# Topological Glass Transition in Entangled Flux State

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We predict a glass transition in the recently discovered flux-fluid state of high-$T_c$ superconductors due to topological entanglements. Two possible relaxation mechanisms are considered—single-flux-line motion with relaxation time $\tau_1 \sim \exp\{\exp[(\Lambda/d)^2]\}$, and collective motion of many flux lines with relaxation time $\tau_{\text{col}} \sim \exp[(\Lambda/d)^6]$, where $\Lambda$ is the magnitude of transverse fluctuations of flux lines and $d$ is the intervortex spacing. This extremely slow relaxation enables the system of entangled flux lines to support supercurrents in the presence of only a few strong pinning centers.

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Regardless of the underlying microscopic mechanism there is an exiting new property of high-$T_c$ superconductors: strong fluctuations due to small intrinsic coherence lengths and high temperatures. In particular, these fluctuations were predicted[1] to be capable of melting the Abrikosov flux lattice. A new entangled flux liquid state occupying a significant part of the phase diagram of $CuO_2$-based superconductors was proposed. The flux decoration experiments[2] support this fascinating idea. In a framework of conventional theory it was unclear how a melted flux state could have no resistance. The reptation model[3] was used to describe the dynamics of this new state leading to small, but finite conductivity.[4] Below we discuss the dynamics of this entangled flux state and conclude that it is frozen. This implies that a system of flux lines has glasslike rather than fluidlike properties, such as finite shear modulus at experimentally relevant time scales. This modulus is due to topological constraints of entangled magnetic fluxes because they cannot cross each other. This topological glass is qualitatively different from the models of pinned vortices, which also exhibit glassy behavior.[5,6]

For a type-II superconductor in a magnetic field $H$ higher than some critical temperature-dependent field $H_{c1}(T)$, it is energetically favorable to allow some field $B$ to penetrate the sample in the form of flux lines each carrying a quanta $\phi_0 = hc/2e$ of magnetic field. This critical field $H_{c1}$ depends on the energy per unit length $\varepsilon_1$ of a single flux line

$$H_{c1} = 4\pi\varepsilon_1/\phi_0 . \tag{1}$$

The number density of these lines is $n = B/\phi_0$ and their interspacing is

$$d = 1/\sqrt{n} = (\phi_0/B)^{1/2} . \tag{2}$$

In conventional superconductors, these lines form an Abrikosov lattice stable up to much higher fields $H_{c2}(T)$ at which superconductivity is destroyed.

Electrical currents flowing past magnetic flux lines exert a Lorentz force on them. If this force causes the flow of flux lines, the phase shift associated with their motion leads to finite electric resistance of the sample. Zero resistivity of the conventional type-II superconductors in a magnetic field $H$ between the two critical values $H_{c1} < H < H_{c2}$ is attributed to the finite shear modulus of the Abrikosov lattice. This lattice is capable of withholding strains due to currents below some critical current density in the presence of a dilute concentration of strong pinning centers.

It was argued theoretically[1,4] and verified experimentally[2] that fluctuations in high-$T_c$ superconductors melt the Abrikosov lattice at temperatures and fields distinctly lower than $H_{c2}(T)$. How can supercurrents exist in a system with melted flux lines? We demonstrate that topological constraints in an entangled flux system lead to an effectively glassy state and finite shear modulus at experimentally relevant time scales.

The reason topological interactions are important is that the energetic cost for intersection of a pair of flux lines even for a minimum distance $l$ (interlayer spacing) along the field direction can be quite high.[1,4] For Bi-Sr-Ca-Cu-O $l \approx 10$ Å, $H_{c1} \approx 80$ G at $T = 77$ K, and this energy cost is

$$2\varepsilon_1 l = H_{c1}\phi_0 l/2\pi \approx 50 k_B T , \tag{3}$$

at least near $H_{c1}$.

