Determination of α_s from a Differential-Jet-Multiplicity Distribution in e^+e^- Collisions at $\sqrt{s}=29$ and 91 GeV

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We measured the differential-jet-multiplicity distribution in e^+e^- annihilation with the Mark II detector. This distribution is compared with the second-order QCD prediction and α_s is determined to be $0.123\pm0.009\pm0.005$ at $\sqrt{s}\approx M_Z$ (at the SLAC Linear Collider) and $0.149\pm0.002\pm0.007$ at $\sqrt{s}=29$ GeV (at the SLAC storage ring PEP). The running of α_s between these two center-of-mass energies is consistent with the QCD prediction.

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The study of jets provides an important laboratory to probe the hard (large momentum transfer) interactions of quarks and gluons. With increasing center-of-mass energy of these hard processes, perturbative QCD effects, masked by fragmentation effects at lower energies, become more visible. One of the main experimental issues for jet analyses is the measurement of the QCD scale parameter $\Lambda_{\overline{\rm MS}}$ ($\overline{\rm MS}$ denotes the modified minimal-subtraction scheme) which determines the coupling strength of the strong interaction at any given mass scale (Q^2). In determining $\Lambda_{\overline{\rm MS}}$ (or α_s), it is better to use observables which are insensitive to fragmentation and higher-order QCD effects. The three-jet event fraction appears relatively insensitive to fragmentation effects, if one chooses a reasonable jet algorithm and if one deals only with hard three-jet events.[1] However, the actual dependence of the three-jet event fraction on the jet-resolution parameter ($u_{\rm cut}$) used to select hard three-jet events is not statistically easy to handle. This problem can be solved by using a differential jet multiplicity as described below.

The purpose of this paper is to present determinations of α_s at two different center-of-mass energies, at the SLAC Linear Collider (SLC) and at the SLAC storage ring PEP. The analysis is performed using the same Mark II detector configuration at both energies and applying the same technique, based on the differential jet multiplicity.

The Mark II detector has been described in detail elsewhere.[2-4] In this analysis, the main drift chamber and barrel and end-cap electromagnetic calorimeters are used. We analyze the data which were collected after the installation of the new drift chamber and of the end-cap shower detector at PEP.[2] The triggers for hadronic events at SLC and at PEP are given elsewhere.[2,3] Trigger efficiencies are close to 100% for multihadronic events so that the analysis is not significantly affected by trigger biases. Events are selected by requiring that the number of charged tracks is at least seven at SLC (at least five at PEP) and that the sum of charged- and neutral-particle energies ($E_{\rm vis}$) is greater than $0.50\sqrt{s}$ at SLC ($0.55\sqrt{s}$ at PEP). In order to reduce the bias due to initial-state radiation and background from two-photon processes for the PEP data, events with large missing energy or with a large-energy photon are eliminated by applying additional cuts described in Ref. 2. For the Z-resonance data such effects are small; hence we do not apply any cuts other than those mentioned above.[3] The detection efficiency for multihadron events is estimated using QCD-based Monte Carlo generators[5-7] to be 0.80 ± 0.02 at SLC (0.51 ± 0.02 at PEP). The integrated luminosities used in the analysis are 19.7 nb^{-1} at SLC and 27 pb^{-1} at PEP. A total of 391 events from the SLC data and 7348 events from the PEP data pass the selection cuts.

The parton-shower models[5,6] are very attractive because they describe the data very well over a wide range of center-of-mass energy using the same parameters,[8] but $\Lambda_{\overline{\rm MS}}$ cannot be uniquely defined in these models which are based on a leading-log approximation. Therefore these models are used only for studying detector effects and for determining efficiencies. Second-order perturbative QCD predictions are directly compared with the data for testing the hard QCD processes and for determining α_s.

We use the algorithm proposed by the JADE Collaboration to define the number of jets (jet multiplicity) in an event.[9] The algorithm proceeds as follows: For each particle (cluster) pair i,j, the scaled invariant mass

$$y_{ij}=\frac{2E_iE_j(1-\cos\chi_{ij})}{E_{\rm vis}^2}$$

is calculated, where E_i and E_j are the energy particles (clusters) and χ_{ij} is the angle between them. The particle (or cluster) pair with the smallest y_{ij} is combined by adding the four-momenta of the two particles (clusters) i and j to form a new cluster $i+j$ ($p^\mu_{i+j}=p^\mu_i+p^\mu_j$). The above clustering procedure is repeated until all the clusters satisfy the condition $y_{ij}>y_{\rm cut}$, where $y_{\rm cut}$ is referred to as the jet resolution. The n-jet fraction $f_n(y_{\rm cut})$ is defined to be the number of n-cluster events obtained with the algorithm, divided by the total number of hadronic events. This jet algorithm has the important feature that mapping from parton jets to hadron jets in Monte Carlo hadronic events is close to one-to-one for reasonably large $y_{\rm cut}$ (≥0.04) values.[1] However, it is not easy to extract α_s by fitting the $f_3(y_{\rm cut})$ [or $f_2(y_{\rm cut})$] distribution because the same events contribute at different $y_{\rm cut}$ values and one must take into account all the correlations in this distribution.

