New Collective Mode and Corrections to Fermi-Liquid Theory in Two Dimensions

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We study a two-dimensional Fermi gas with an arbitrary short-range repulsive interaction. We show that the vertex part has an unusual singularity in the particle-particle channel for all momenta $q < 2k_F$, which is not found in higher dimensions. We interpret this as a collective mode representing a bound excitation of two holes. This mode, however, does not lead to an instability, or to a breakdown of Fermi-liquid theory in the low-density limit. The resulting Fermi-surface phase shift vanishes, and a $|\omega|^{5/2}$ correction to $\Sigma''(k_F,\omega)$ is obtained. The 2D case is also contrasted with a similar calculation in 1D.

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There have been recent suggestions[1,2] that the normal state of the high-temperature superconductors is not an ordinary Landau Fermi liquid. While interacting fermion systems in one dimension are known to display non-Fermi-liquid behavior, there are no simple examples in higher dimensions of systems which do not have a broken symmetry and yet differ from a Landau Fermi liquid. Anderson has recently suggested[3] that the 2D Hubbard model may have a non-Fermi-liquid ground state due to a "nonrenormalizable phase shift" at the Fermi surface coming from "antibound states above the upper band edge."

Another motivation for the present work comes from the study of models with attractive interactions. Randeria, Duan, and Shieh have shown[4] that the existence of a two-body bound state in vacuum is a necessary condition for an s-wave pairing instability in two dimensions. This led them to suggest[4] that the presence of some bound pairs coexisting with unbound fermions might be responsible for the anomalous normal-state properties of short-coherence-length superconductors. Schmitt-Rink, Varma, and Ruckenstein further studied attractive models at finite temperatures and argued[5] that there are two-particle bound resonances for all center-of-mass momenta $q > 2k_F$. The question of whether or not these bound states lead to a breakdown of Fermi-liquid theory in the weak-coupling regime is not clear[6] at the present time. It is clearly of great interest to know if such models with attractive interactions provide a microscopic basis for the marginal Fermi-liquid theory of Varma *et al.* as conjectured in Ref. 2. It thus seemed important to us to study a Fermi gas with repulsive interactions more carefully, before returning to the attractive case, to see if any of these subtleties showed up there as well.

In this paper we study a 2D Fermi gas with arbitrary short-range repulsive interactions. The 3D version was studied many years ago by Galitskii,[7] and it provided one of the first examples of a Landau Fermi liquid in which various quantities of interest could be explicitly calculated. We find that the 2D case is rather different from 3D, the main difference coming from the finite density of states at the bottom of the band. The vertex part in the particle-particle channel has, in addition to the usual branch cut representing scattering states, an isolated pole below the bottom of the two-particle band for every center-of-mass momentum $q < 2k_F$. We show that this collective mode is simply a bound excitation of two holes.

The existence of these collective modes necessarily requires us to go beyond any finite order in perturbation theory. The energy of these modes relative to the two-particle continuum has an essential singularity in the strength of the repulsive interaction. This "breakdown" of perturbation theory does not of course necessarily imply a violation of Fermi-liquid theory (FLT). (As is well known, collective modes like zero sound and paramagnons only lead to corrections to Fermi-liquid behavior in 3D and not to a qualitative change.)

We investigate the effect of these collective modes on the validity of FLT in the low-density limit. We show below that in 2D the collective modes lead to a $|\omega|^{5/2}$ correction to the standard 2D result[8] for the imaginary part of the single-particle self-energy $\Sigma'' \sim \omega^2 \ln|\omega|$, and that the phase shift vanishes at the Fermi surface. In addition, we also look at the 1D problem, where we again find the pole corresponding to this collective mode, but in 1D the pole does not directly lead to any correction to Σ'' and the Fermi-surface phase shift is again found to be zero. Thus, in both 1D and 2D, the corrections to Σ'' due to the new poles are subdominant to the contributions due to two-particle scattering which lead to a violation of FLT in 1D but not in 2D.