In Fig. 1 we present a schematic drawing of melted flux lines in a slab of thickness $L$ aligned with $CuO_2$ planes. These lines begin to entangle as soon as the relative displacement of their ends $\Lambda$ in the $a$-$b$ plane (tangential to the magnetic field) exceeds the mean spacing $d$ between the flux lines. In equilibrium the root-mean-square projection of the end-to-end vector of a flux line in a liquid state onto the $a$-$b$ plane grows with the

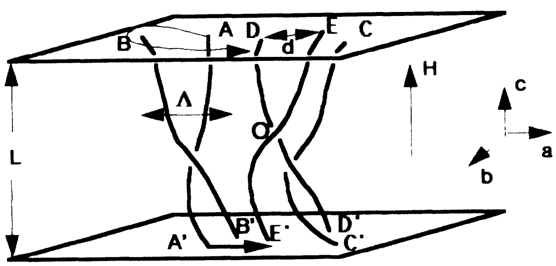


FIG. 1. Schematic of a disordered state of flux lines. The interline spacing is $d$ and the transverse fluctuations of fluxes is $\Lambda$.

(a) (b)

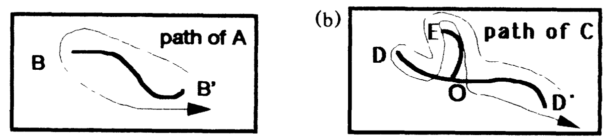


FIG. 2. (a) Projection of the flux $BB'$ onto the top plane and the path of the upper end $A$ of the flux line $AA'$ around point $B$ (thin line). (b) Projections of the flux line $DD'$ and the top part $OE$ of the line $EE'$ onto the upper plane. The path of the top end $C$ (thin line) becomes significantly more complicated than the path of the end $A$ in (a).

slab width $L$ as[4]

$$\Lambda_{\rm eq} = (2\pi k_B T L/\varepsilon_3)^{1/2}, \tag{4}$$

where energy per unit length $\varepsilon_3$ is much smaller than $\varepsilon_1$, reflecting the weak coupling between the planes. This distance could be as large as $\Lambda_{\rm eq} \approx 1$ $\mu$m for sample thickness $L = 0.01$ cm at temperature $T = 77$ K, and $\varepsilon_3 \approx 0.01\varepsilon_1 = 0.01 H_{c1}\phi_0/4\pi$ with $H_{c1} = 100$ G. We argue below that if the tangential displacement $\Lambda$ is larger than a few interflux spacings $d$, the relaxation time of this entangled state is practically infinite.

We shall analyze two possible mechanisms of relaxation—a single-flux-line motion and collective modes. Let us consider first a single-flux-line relaxation mechanism and calculate the time during which a typical flux line diffuses a distance of order $\Lambda$ along the $a$-$b$ plane. In order to move a distance of the order of its own size, it has to disentangle from the neighboring flux lines. It turns out that the trajectory of such disentanglement is extremely complex. Since the $a$-$b$ projection of a flux line covers the area of order $\Lambda^2$, it is entangled with approximately $n\Lambda^2 = (\Lambda/d)^2$ neighboring lines, where $n$ is the density of lines and $d$ is their average spacing. All these lines are entangled with each other as well.

Consider the simplest configuration of an entangled pair of flux lines, such as lines $AA'$ and $BB'$ in Fig. 1. In order for the line $AA'$ to follow the motion of its lower end $A'$ to the right, the top end $A$ has to go around point $B$ in the counterclockwise direction. In Fig. 2(a) we sketch the projection of the flux line $BB'$ onto the top plane and the path of the end $A$ of the line $AA'$ around point $B$.

A simple configuration of three entangled flux lines $CC'$, $DD'$, and $EE'$ is shown in Fig. 1. The path that the top end $C$ of the line $CC'$ needs to cover in order to follow the lower end $C'$ to the right and disentangle from lines $DD'$ and $EE'$ is presented in Fig. 2(b). Note that $C$ has to go twice around point $E$ (once on the way towards point $D$ and once on the way back).

