Why Do Quarks Behave Like Bare Dirac Particles?

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An explanation is offered why quarks in the constituent quark model should be treated as particles with axial coupling $g_A=1$ and no anomalous magnetic moment.

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The strong interactions give the u and d quarks of the constituent quark model masses that are very much larger than those in the Lagrangian of quantum chromodynamics. One might naturally suppose that since the constituent quark's mass can be completely altered by the cloud of gluons and quark-antiquark pairs it contains, the same might be true of other properties of the quark, such as its magnetic moment and all others of its electroweak couplings and form factors. Yet most calculations using the constituent quark model (and also the bag model) seem able to account for the electroweak properties of hadrons by treating the constituent quark as a bare Dirac particle, with the same electroweak properties as for the quarks in the standard SU(3)⊗SU(2)⊗U(1) Lagrangian. This paper offers an explanation of why the constituent quark has such simple electroweak couplings.

For simplicity, consider quantum chromodynamics with just two massless quarks u,d. We start with the observation that since the constituent quark model is supposed to incorporate the spontaneous breakdown of SU(2)⊗SU(2) chiral symmetry of QCD, we should include among its degrees of freedom not only quarks, but also pions,[1] which are not well described anyway as nonrelativistic quark-antiquark bound states. We can then take over into the constituent quark model the whole familiar apparatus of soft-pion theorems, nonlinear Lagrangians, and sum rules. Now, in general, if the Adler-Weisberger sum rules for pion scattering on arbitrary targets can be saturated with narrow one-particle states, then the complete set of these sum rules can be put in the Lie-algebraic form[5]

$$[X_a,X_b]=i\epsilon_{abc}T_c\,, \tag{1}$$

where T_a is the isospin matrix, and X_a is the matrix whose elements give the "reduced amplitudes" for emission of a pion with isospin index a between one-particle states.[2] Also, in the narrow-state approximation the sum rules[3] that follow from soft-photon theorems[4] and the dispersion relations for neutral-pion photoproduction and photon scattering on an arbitrary target take the algebraic form

$$[\kappa,X_3]=0\,, \tag{2}$$

$$[\kappa,\kappa^\dagger]=0\,. \tag{3}$$

Here κ is the momentum-independent matrix whose nondiagonal elements give the amplitudes for collinear one-photon transitions:

$$\langle \mathbf{p}',F,\lambda'|\mathbf{J}_x+i\mathbf{J}_y|\mathbf{p},I,\lambda\rangle\equiv\frac{i\sqrt{2}(m_F^2-m_I^2)(F,\lambda'|\kappa|I,\lambda)}{(2\pi)^{3/2}\sqrt{4E_FE_I}}\,, \tag{4}$$

where $\mathbf{J}$ is the electric current, λ' and λ are the final and initial helicities, and the final and initial momenta $\mathbf{p}'$ and $\mathbf{p}$ are taken in the $-z$ direction. The diagonal elements of κ are defined by

$$(I,\lambda'|\kappa|I,\lambda)\equiv[\tfrac{1}{2}(j_I-\lambda)(j_I+\lambda+1)]^{1/2}\times\delta_{\lambda',\lambda+1}(\mu_I/j_I-e_I/m_I)\,, \tag{5}$$

where μ_I, e_I, j_I, and m_I are the magnetic moment, electric charge, spin, and mass of particle I. In the spirit of the constituent quark model, we should consider these sum rules to be valid in the one-quark sector as well as in color-neutral sectors.

If we are willing to assume that in the constituent quark model there are no color triplet particles besides the quark itself, then our work is done. The only representations of SU(2)⊗SU(2) that contain only a *single* isospin-$\frac{1}{2}$ representation of the unbroken SU(2) subgroup are $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$, corresponding to the possibilities $X_a=T_a$ and $X_a=-T_a$. Parity conservation tells us that whichever possibility is realized for $\lambda=\frac{1}{2}$, the opposite applies for $\lambda=-\frac{1}{2}$. If $X_a=\pm T_a$ for $\lambda=\frac{1}{2}$, then the axial coupling constant g_A of the quark is ± 1. Also, by taking the expectation value of Eq. (3) between one-quark states, we easily see that $\kappa=0$, giving a constituent quark magnetic moment with the Dirac value $2e_q/m_q$. (This is *not* the same as the usual argument for the Dirac moment of the electron from renormalizability. The only assumption we are making regarding high-energy behavior is that certain dispersion relations should have no subtractions; the same argument would apply for particles of arbitrary spin, even where no renormalizable theories exist.)