Based on an analogy to the problem of noninteracting fermions scattering off a single impurity,[9] Anderson[3] has suggested that a nonzero phase shift at the Fermi surface should be related to a violation of Fermi-liquid theory.

The relationship [see Eq. (11) below] between the scattering phase shift and the energy of the system is formally identical to that in the single-impurity problem (Fumi's theorem). However, we show below that the connection between the phase shift and the self-energy [see Eq. (13)] is rather different from that in the single-impurity problem. Thus the quasiparticle residue Z in the interacting Fermi gas is apparently not completely determined by the value of the phase shift at the Fermi surface. Even though the Fermi-surface phase shift vanishes in both 1D and 2D, Z can behave qualitatively differently for these two cases.

Consider a 2D gas of fermions of mass m with an arbitrary two-body repulsive interaction of range R which is much smaller than the average interparticle spacing k_F^{-1}. For energies much less than $\epsilon_R = \hbar^2/2mR^2$, the interaction is described[10] by an energy parameter $E_a > \epsilon_R$ which characterizes the s-wave t matrix[11] for the two-body problem,

$$T_{\mathbf{k},\mathbf{k}}^{-1}(\omega) \simeq \tau_0^{-1}(\omega) = C[\ln|E_a/\omega| + i\pi\Theta(\omega)] , \tag{1}$$

where $C = mL^2/4\pi\hbar^2$ with L^2 the size of the box.

In the many-body system the diluteness parameter $k_F R \ll 1$ allows one to restrict attention to the ladder diagrams.[7] The vertex part Γ, for center-of-mass momentum $\mathbf{q}$ and energy ω, is then given by

$$\Gamma^{-1}(\mathbf{q},\omega) = \tau_0^{-1}(\omega') + \chi_0(0,\omega') - \chi(\mathbf{q},\omega) , \tag{2}$$

where the result is independent of the choice of ω'. The function χ is defined as

$$\chi(\mathbf{q},\omega) = -\sum_{\mathbf{k}} \frac{1-f(\epsilon_+ - \mu) - f(\epsilon_- - \mu)}{\epsilon_+ + \epsilon_- - 2\mu - \omega - i\eta} , \tag{3}$$

with $\epsilon_\pm = \epsilon_{\pm\mathbf{k}+\mathbf{q}/2}$, and χ_0 is given by setting the Fermi functions f and the chemical potential μ to zero in χ. Introducing an ultraviolet cutoff Λ in the χ's and using (1), we find

$$\begin{aligned}\Gamma^{-1}(\mathbf{q},\omega) &\equiv C[A(\mathbf{q},\omega) + iB(\mathbf{q},\omega)] \\ &= C\ln(E_a/2\Lambda) - \chi(\mathbf{q},\omega) .\end{aligned} \tag{4}$$

From the graphical solution of $\Gamma^{-1} = 0$ shown in Fig. 1, it is easy to see that, for any q, Γ has a branch cut along the real ω axis for $\omega > \omega_q^*$, where the bottom of the two-particle band ω_q^* is given by

$$\omega_q^* \equiv \min_k[\epsilon_+ + \epsilon_- - 2\mu] = q^2/4m - 2\mu . \tag{5}$$

(The branch cut exists in any dimension since $B \neq 0$ only for $\omega > \omega_q^*$.) In addition, in 2D Γ has an isolated pole at $\omega_b(q) < \omega_q^*$, coming from the nonzero density of states at the bottom of the band ($\mathbf{k} = 0$). It is also clear from Fig. 1 that this solution exists for all $q < 2k_F$, for it is only then that the bottom of the band ω_q^* lies below the Fermi surface $\omega = 0$.

