Weak Localization in a Distribution of Magnetic Flux Tubes

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Thin gates of type-II superconducting materials have been prepared on top of the two-dimensional electron gas in a GaAs/AlGaAs heterostructure. In an applied magnetic field the flux distribution at the electron gas takes the form of flux tubes which are much narrower than an electron phase coherence length. We observe a qualitatively new weak-localization magnetoconductance for small fields proportional to $|B|$, in contrast to the B^2 homogeneous result and in semiquantitative agreement with the theory of Rammer and Shelankov.

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In the last ten years since the publication of the scaling theory of localization[1] considerable progress has been made in the understanding of electrical transport in disordered two-dimensional systems. We now know that when disorder is weak there is a quantum correction to the Drude conductivity due to the interference of waves propagating in a random medium. A quasiclassical understanding may be obtained by considering an electron diffusing along closed classical paths which return to the starting position. In general, due to the random positions of the impurities, only time-reversed paths will be mutually phase coherent upon returning. These time-reversed paths interfere constructively, leading to coherent backscattering and the weak-localization correction to the conductivity.[2] An applied field breaks the time-reversal symmetry of these loops, leading to a phase shift of $\exp\{i2\pi\Phi/\Phi_0\}$ between them, where Φ is the flux enclosed by the trajectory, and Φ_0 $(=h/2e)$ is the flux quantum. This phase shift suppresses the localization correction to the conductivity and gives rise to negative magnetoresistance as expressed for a doubly spin degenerate system by[3]

$$\sigma_L(B,T)-\sigma_L(0,T)=\alpha\frac{e^2}{2\pi^2\hbar}\left[\psi\left(a_i+\frac{1}{2}\right)-\psi\left(a_e+\frac{1}{2}\right)+\ln\left(\frac{\tau_i}{\tau_e}\right)\right], \tag{1}$$

where α is a constant of order 1 whose value depends on whether inelastic, spin-orbit, or magnetic scattering processes dominate, ψ is the digamma function, and $a_{i,e}=\hbar/4eBD\tau_{i,e}$. D is the electron diffusion constant and $\tau_{i,e}$ are the inelastic and elastic scattering times, respectively. For magnetic fields very much less than the characteristic "inelastic" field $B_i=\hbar/4eD\tau_i$ Eq. (1) simplifies to yield a quadratic correction to the conductance,

$$\sigma_L(B,T)-\sigma_L(0,T)\approx\frac{1}{24}\alpha\frac{e^2}{2\pi^2\hbar}\left(\frac{B}{B_i}\right)^2. \tag{2}$$

Equation (1) was developed for a homogeneous magnetic field, and one would expect a qualitatively different result if the field were present in the form of flux tubes containing integral numbers of flux quanta whose radii were much smaller than an electron phase coherence length ($l_i=\sqrt{D\tau_i}$). A novel situation now arises where a classical path could entirely encompass a number of flux tubes (containing N flux quanta) leading to a trivial phase shift of $\exp\{i2\pi(N\Phi_0)/\Phi_0\}$ between time-reversed paths, and preserving phase coherence. In fact, only paths that cut through a flux tube will contribute to the magnetoresistance. This situation has a very simple physical realization. The gate of a silicon metal-oxide-semiconductor field-effect transistor (MOSFET) could be fabricated from a type-II superconducting material; some measurements on Nb-gated MOSFETs have been published previously.[4] Alternatively, as described here, one could evaporate a type-II superconducting film on top of a GaAs/AlGaAs-heterostructure two-dimensional electron gas (2DEG). In fields above its lower critical field H_{c1} an ideal type-II superconductor enters the mixed state when flux penetrates the material as a hexagonal lattice of narrow flux tubes, each containing a single flux quantum. The very close proximity of the 2DEG to the superconducting film (80 nm here) results in the magnetic-field distribution in the superconductor being projected intact down onto the electronic system.