It is easy to see that in the dense system lines $DD'$ and $EE'$ would be entangled with other flux lines and the total path of the end $C$ in the top plane can be represented by a random Cayley tree. The trunk of the tree is a random walk with $(\Lambda/d)^2$ steps each of size $d$ and root-mean-square end-to-end distance $\Lambda$. There are random branches leaving the trunk with branch points spaced on average distance $d$ apart. Each of these branches is a random walk itself with step size $d$ and could branch as well.

The total length of the trajectory of point $C$ is of the order of the mass of the random Cayley tree, which is an exponential function of the length of the trunk (number of the branch generations) $\exp[(\Lambda/d)^2]$. The number of free branch ends in this tree $\sim\exp[(\Lambda/d)^2]$ many times exceeds the number of ends of the lines entangled with $CC' \sim (\Lambda/d)^2$. This can be explained by the fact that point $C$ has to go around point $E$ not only on its way around point $D$ itself, but also on its way around end $F$ of some other line $FF'$ (not shown in the figures) also entangled with $DD'$ and $EE'$. Therefore, point $C$ has to go many times around the ends of all the lines it is trying to disentangle from.

Thus, in order to disentangle from just one of its neighbors, the end of the flux line has to move along the trajectory of length $l_c \sim d\exp[(\Lambda/d)^2]$. Note that this trajectory is unique and the *a priori* probability to find this unique trajectory is exponentially small $P \sim \exp(-l_c/d)$. If the end of the flux line makes a wrong turn and passes one of its neighbors on the wrong side, it has to trace the trajectory back and correct this mistake in order for it to disentangle from the neighbor. Therefore, the disentanglement time $\tau_1$ of a single-flux line is

$$\tau_1 \sim \exp\{\exp[(\Lambda/d)^2]\}. \tag{5a}$$

So far we have analyzed the motion of a single-flux line assuming others to be fixed. Next, we consider collective modes that allow a given flux line to disentangle and completely change its configurations with the help of surrounding flux lines that rearrange appropriately. Let us estimate the fraction of the system that has to be involved in a collective attempt to release an entanglement distance $s\xi_z$ away from the nearest surface, where $\xi_z \approx d^2\varepsilon_3/k_B T$ is the average vertical distance between entanglements, and $s$ is the number of entanglements between a chosen one and the surface along a typical flux line.

Consider an entanglement between lines $AA'$ and $BB'$

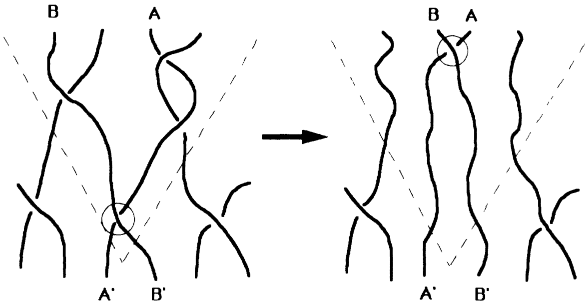


FIG. 3. Collective disentanglement mode.

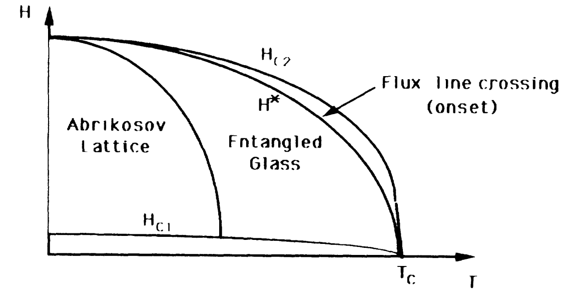


FIG. 4. Schematic phase diagram.