The weak point in the above argument is our assumption that the Adler-Weisberger and Drell-Hearn sum rules in the one-quark sector are saturated with the one-quark state itself. In using the constituent quark model, it is generally tacitly assumed that physical hadronic states, in general, contain just quarks, antiquarks, and perhaps pions and glueballs, but no gluons or colored bound states of gluons. With this assumption, we can adapt the usual topological arguments[6] to show that in the limit $\mathcal{N}_{\text{color}} \to \infty$, the only intermediate states in pion-quark or photon-quark scattering are single-particle states with the color and isospin of a quark. (The QCD graphs of leading order in $1/\mathcal{N}_{\text{color}}$ for matrix elements of any number of currents between one-quark states are planar when drawn with all gluon lines on the same side of the quark line. The only way to divide such a graph into two *connected* parts, to each of which is attached one of the two external quark lines, is to cut through any number of gluon lines, and just once through an internal quark line.) But this still leaves open the possibility that the intermediate states may be excited states of the quark, perhaps of different spin. It is, in fact, usually assumed in the constituent quark model that the quark has no excited states, but it is not clear why this should be the case.

Fortunately, we can use the "large-$\mathcal{N}_{\text{color}}$" approximation and one other assumption to show that even if the constituent quark does have excited states, they cannot contribute to the commutators in Eqs. (1)-(3). First, note that since the quark and its excited states can only have isospin $\frac{1}{2}$, they must all be linear combinations of states belonging to the $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ representations of $SU(2)\otimes SU(2)$. The absence of Regge trajectories with isospin 2 allows the derivation of superconvergence relations,[7] which in the narrow-state approximation tells us that the mass-squared matrix behaves as a sum of $(0,0)$ and $(\frac{1}{2},\frac{1}{2})$ terms.[2] In a basis in which we list all the linear combinations of states belonging to the $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ representations in that order, these two terms in the mass matrix must take the supermatrix form

$$m_S^2=\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix},\quad m_V^2=\begin{pmatrix} 0 & C \\ C^\dagger & 0 \end{pmatrix}. \tag{6}$$

In this basis, a mass eigenstate like the quark with helicity $\lambda=\frac{1}{2}$ is a linear combination of $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ terms with coefficients u_k and v_k, such that the column (u,v) is an eigenstate of $m_S^2+m_V^2$. Acting on $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ terms, X_a is $+T_a$ and $-T_a$, respectively, so such a particle has axial coupling

$$g_A=\frac{\sum_k(|u_k|^2-|v_k|^2)}{\sum_k(|u_k|^2+|v_k|^2)}, \tag{7}$$

which, in general, could have any value between $+1$ and -1.

To go further, we adopt an approximation[2] that seems to work well in ordinary hadronic physics, that forward pion-quark scattering becomes purely elastic at high energy. It follows[2] that the two Hermitian matrices in (6) must commute, so that the column (u,v) is a simultaneous eigenvector of both matrices, say, with eigenvalues μ_0^2 and Δ, respectively. But then u is an eigenvector of A and $CC^\dagger$, and v is an eigenvector of B and $C^\dagger C$, in both cases with eigenvalues μ_0^2 and Δ^2, respectively. Eigenvectors with different values of μ_0^2 and Δ^2 have u's and v's both orthogonal, so that the matrix elements of X_a vanish between such eigenstates. We see then that the sets of states that can be connected by strong pion emission and absorption can only be of three types. For $\Delta\neq 0$, we have a pair of states with squared masses $\mu_0^2\pm|\Delta|$ and $v=\pm C^\dagger u/|\Delta|$, for which Eq. (7) gives $g_A=0$. For $\Delta=0$, we can have either a pure $(\frac{1}{2},0)$ eigenstate with $v=0$ and $g_A=+1$, or a pure $(0,\frac{1}{2})$ eigenstate with $u=0$ and $g_A=-1$. I do not know why the quark should have $g_A=+1$ rather than $g_A=0$ or $g_A=-1$, but these are the only three possibilities.

We can now also understand the quark's magnetic moment, by simply taking the matrix element of the photoproduction sum rule (2) between quark states with helicities $\lambda=+\frac{1}{2}$ and $\lambda=-\frac{1}{2}$. For $g_A=1$, these states have $X_a=T_a$ and $X_a=-T_a$, respectively, so (2) tells us that this matrix element of the *anticommutator* $\{T_3,\kappa\}$ vanishes. We can conclude that the matrix element (5) must vanish for a quark with a definite value for T_3, so that both u and d quarks must have vanishing anomalous magnetic moment.

The reader may perhaps wonder at this point why the same arguments do not apply to the nucleon. In the large-$\mathcal{N}_{\text{color}}$ approximation, the intermediate states contributing to pion-nucleon or photon-nucleon scattering are those that can be formed from just $\mathcal{N}_{\text{color}}$ quarks. Even for $\mathcal{N}_{\text{color}}=3$, this allows isospins $\frac{1}{2}$ and $\frac{3}{2}$, so the nucleon and its excited states are linear superpositions of $SU(2)\otimes SU(2)$ representations $(\frac{3}{2},0)$, $(0,\frac{3}{2})$, $(1,\frac{1}{2})$, $(\frac{1}{2},1)$, $(\frac{1}{2},0)$, and $(0,\frac{1}{2})$. This seems to be too complicated to allow simple conclusions about the nucleon and its excited states to be drawn from the arguments of this paper.

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