To overcome this difficulty, a differential jet multiplicity is defined in the following way. The clustering is terminated when the number of jets has reached a preselected value n, irrespective of y_{ij} values. For each event, particles are assigned to n jets using this method and y_n is defined to be the minimum value of the y_{ij}'s ($i\neq j$, $i,j=1,2,\ldots,n$). In other words, y_n is the $y_{\rm cut}$ value corresponding to the transition from n jets to $(n-1)$ jets for a given event. The distribution function of y_n is denoted $g_n(y_n)$. Integrating $g_3(y_3)$ over y_3 from 0 to $y_{\rm cut}$, one recovers $f_2(y_{\rm cut})$ because all the events with $y_3<y_{\rm cut}$ are categorized as two-jet events for the given jet resolution $y_{\rm cut}$. Hence,

$$g_3(y_3)\,|_{y_3=y_{\rm cut}}=\frac{\partial}{\partial y_{\rm cut}}f_2(y_{\rm cut})\,.$$

Similarly,

$$g_4(y_4)\,|_{y_4=y_{\rm cut}}=\frac{\partial}{\partial y_{\rm cut}}[f_2(y_{\rm cut})+f_3(y_{\rm cut})]\,.$$

Note that only the leading term ($\propto\alpha_s^2$) is available for g_4 in second-order QCD calculations. Therefore we restrict our analysis to the differential jet fraction $g_3(y_3)$ to determine α_s.

Detector effects, biases due to event selection, and initial-state radiation effects are corrected with bin-by-bin correction factors. In the range $0.04\leq y_3\leq0.14$, the corrections are typically less than 5% for SLC data (10% for PEP data). The bin-to-bin systematic errors due to the variation of the correction factors for various models[5-7] are less than 4% at SLC (3% at PEP). These errors slightly increase with y_3. The overall normalization uncertainty in the correction factors is estimated to be 2% at SLC (3% at PEP). The corrected $g_3^{\rm corr}(y_3)$ distributions for the two data samples are shown in Fig. 1. Also shown in the figure are the QCD predictions for three $\Lambda_{\overline{\rm MS}}$ values, as obtained by differentiating the function f_2 calculated by Kramer and Lampe, in the $\overline{\rm MS}$ scheme, for $y_3\leq0.14$.[10] The shape of the distributions, which depends only slightly on $\Lambda_{\overline{\rm MS}}$, is well described by the QCD predictions.

Corrections are not applied for fragmentation effects. Rather, they are accounted for as systematic errors. These errors are estimated as follows. Using the same jet algorithm, and for a given fragmentation model, the distributions of y_3 at the parton level ($g_3^{\rm partons}$) and after fragmentation ($g_3^{\rm hadrons}$) are obtained. The systematic errors are then derived, for a given y_3, from the differences $|g_3^{\rm partons}-g_3^{\rm hadrons}|$ for various models.[5-7] In Fig. 2, the ratio $g_3^{\rm partons}/g_3^{\rm hadrons}$ is shown as a function of y_3 for two models.[5,7] In the range $0.04\leq y_3\leq0.14$, the

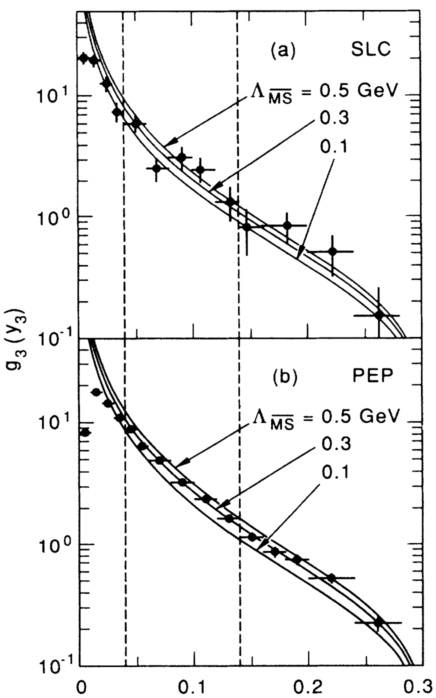

FIG. 1. The experimental distributions of y_3 at (a) $\sqrt{s}=91$ GeV and (b) $\sqrt{s}=29$ GeV. Only the statistical errors are indicated in the figures. The curves below $y_3=0.14$ indicate the QCD predictions with $\Lambda_{\overline{MS}}=0.1$, 0.3, and 0.5 GeV for $Q^2=s$. The y_3 range used in the fit for the determination of α_s is defined by the two dashed lines. The curves above $y_3=0.14$ are extrapolated from the QCD predictions in the low-y_3 range.