in Fig. 3 outlined by the circle. In order to collectively release it, flux lines $AA'$ and $BB'$ have to disentangle from their own neighbors, while those have to disentangle from their own prospective neighbors, etc. Thus, in order to collectively release the chosen entanglement (circled in Fig. 3), it is necessary to push to the surface all entanglements inside a cone (dashed line in Fig. 3). The volume of this cone is $\sim s^3\xi_z d^2$ and it contains $\sim s^3$ entanglements. As a result of this collective mode all lines inside this cone are straightened (since all the entanglements inside it are pushed to the surface) leading to an increase of free energy (decrease of entropy) of $kT$ per each entanglement involved. Therefore, the relaxation time of this collective mode is $\tau_s \sim \exp(s^3)$. The hardest entanglements to be released are located in the middle of a flux line with $s \sim (\Lambda/d)^2$; therefore, the longest relaxation time of these collective modes is[7]

$$\tau_{\rm col} \sim \exp[(\Lambda/d)^6] . \tag{5b}$$

From any practical point of view for $\Lambda/d \gtrsim 2$–$4$ both collective and single-flux-line relaxation times [Eqs. (5a) and (5b)] can be considered *infinite*. Thus, as soon as lines begin to entangle, the system becomes glassy. In this glassy phase the ends of the flux lines freely move on the scale of interline spacing $d$ and pass by each other. But at times smaller than $\tau_s$ the trajectory of each end is localized in the region of size $\sim d\sqrt{s} \lesssim \Lambda$.

Next, we would like to determine the conditions under which this glassy entangled system of vortex lines melts by the virtue of flux-line crossing and interchanging. The interaction potential per unit length for a pair of vortex lines is $V(r) = (\phi_0^2/8\pi^2\lambda^2)K_0(r/\lambda)$, where $K_0$ is a modified Bessel function [$K_0(x) \approx -\ln(x)$ for small $x$]. In order for two flux lines to cross, the distance between them has to decrease from their average separation $d$ to the core diameter $\xi$. Thus, the crossing energy per unit length is of the order $E_x = (\phi_0^2/8\pi^2\lambda^2)\ln(d/\xi)$. We assumed that both the intervortex distance $d$ and the coherence length $\xi$ are much less than the London penetration depth $\lambda$. In order to estimate this energy, we can compare it with the vortex line tension $\varepsilon_l = (\phi_0^2/16\pi^2\lambda^2)\ln(\lambda/\xi)$, which is related to the critical field $H_{c1}$ by Eq. (1). Therefore, the flux crossing energy for a typical crossing distance $l$ is

$$\begin{aligned} E_x l &= 2\varepsilon_l l \ln(d/\xi)/\ln(\lambda/\xi) \\ &= 2\varepsilon_l l \ln(H_{c2}/H)/\ln(H_{c2}/H_{c1}) . \end{aligned} \tag{6}$$

Note that we recover Eq. (3) for $H \approx H_{c1}$ ($d \approx \lambda$). The second equality in Eq. (6) was obtained using relations similar to Eq. (2) connecting distance scales with corresponding magnetic fields $d \approx (\phi_0/H)^{1/2}$, $\lambda \approx (\phi_0/H_{c1})^{1/2}$, and $\xi \approx (\phi_0/H_{c2})^{1/2}$. The magnetic field $H^*$ at which the entangled vortex glass begins to melt corresponds to the flux crossing energy $E_x l$ of the order of $k_B T$,

$$\frac{H^*}{H_{c2}} = \exp\left[\frac{2\pi k_B T}{H_{c1}\phi_0 l}\ln\left(\frac{H_{c1}}{H_{c2}}\right)\right] . \tag{7}$$

In the present paper we approximate the flux crossing distance $l$ by the shortest physical scale in the problem (interlayer spacing or core diameter) $l \approx 10$ Å. Simple estimates show that at low enough temperatures ($T < 3$ K) flux lines are stiffer and the crossing distance $l$ can be larger than 10 Å and grows with decreasing temperature. In this low-temperature region there is even higher-energy cost for flux line crossing, but we do not consider this regime any further.