A theoretical analysis of weak localization in a distribution of flux tubes has been made by Rammer and Shelankov.[5] They found, perhaps unexpectedly, that the magnetoconductance for an isolated flux tube of radius r_0 containing N quanta of flux does not depend on N to leading order. The authors give closed results for the macroscopic magnetoconductance in a few simplifying limits. For the case corresponding to this experiment when the flux tubes are small ($r_0\ll l_i$), contain N flux quanta, and have a large mutual separation [$l_B=(2N\Phi_0/\sqrt{3}B)^{1/2}>l_i$] in the low-field limit ($B\ll NB_i$), they find

$$\sigma_L(B,T)-\sigma_L(0,T)\approx\alpha\frac{e^2}{2\pi^2\hbar}\frac{1}{N\ln(B_0/NB_i)}\left(\frac{|B|}{B_i}\right), \tag{3}$$

where the magnetic field within the flux tubes is assumed to be constant and equal to B_0.

Clearly, in this limit the magnetoconductance is a lot stronger in the inhomogeneous case and depends linearly on $|B|$. At higher fields ($B \gg NB_i$) a typical path encircles many tubes, the nonuniformity of the field becomes unimportant, and one returns to the homogeneous result. In this Letter we demonstrate experimentally the validity of Eq. (3) and are able to achieve semiquantitative agreement between the calculated and the observed behavior in the low-field limit.

Our chosen system was a GaAs/AlGaAs heterostructure with a thin metallic film deposited on top. Superconducting lead (Pb) gates 200 nm thick were thermally evaporated at ambient temperature onto structures on one half of a chip, and 200-nm-thick gold (Au) gates were electron-beam evaporated onto the other half. In this way the influence of the superconducting film could be directly compared to that of a normal film in essentially identical Hall bars on the same chip. The radius of a flux vortex in the type-II superconductor ($\sim\lambda_L$) must be smaller than that inelastic-scattering length in the 2DEG before one enters a new transport regime. Therefore, a type-II superconducting film with the minimum possible λ_L was chosen. Bulk Pb is a type-I superconductor, but in thin-film form becomes type II as a consequence of an increase in λ_L due to surface and grain-boundary scattering, and because demagnetization effects make large type-I domains energetically unfavorable.[6] In decoration experiments on comparable Pb films deposited under almost identical conditions, Dolan[7] found that flux is indeed frequently present in the form of vortices of single flux quanta at low fields (<50 G) (mixed state) and of multiple flux quanta at higher fields (transitional state). Figure 1 shows the results of a calculation of the magnetic-field distribution across a flux-oid diameter using the Clem variational model[8] where the variational core radius has been assumed to be equal to the superconducting coherence length ξ. Values for λ_L (51 nm) and ξ (80 nm) have been inferred for a comparable Pb film from Ref. 7. Assuming that the flux vortices in the superconductor occupy an ordered lattice with wave vectors $\mathbf{q}$ one can show by Fourier transformation of Maxwell's equations that successive Fourier components of the inhomogeneous field distribution a distance z away from the surface are damped out by a factor $\exp(-|\mathbf{q}|z)$.[5] The distribution for a mean applied field of 10 G at a distance of 80 nm from the superconductor has also been calculated in Fig. 1. Clearly, the field at the 2DEG ($z=80$ nm) is still strongly inhomogeneous, the peak field at the core (185 G) being almost 20 times larger than the mean applied field, and the radius at half maximum (110 nm) is less than a third of the inelastic-scattering length in the 2DEG used here.

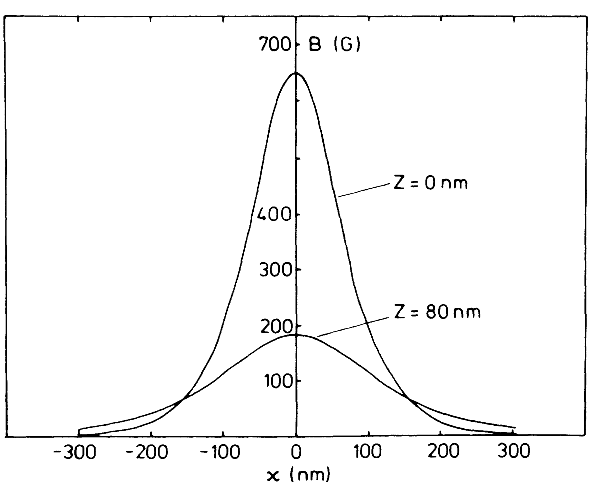

FIG. 1. Calculated magnetic-field distribution across a vortex at the surface of a superconducting film and 80 nm away from it.

