slightly altered as α_s, M_Z, c_3, and m_H are varied. The results obtained from the fit by varying these input parameters one at a time while holding the others fixed (from the set of values $\alpha_s = 0.12$, $c_3 = 30$, $M_Z = 91.177$ GeV, and $m_H = 100$ GeV) are displayed in Table II. Note that as m_H increases, larger values of m_t become more favorable in the fit to the full data set. However, for larger values of m_H the overall value of the maximum of the likelihood function decreases, which indicates a slight preference for smaller values of m_H.

In order to demonstrate the consistency between the data and the SM, we show in the last column of Table I the best-fit values for the various measurements assuming $M_Z = 91.177$ GeV, $m_H = 100$ GeV, $\alpha_s = 0.12$, $c_3 = 30$, and the most likely value from the full data set $m_t = 151$ GeV. A comparison of these predictions with the weighted average of the data (as shown in the table) leads to the conclusion that the predicted values from the fit essentially lie within the 1σ errors of the corresponding experimental quantities.

In summary, the SM predictions are in good agreement with the latest experimental determinations of the properties of the W and Z bosons and $\sin^2\theta_W$. The re-

TABLE II. Sensitivity of the most likely value of m_t and the 95%-C.L. upper limit on m_t due to different choices for the input parameters M_Z, m_H, c_3, and α_s from our full fit to the data.

M_Z (GeV)	m_H (GeV)	c_3	α_s	Most likely m_t (GeV)	m_t (GeV) 95% C.L.
91.177	42	30	0.12	142	192
91.177	100	30	0.12	151	200
91.177	1000	30	0.12	183	225
91.146	100	30	0.12	159	205
91.208	100	30	0.12	143	194
91.177	100	30	0.10	168	212
91.177	100	30	0.14	132	186
91.177	100	0	0.12	155	203
91.177	100	64.3	0.12	146	196

sults of this analysis show that reasonably large values of m_t are preferred by the existing data, suggesting that the top quark may be sufficiently massive as to avoid detection at the Fermilab Tevatron collider until very high luminosities are obtained.[25] A reduction of the uncertainties in the data would not only help in pinning down the mass of the top quark from radiative corrections, but could also be used to probe whether this value is consistent with the value obtained by direct measurements at the Tevatron collider. We anxiously await further precision measurements from the colliders in order to localize the missing elements of the SM.

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