In order to focus on the validity of Fermi-liquid theory we will limit the discussion to zero temperature in the remainder of this paper; this will also allow us to obtain analytical results for many of the quantities of interest. Details will be reported elsewhere.[12] For convenience we scale all energies with 2μ and introduce the notation $\tilde{\omega} = \omega/2\mu$, $\tilde{E}_a = E_a/2\mu$, and $u_q = \omega_q^*/2\mu$. At $T = 0$, one finds

$$\begin{aligned}A(\mathbf{q},\omega) = {} & \ln[\tilde{E}_a(u_q - \tilde{\omega})] \\ & - 2\ln\{[(u_q+1)(u_q - \tilde{\omega}) + \tilde{\omega}^2/4]^{1/2} - \tilde{\omega}/2\} ,\end{aligned} \tag{6}$$

which is valid only for $\tilde{\omega} < u_q$. Since $B = 0$ in this region, solving $A(\mathbf{q},\omega) = 0$ we find that Γ has a pole at

$$\omega_b(q) = -|\omega_q^*| - |\omega_q^*|^2/E_a \tag{7}$$

to leading order in $1/\tilde{E}_a$, with residue $R_b(q) = -|\omega_q^*|^2/CE_a$, for all $q < 2k_F$. Note that $\omega_b(q)$ has an essential singularity in the potential through the dependence of E_a on the interaction.[10]

These particle-particle channel poles are collective in that they exist only in the presence of a Fermi surface. We interpret these states as bound excitations of holes. Since the excitation energy of a hole state increases as ω decreases below the Fermi surface $\omega = 0$, these collective excitations have an energy $|\omega_b(q)|$, which vanishes at $q = 2k_F$ and is maximum at $q = 0$. In terms of holes it is easy to see that these modes are high-energy states even though the poles occur below the bottom of the two-particle band. We will show below that they lead to an increase in the total energy of the system.

The poles are on the real axis, and their existence does not lead to an instability of the system. This is rather different from the usual situations in which collective ex-

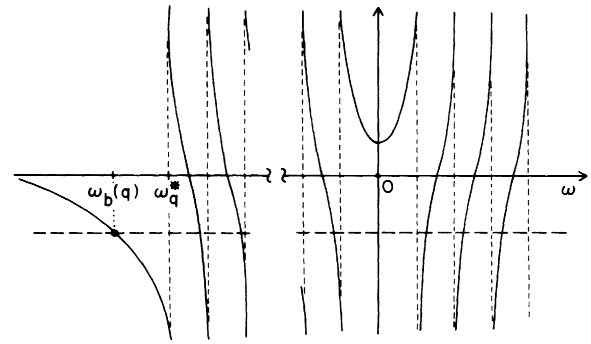

FIG. 1. Graphical solution of $-\chi(\mathbf{q},\omega) = -1/V$ for fixed $\mathbf{q}$ in a finite box. For a repulsive interaction $V > 0$, the right-hand side (shown as the horizontal dashed line) intersects $-\chi$ (the solid curves) at a series of closely spaced points within the two-particle band $\omega \geq \omega_q^*$. These solutions represent two-particle scattering states and lead to a branch cut in Γ. In addition, in 2D, with a nonzero density of states at the bottom of the band, there is an isolated solution below the band at $\omega_b(q)$ for all $q < 2k_F$. This is an isolated pole in Γ representing the collective mode described in the text.

citations in normal systems are in the particle-hole channel, and poles in the particle-particle channel for attractive interactions lead to instability (superconductivity).

It might be useful to contrast this mode with other bound states that have been discussed recently in connection with possible violations of FLT. First, these $q < 2k_F$ poles are collective in nature in contrast to the $q > 2k_F$ bound states[5] in attractive models which, for large q, are simply independent two-particle bound states. Second, viewed as bound states of holes, these poles are analogous to antibound states[3] above the upper band edge in the 2D Hubbard model. It must be emphasized that these collective bound states exist even in a continuum model and do not require an upper band edge; the lower band edge (for the particles), which is essential for stability, acts like an upper band edge for the holes.