Modern GaAs-based 2DEGs can have low-temperature mobilities greater than 10^7 cm^{2}V^{-1}s^{-1}, far in excess of what is useful for studying weak localization. Consequently, the quality of the sample used here was deliberately degraded by using a very short (30 nm) GaAs buffer layer between the wafer substrate and the 2DEG. This would normally be at least an order of magnitude thicker to prevent the possibility of unwanted impurities riding at the growth surface reaching the active region of the device near the GaAs/AlGaAs interface. At low temperatures it was found that the presence of the evaporated gates lead to the depletion of the 2DEG below them. This was overcome by a short photoexcitation with a red light-emitting diode which established a stable electron density by virtue of the persistent photoconductivity effect.[9] In the measurements presented in this Letter the Pb-gated Hall bar had a mobility of 17000 cm^{2}V^{-1}s^{-1} and a carrier concentration of 1.98×10^{11} cm^{-2} at 4.2 K and was quite comparable with the Au-gated structure ($\mu=17700$ cm^{2}V^{-1}s^{-1} and $n_{2D}=1.86\times10^{11}$ cm^{-2}).

The completed devices took the form of 100-μm-wide Hall bars with three pairs of voltage contacts along the sides. They were mounted inside a temperature-controllable sample holder incorporating a high-purity-copper-wire solenoid. The entire insert was immersed in a bath of liquid helium, and the sample temperature stabilized to better than ±5 mK with a commercial controller equipped with a capacitance thermometer. The field at the sample was calculated taking into account the finite wire thickness and solenoid length, and was cross-checked with a Hall-probe gaussmeter. All measurements were performed at 15 Hz with an rms ac current of 300 nA. The Hall voltage (V_H) was measured directly at the sample, and changes in the longitudinal resistivity (ρ_{xx}) detected with a sensitive ac resistance bridge.

Figure 2 shows some selected measurements of the Hall voltage at the 2DEG in the Pb-gated structure at

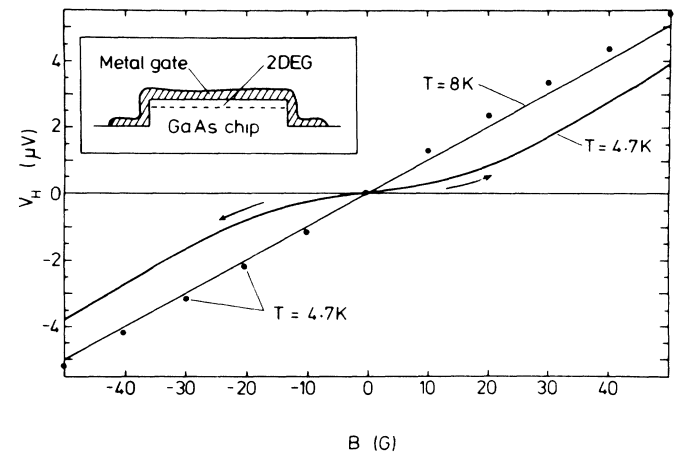

FIG. 2. Measured Hall voltage of a sample with a Pb gate as a function of magnetic field for three distinct conditions (see text). Inset: Cross section through a sample.

