Modulational Instability of Copropagating Frequencies for Normal Dispersion

In a recent Letter,[1] Agrawal has shown that nonlinear optical propagation can exhibit modulational instability (MI) in the normal dispersion region. This is significant in that, although there was known for some time to be MI in the anomalous dispersion region,[2] the normal region was thought to be stable. Agrawal mathematically shows that cross-phase modulation of two distinct copropagating frequencies, when described by two incoherently coupled equations (ICE's), can yield MI in the normal dispersion region. For these ICE's he shows that even in the realistic case of the two frequencies traveling at different group velocities MI is present. As the group-velocity mismatch $\delta = v_{g2}^{-1} - v_{g1}^{-1}$ increases it was shown that the peak gain increases and the frequency which exhibits maximum gain varies according to $\omega_{\max\,\mathrm{gain}} \cong \delta/\beta$, where $\beta = d^2k/d\omega^2$ is the dispersion. In the examples that Agrawal considered it was found coincidentally, and left unexplained, that $\omega_{\max\,\mathrm{gain}}$ was approximately equal to the frequency difference between the two copropagating waves, $\Delta\omega = \omega_2 - \omega_1$. That is, each wave develops a sideband which is (nearly) degenerate with the copropagating wave.

In this Comment I show that, although Agrawal's analysis of the ICE's is entirely correct, these equations have implicit in them assumptions which are strongly violated by the solutions he finds. To correctly solve the problem one must integrate the nonlinear Schrödinger equation (NLSE). Numerical integrations of the NLSE show that the actual gain is greatly reduced from that calculated by Agrawal.

The analysis of two copropagating frequencies in a nonlinear medium is approached by substituting the sum of the two waves $A_i = \exp\{-i(\omega_i t - k_i z)\}$ in the NLSE (modified for higher-order dispersion, if necessary), and collecting similar frequency terms. One obtains Agrawal's ICE's [Eq. (1) of Ref. 1] with a coherent coupling term $-\gamma A_j^2 A_{3-j}^* \exp\{\pm i(\Delta\omega t - \Delta k z)\}$ on the right-hand side of the ICE's for $j=1$ and 2, respectively. If the bandwidths of A_1 and A_2 (which include sidebands at the modulation frequency) are $\ll \Delta\omega$, then one expects that these coherent terms will not be phase matched and one might argue that they can be ignored. However, in the examples which Agrawal discusses, the modulation frequency approaches $\Delta\omega$ so that the bandwidth of the A_i's are $\sim\Delta\omega$, and the coherent terms will have a component which is certainly phase matched, and cannot be ignored. In other words, optical nonlinear propagation is correctly described by the NLSE, and, by neglecting coherent coupling terms, the NLSE may be reduced to Agrawal's ICE's. For the cases Agrawal examines this reduction is unjustified.

The true solution of this problem is found by a numerical integration of the NLSE, where one uses the sum of two strong cw frequencies and a weak modulational perturbation as the input field. In this way all coherent coupling between the waves is included. An example of such a calculation is shown in Fig. 1 (solid curve), where the growth of the spectral intensity as a perturbation frequency of 0.5 THz is plotted versus the propagation distance. To facilitate comparison, this calculation uses Agrawal's parameters: $\gamma = 0.015\ \mathrm{m}^{-1}/\mathrm{W}$, $\beta = 0.06\ \mathrm{ps}^2/\mathrm{m}$ is assumed constant, each strong wave has a power of 100 W, and the frequency difference between them is 2.6 THz ($\delta = 1$ ps/m). This result is compared with the numerical integration of Agrawal's ICE's for the same initial conditions (dashed curve). One sees that the solution of the NLSE is oscillatory and does not exhibit the gain ($0.7\ \mathrm{m}^{-1}$) present in Agrawal's solution. This type of oscillatory behavior is typical of that observed over the range of relevant perturbation frequencies. Varying the powers and frequency difference of the strong waves yields similar results. Therefore, one sees that the coherent coupling terms have a significant effect, in general, and thus the ICE approach will lead to unphysical results.

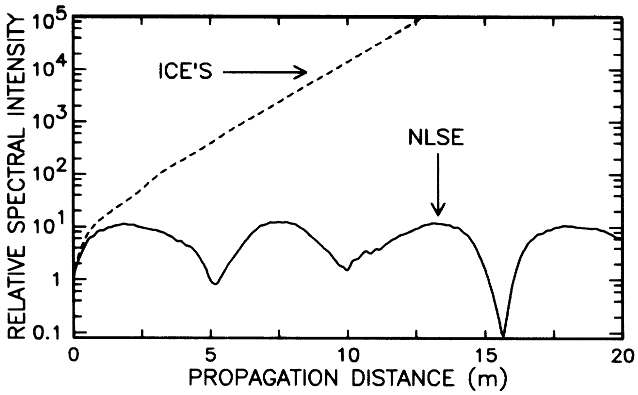

FIG. 1. Calculated growth of a weak perturbation at 0.5 THz on two strong (100 W) waves separated by 2.6 THz. The solid curve is the integration of the NLSE (which includes all coherent coupling terms), and the dashed curve is for the ICE's of Ref. 1.

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[1]G. P. Agrawal, Phys. Rev. Lett. **59**, 880 (1987).
[2]A. Hasegawa and W. F. Brinkman, IEEE J. Quantum Electron. **16**, 694 (1980).