We next turn to a calculation of the phase shift defined by $\Gamma(\mathbf{q},\omega) = |\Gamma| \exp[i\delta(\mathbf{q},\omega)]$. Many of the quantities of interest can be expressed in terms of the phase shift as shown below. We now need a more general expression for A and B than was obtained earlier [since (6) was valid only for $\tilde{\omega} < u_q$]. The angular integration in (3) can be performed to obtain

$$A(\mathbf{q},\omega) = \ln \frac{\tilde{E}_a |\tilde{\omega} - u_q|}{(\xi_- - \tilde{\omega})^2} - \int_{\xi_-}^{\xi_+} \frac{dy}{y - \tilde{\omega}} F(y;u_q) , \tag{8}$$

$$B(\mathbf{q},\omega) = \pi\Theta(\tilde{\omega} - u_q) \times \begin{cases} -1, & \text{for } \tilde{\omega} < \xi_-, \\ 1 - F(\tilde{\omega};u_q), & \text{for } \xi_- < \tilde{\omega} < \xi_+, \\ 1, & \text{for } \xi_+ < \tilde{\omega}. \end{cases} \tag{9}$$

Here the function F is given by

$$F(y;u_q) \equiv \frac{2}{\pi} \arccos \left(\frac{y}{2\sqrt{(y-u_q)(u_q+1)}} \right) \tag{10}$$

and $\xi_\pm = 2(u_q + 1 \pm \sqrt{u_q+1})$. The virtue of these expressions is that they make explicit the locations $\tilde{\omega} = \xi_\pm$ of all the features in the phase shift, and they allow us to make asymptotic expansions. For $u_q > 0$ the resulting phase shift δ is negative within the two-particle band (for $\tilde{\omega} > u_q$) and vanishes below the band. For $u_q < 0$, or equivalently $q < 2k_F$, things are more interesting. The phase shift is negative for $\tilde{\omega} > 0$, with δ increasing as $\tilde{\omega}$ decreases below zero, showing a shoulderlike feature at ξ_- and finally attaining its maximum value of π at the bottom of the band u_q. Between the bottom of the two-particle band and the collective pole energy, given by (7), i.e., $\omega_b(q)/2\mu \le \tilde{\omega} \le u_q$, the phase shift $\delta \equiv \pi$, reflecting the bound state. Finally, $\delta \equiv 0$ for $\omega < \omega_b(q)$.

We now show, using the phase shifts, that the collective poles lead to an increase in total energy and the chemical potential of the system. Within the ladder approximation Nozieres and Schmitt-Rink[13] have expressed the thermodynamic potential in terms of the two-body phase shift. At $T=0$ we have

$$\Omega(\mu) - \Omega_0(\mu) = \frac{1}{\pi} \sum_{q < 2k_F} \int_{-\infty}^{0} d\omega\, \delta(\mathbf{q},\omega) , \tag{11}$$

where the subscript 0 is used for the noninteracting system. It can be easily shown that the change in the ground-state energy due to interactions $E(N) - E_0(N) \ge \Omega(\mu_0) - \Omega_0(\mu_0)$, where μ_0 is the chemical potential of the noninteracting system. From the non-negativity of the phase shift below the Fermi surface and (11), it then follows that the energy of the system increases.[14] While the contribution[15] of the two-particle continuum ($\omega_q^* < \omega$) has to be evaluated numerically, that coming directly from the bound states $[\omega_b(q) < \omega < \omega_q^*]$ is simply $\Delta\Omega_b/L^2 = 8m\mu^3/3\pi\hbar^2 E_a$.

The chemical potential of the interacting gas is determined by calculating the function $N(\mu) = -\partial\Omega/\partial\mu$ and solving for μ for a given number density of fermions. The correction coming directly from the bound states is clearly $\Delta N_b/L^2 = -8m\mu^2/\pi\hbar^2 E_a$. We have calculated the correction due to the two-particle continuum states numerically and find[12] that ΔN is indeed negative and hence $\mu > \mu_0$. The numerical result from the continuum does not differ much from the low-order perturbative result of Ref. 16, where a 2D Fermi gas was studied without appreciating the existence of the collective modes.