bin-to-bin systematic errors associated with fragmentation effects are 3%–5% at SLC (5%–10% at PEP). The normalization uncertainty is estimated to be 2% at SLC (4% at PEP).[11] The systematic errors estimated by varying the fragmentation parameters are significantly smaller than the errors mentioned above.

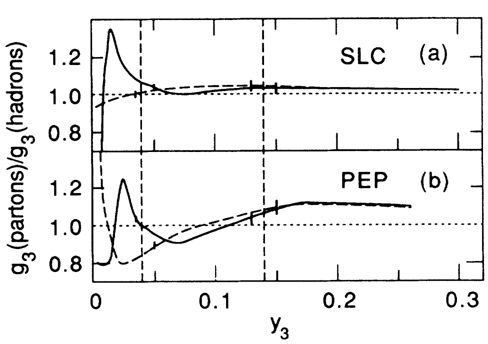

FIG. 2. The ratio $g_3^{\text{partons}}/g_3^{\text{hadrons}}$ as a function of y_3 for partons and for hadrons (after fragmentation and decay of unstable particles) at (a) $\sqrt{s}=91$ GeV and (b) $\sqrt{s}=29$ GeV. The solid curve corresponds to the Lund model based on $O(\alpha_s^2)$ matrix element and the dashed curve to the Lund parton-shower model. The error bars indicate the Monte Carlo statistical errors.

The α_s value is obtained from a fit of the corrected $g_3(y_3)$ distribution by the $O(\alpha_s^2)$ QCD prediction.[10] The fit is performed within the range of $0.04 \le y_3 \le 0.14$ using a likelihood method which accounts for the statistical errors and the various systematic errors. The lower y_3 limit of the fitted range is chosen in order to limit the fragmentation effects, while the upper limit arises only because the QCD prediction for $y_3 > 0.14$ is not available in Ref. 10. Choosing the renormalization point Q^2 to be s, we obtain

$$\alpha_s = 0.123 \pm 0.009 \pm 0.005 \text{ at SLC},$$

$$\alpha_s = 0.149 \pm 0.002 \pm 0.007 \text{ at PEP (Ref. 12)}.$$

The running of α_s from 29 to 91 GeV is consistent with the QCD prediction, as shown in Fig. 3. The running of α_s with Q^2 is governed by the renormalization-group-equation (RGE) which, to second order in α_s, is given by

$$\frac{\partial}{\partial \ln Q^2}\frac{\alpha_s}{2\pi} = -b_0\left(\frac{\alpha_s}{2\pi}\right)^2\left(1+b_1\frac{\alpha_s}{2\pi}\right).$$

The coefficients b_0 and b_1 do not depend on the renormalization scheme chosen; hence they represent fundamental physical quantities. Denoting by n_f the effective number of flavors at a given Q^2, QCD predicts $b_0 = (33-2n_f)/6$ and $b_1 = (153-19n_f)/(33-2n_f)$. The RGE can be integrated to express b_0 in terms of our two measurements of the coupling constant α_s^{SLC} and α_s^{PEP} and of the $\ln Q^2$ variation $\Delta \ln Q^2 = 2\ln(91/29) = 2.29$. One gets

$$b_0 = \frac{F(\alpha_s^{\text{SLC}}) - F(\alpha_s^{\text{PEP}})}{\Delta \ln Q^2},$$

with

$$F(\alpha_s) = \frac{2\pi}{\alpha_s} - b_1 \ln\left(\frac{2\pi}{\alpha_s} + b_1\right).$$

This formula leads to $b_0 = 3.4^{+2.1}_{-1.4}$ where the errors take into account the partial cancellation of the systematic uncertainties. This value, which is almost independent of b_1, is in good agreement with the QCD prediction of $b_0 = 3.83$ for $n_f = 5$.