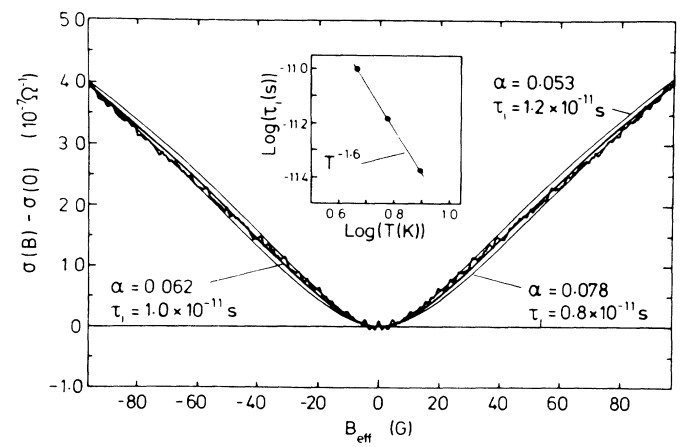

FIG. 3. Plot of magnetoconductance of a structure with a Au gate as a function of $B_{\rm eff}$ at 4.6 K. Smooth curves are fits to the data with Eq. (1). Inset: T dependence of τ_i in this sample.

very low fields which reflect phenomena characteristic of flux motion in type-II superconducting films. The straight line was measured at 8 K, which is above the superconducting critical temperature of Pb ($T_c = 7.2$ K), and represents the classical Hall voltage of the sample. The heavier curves were measured by cooling to 4.7 K at $B = 0$ G and subsequently sweeping the field either up or down. Note that the Hall voltage is initially strongly suppressed due to delayed flux entry into the Pb film. The isolated points represent the Hall voltage after cooling down through T_c in an applied field. Note that these essentially follow the classical linear behavior because the metastable barriers to flux entry have been removed. The Hall voltage measured in this way always slightly exceeds the value at 8 K by a constant amount due to a small Meissner effect in the gate material cladding the vertically etched walls at the edge of the Hall bar (see inset of Fig. 2). Figure 2 clearly demonstrates that the mean field at the 2DEG can, in general, differ from the externally applied field due to the close proximity of the superconducting film. It suggests using the Hall voltage to define an operational effective mean field $B_{\rm eff} = [V_H(T,B)/V_H(8\ {\rm K},B)]B$, where B is the externally applied field.

Figure 3 shows a plot of the magnetoconductance $[\sigma(B) - \sigma(0) = \Delta\sigma_{xx} = -\Delta\rho_{xx}/\rho_{xx}^2]$ for the Au-gated sample at 4.6 K as a function of $B_{\rm eff}$. The effective field was established by measuring the Hall voltage immediately following the measurement of ρ_{xx} under identical conditions. Clearly, when the gate material is normal $B_{\rm eff} = B$, but for consistency the same procedure was always followed. Upon field reversal, measurements of ρ_{xx} showed slight asymmetries due to a few percent of the Hall voltage being mixed into the signal as a result of very weak spatial inhomogeneities in the 2DEG. The data in Fig. 3 have been symmetrized after expression in terms of $B_{\rm eff}$. The lines through the data are fits of Eq. (1) with different values of α and τ_i, D and τ_e being calculated from the low-temperature Hall mobility. The inset shows best-fit values of τ_i plotted against the three different measurement temperatures on a log-log plot. Values of τ_i are very comparable with those found by Lin *et al.*[10] in similar samples. However, the best-fit value of α (0.062 ± 0.005) is a factor of 10 smaller than those authors observed and the $\tau_i \sim T^{-(1.6\pm0.1)}$ power-law behavior observed here differs from their clear T^{-1} dependence. There are many possible explanations for this, the dominance of magnetic or spin-orbit scattering, Maki-Thompson-type fluctuations,[10] or spatial electron-density inhomogeneities. In view of the increased likelihood of additional impurities having reached the active region of these devices we consider the first two scattering mechanisms to be most likely. Attempts to fit our data with a full theory containing these additional scattering processes[11] indicate that magnetic scattering appears most plausible, and fair agreement can be obtained if a magnetic scattering time of about 3×10^{-12} s is assumed.