Let us next look at the phase shift at the Fermi surface ($\tilde{\omega} = 0$). For $0 < q < 2k_F$, δ goes to zero linearly with a negative slope which depends on q. For $q=0$, the phase shift vanishes singularly,

$$\delta(0,\tilde{\omega}) \simeq \frac{-\pi\, \mathrm{sgn}\tilde{\omega}}{\ln(\tilde{E}_a/\tilde{\omega}^2)}, \quad \tilde{\omega} \to 0 . \tag{12}$$

This singularity is not special to 2D and comes from the discontinuity in the occupation factors in (3) at the Fermi surface which exists in all dimensions.[17] Finally, for $q = 2k_F$, the phase shift is $\delta \simeq -\tilde{\omega}^{1/2}/\ln\tilde{E}_a$ for $\tilde{\omega} \to 0^+$ and identically zero for $\tilde{\omega} \to 0^-$. Thus we find that $\delta(q,\omega=0) = 0$ for all q.

The single-particle self-energy is obtained from the vertex part Γ by joining two of the external legs, which at $T=0$ yields

$$\Sigma(\mathbf{k},\omega + i\eta) = \frac{1}{\pi} \sum_{q < 2k_F} \int ds \frac{\Theta(\mu - \epsilon_{\mathbf{k}-\mathbf{q}}) - \Theta(-s)}{\omega + \epsilon_{\mathbf{k}-\mathbf{q}} - \mu - s + i\eta} \times \mathrm{Im}[\Gamma(\mathbf{q},s)] , \tag{13}$$

where the spectral weight $C\,\mathrm{Im}[\Gamma] = -B/(A^2+B^2) = -\partial\delta/\partial\ln\tilde{E}_a$. As before we can separate the contributions of the pole (denoted by a subscript b) and the continuum. We find the pole contribution to be $\mathrm{Im}[\Gamma]_b = (|\omega_q^*|^2/CE_a)\pi\delta(\omega - \omega_b(q))$. A straightforward but lengthy calculation then shows that the low-lying collective excitations near $q=2k_F$ provide a correction to the imaginary part of the self-energy that goes as $\Sigma_b''(k_F,\omega)\sim\mu|\tilde{\omega}|^{5/2}/\tilde{E}_a$ for $\omega\rightarrow 0^-$. The continuum gives the well-known[8] 2D result $\omega^2\ln\omega$ whose coefficient in the low-density limit[16] is of order $(\ln\tilde{E}_a)^{-2}$. Thus even with the new bound excitations, the quasiparticles remain well defined with an infinite lifetime as one approaches the Fermi surface, and a nonzero quasiparticle residue Z.

Finally, it is interesting to ask what one obtains by using the above techniques in one dimension. From the vertex part (4) in 1D we again find a collective bound pole below the two-particle continuum for $q<2k_F$. The resulting Fermi-surface phase shift also vanishes in 1D, but now phase-space restrictions lead to a vanishing contribution from these poles to $\Sigma''(k_F,\omega)$ within the low-density regime. The continuum contribution, however, gives $\Sigma''(k_F,\omega)\sim\omega$, and $Z=0$, also for simple phase-space reasons. Thus, in one dimension, the collective poles do not appear to play a role in the breakdown of Fermi-liquid theory. (This is perhaps not altogether surprising in view of the fact that many of the commonly used 1D approximations ignore the band edges, and thus would not even obtain the collective mode.)

To conclude, we have shown that a new collective mode, which is a bound excitation of hole pairs, exists in two dimensions where there is a nonzero density of states at the bottom of the band. Since these arise from nonperturbative effects in the particle-particle channel, we used the ladder approximation which is reasonable for low densities. The connection, if any, between the existence of such bound excitations and the breakdown of FLT is not clear. Our results suggest that either there is no breakdown of Fermi-liquid theory in 2D, or else that it is sufficiently subtle that it is not enough to go beyond any order in perturbation theory in the interaction but it is also necessary to go beyond the low-density approximation. Of course, a lot of effort has recently gone into attacking the Hubbard model near half filling, which is the opposite extreme in terms of density to the problem studied here. But there one has other instabilities—like magnetism, metal-insulator transition, and possibly, superconductivity—to contend with.

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