To express the α_s measurements in terms of the QCD scale parameter $\Lambda_{\overline{MS}}$, we use the approximation solution of the RGE given in Ref. 13. We obtain $\Lambda_{\overline{MS}} = 0.29^{+0.17+0.11}_{-0.12-0.06}$ GeV at SLC, and $\Lambda_{\overline{MS}} = 0.28^{+0.02+0.08}_{-0.02-0.07}$ GeV at PEP, in agreement with the value $0.33 \pm 0.04 \pm 0.07$ GeV previously obtained using the energy-energy correlation by Mark II at 29 GeV.[14]

In finite-order perturbative calculations, there is an ambiguity due to the renormalization scale Q^2. Recently, triggered by the work of Kramer and Lampe,[15] several experimental papers were published in an attempt to optimize Q^2 for the determination of $\Lambda_{\overline{MS}}$.[16–18] The simultaneous determination of Q^2 and $\Lambda_{\overline{MS}}$ using jet multiplicity favors very small Q^2 values,[18] but the result is very sensitive to the four-jet fraction which does not have the next-to-leading order term in the $O(\alpha_s^2)$ calcu-

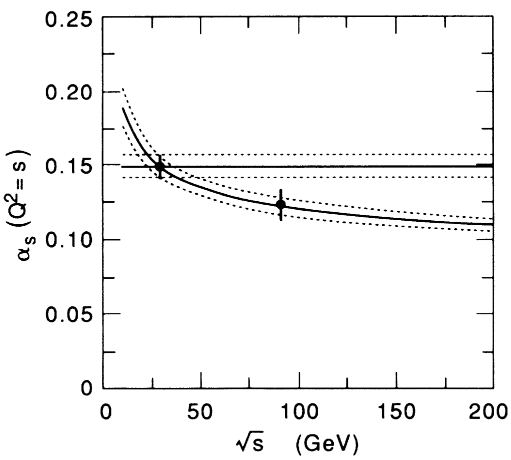

FIG. 3. The strong coupling $\alpha_s(Q^2=s)$ as a function of $\sqrt{s}$. The errors include statistical and systematic uncertainties added in quadrature. Also shown are the extrapolations of the α_s measurement at $\sqrt{s}=29$ GeV to higher energies using the formula of Ref. 13, or assuming a constant α_s. The dotted lines indicate the extrapolation of the measured $\alpha_s \pm 1\sigma$ from 29 GeV.

lation. If a variable with next-to-leading order terms is used, the Q^2 ambiguity is large. Several prescriptions have been proposed to assign Q^2 a particular value.[19-21] For the purpose of illustrating and exploring the effect of the choice of Q^2, we use the Brodsky-Lepage-Mackenzie (BLM) method[21] to eliminate the Q^2 ambiguity for g_3 at each y_3 value. The Q value prescribed by the BLM method (Q^*) is 4 GeV (1.3 GeV) at $y_3=0.05$ and increases to 6 GeV (2.0 GeV) at $y_3=0.10$ for $\sqrt{s}=91$ GeV ($\sqrt{s}=29$ GeV). In this picture, the smallness of Q^2 might be understood in terms of the typical momentum scale involved in the vacuum polarization loops; the energy scale is related to the allowable invariant mass (virtuality) of gluons, which can be as small as a few GeV. Choosing $Q^2=(Q^*)^2$ at each value of y_3 and $\sqrt{s}$, and n_f values appropriate to the small Q^* values ($n_f=4$ for SLC and $n_f=3$ for PEP), the $\Lambda_{\overline{\rm MS}}$ values obtained using the BLM method are $0.17^{+0.08}_{-0.06}{}^{+0.05}_{-0.003}$ GeV at SLC, and $0.17^{+0.01}_{-0.01}{}^{+0.03}_{-0.03}$ GeV at PEP. The range of the $\Lambda_{\overline{\rm MS}}$ values discussed in this Letter implies that the uncertainty induced by the Q^2 ambiguity is in excess of the systematic errors arising from the fragmentation effects.

In conclusion, we have presented the measurement of the coupling strength of the strong interaction in e^+e^- annihilation at $\sqrt{s}\approx M_Z$ (SLC) and at $\sqrt{s}=29$ GeV (PEP) using the differential jet multiplicity g_3. The method is relatively insensitive to fragmentation effects and statistically easy to handle. In the framework of second-order QCD calculations and for $Q^2=s$, the measured values of α_s are $0.123\pm0.009\pm0.005$ at $\sqrt{s}=91$ GeV and $0.149\pm0.002\pm0.007$ at $\sqrt{s}=29$ GeV. The running of α_s from 29 to 91 GeV is seen and is consistent with the QCD prediction. For comparison, results for $\Lambda_{\overline{\rm MS}}$ have been also presented at considerably smaller values of the renormalization point (Q^2), as suggested, for example, by the Brodsky-Lepage-Mackenzie method.

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