Expanding the exponential in Eq. (7) to the first order, we get a rough estimate of the melting transition $H^*$

$$(H_{c2} - H^*)/H_{c2} \approx 0.1 T/T_c . \tag{8}$$

Thus, we obtain a phase diagram schematically drawn in Fig. 4. The above estimates are based on a three-dimensional continuum theory of anisotropic superconductivity, applicable if the distance $d$ between flux lines is sufficiently large. Near $H_{c2}$ this distance is small and it may be more convenient to describe fluxes using quasi-two-dimensional models such as the pancake model.[8] This may lead to some changes in the coefficients in the above equations.

If we start from an entangled glass state and decrease temperature $T$, the flux lines get stiffer and reduce the transverse fluctuation $\Lambda$, pushing entanglements out of the system to the surface. At first the entanglements adjacent to the surface are pushed out. This allows the deeper entanglements to move closer to the surface and be pushed out plane after plane. At lower temperatures we recover the Abrikosov lattice. Note that this sequential disentanglement process does not contain any exponentials, and is much faster than the relaxation processes discussed above [Eqs. (5a) and (5b)]. Nevertheless, this sequential process could be slower than cooling of the sample. It is remarkable that a few strong pinning centers can cause irreversibility and hysteresis effects by blocking the whole system of entanglements inside the sample.

For the sake of simplicity we have not discussed the effect of impurities. As was shown by Larkin and Ovchinikov,[9] the presence of impurities destroys the order of the Abrikosov lattice and smears out the melting transition. It was suggested by Fisher[6] that collective pinning of vortex lines by impurities leads to a glassy state. All arguments presented in this Letter should apply to this state, which may correspond to a typical experimental situation. The main issue is whether flux-line crossing or depinning is more essential for the resistance. The experimental data on proton-irradiated crystals[10] support the importance of topological interactions. It was demonstrated[10] that the increasing concentration of strong impurities does not affect the position of the irreversibility line (since it is controlled by line crossing), while increasing the critical current (at which the whole system of entangled flux lines is pulled off impurities).

In summary, we have demonstrated that the lifetime of an entanglement above the melting transition of the Abrikosov lattice is practically infinite. Thus, at any experimental time scale the entangled flux lines form a three-dimensional network (topological glass) with a finite shear modulus. A few strong pinning centers can prevent this system from sliding, leading to the absence of resistance. This glass is different from the ones discussed in the literature since it is due to topological entanglements rather than impurities.

Because of the line-crossing mechanism with characteristic energy $E_x l$ [Eq. (6)] the conductivity appears near $H^*(T)$. Therefore, it is not a sharp transition line and its position may depend on the frequency, as observed in some experiments.[11] Below this transition relaxation time diverges very quickly as $\exp(E_x l/k_B T)$ and becomes much longer than the time estimated[4,12] on the basis of the reptation model.[3]

In thin films the entanglement effect should disappear and the melting transition would coincide with the superconductivity transition. This occurs when $\Lambda_{eq}$ defined in Eq. (4) becomes smaller than intervortex spacing $d$,

$$L < \varepsilon_3 \phi_0 / 2\pi k_B T H \,. \tag{9}$$

As mentioned above, at temperature $T = 77$ K we can take $\varepsilon_3 \approx 0.01\varepsilon_1 = 0.01 H_{c1}\phi_0/4\pi$ and obtain the condition $L < 10^{-4}\phi_0^2/k_B T \approx 10$ $\mu$m near $H_{c1} \approx 100$ G and $L < 1000$ Å for fields of order 10 kG. It was observed experimentally[13] that the irreversibility line becomes thickness dependent in films thinner than $\approx 1000$ Å in a magnetic field $H = 7$ kG.

It is interesting to note that systems of nearly parallel nonintersecting lines are often encountered in physics. Examples of such systems are parallel dislocation lines in deformed crystals, vortex lines in superfluids, and flux lines in neutron stars.[14] In all these cases, topological constraints may significantly influence the behavior of the systems. We believe that the most interesting and easily studied is the topological glass transition in some polymeric systems, such as grafted polymers and block copolymers. We will discuss it in a future publication.[15]

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