Figure 4 shows magnetoconductance data for the Pb-gated sample plotted against $B_{\rm eff}$ at three different temperatures. In each case the sample was cooled at $B = 0$ and either swept up or down in field. Thus each curve is a composite of two sweeps which have again been symmetrized, and plots at different temperatures have been vertically offset for clarity. Notice that the behavior at low fields is qualitatively different from that shown in Fig. 3 below T_c but identical above T_c. The best-fit of Eq. (1) to the data at 8.2 K is shown as a dotted line and yields a slightly different value of α (0.086 ± 0.005) from the equivalent Au-gated data but a very similar value for τ_i (3.7×10^{-12} s). The dotted lines at temperatures below T_c are plots of Eq. (1) using $\alpha = 0.086$ and assuming τ_i is the same as for the Au-gated samples at the same temperature. Notice that while the data show strong deviations from the dotted lines near $B = 0$ G, they approach one another asymptotically at high fields

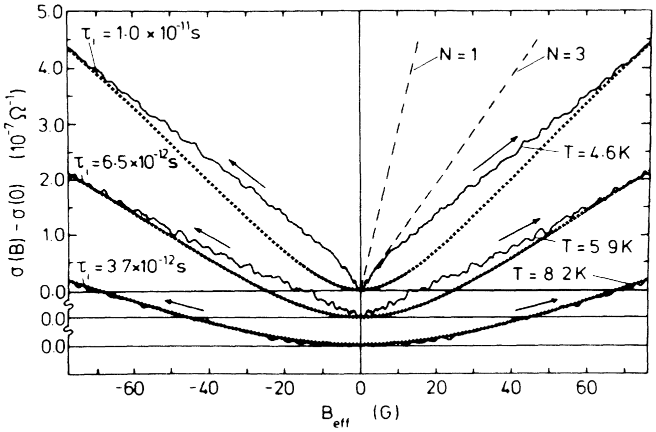

FIG. 4. Plots of magnetoconductance of a structure with a Pb gate as a function of B_{eff} at three different temperatures. Dotted lines are plots of Eq. (1) (see text). Dashed lines are plots of Eq. (3) for $N=1$ and 3 at 4.6 K.

giving us confidence in our analysis procedure.

The measurements in Fig. 4 at 4.6 and 5.9 K show a pronounced linear behavior for low fields (< 10 G) as expected from Eq. (3). At 4.6 K the conditions for Eq. (3) to be valid are reasonably well satisfied when $B_{\text{eff}} \ll NB_i \sim N(15\text{ G})$, i.e., $r_0(110\text{ nm}) < l_i(340\text{ nm})$ and $l_B(420\sqrt{N}\text{ nm at }100\text{ G}) > l_i$. The dashed lines are calculations of Eq. (3) for $N=1$ and 3 at 4.6 K assuming that B_0 is equal to N times the peak vortex field in Fig. 1. Note that the $N=3$ line represents a good asymptote to the low-field data suggesting that on average the flux is present in vortices containing triple flux quanta, in agreement with frequent observations of Ref. 7 at higher fields, but in apparent disagreement with their low-field results. However, quantitative fits should probably not be expected since the theoretical model of the vortex as being a cylinder of uniform flux density is clearly not very realistic. This is reflected in the data taken at 5.9 K where the magnetoconductance increases much more weakly with field near $B=0$ G. Taken at face value this implies the presence of vortices containing more than twice as many quanta of flux as at 4.6 K. In reality, the weakening is almost certainly due to a smearing of the inhomogeneous field distribution and a breakdown of the validity of Eq. (3).

In conclusion, we have studied weak localization in a distribution of magnetic flux tubes which were narrower than the electron phase-coherence length. We observe a qualitatively new magnetoconductance at low fields which depends on $|B|$ and is in semiquantitative agreement with the theory of Rammer and Shelankov.[5] To achieve a quantitative comparison it will be necessary to generalize the theory to include the real magnetic-field distribution of a flux vortex. Hybrid semiconductor/superconductor structures of the type studied here have potential for making detailed studies of flux structures in superconductors. In particular, σ_{xx} probes the inhomogeneous field distribution on a scale of the inelastic length l_i. A voltage bias between the superconducting gate and the 2DEG could be used to tune l_i and infer the internal field distribution of a vortex.

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