Deterioration and Improvement of Hot Plasma Confinement in Magnetic Fusion Devices

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A simple physical picture of plasma transport in magnetic fusion devices allows us to deduce the decreasing energy confinement time with added heating power and the increasing confinement time with added plasma current. The first is due to Bohm-like diffusion and the second is due to the characteristics of Z pinches and a decrease of deposited power due to enhanced radiation loss. The addition of momentum into a plasma by flowing current that includes noninductive current drive may lead to a Z-pinch effect that compensates for the decrease in confinement time caused by the heating.

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The scaling of plasma confinement times with various plasma parameters can be estimated empirically from an examination of the large body of accumulated data, an example being Kaye-Goldston scaling in tokamaks.[1] Before the invention of the tokamak, plasma energy confinement times were determined by Bohm-like diffusion; that is, the cross-field diffusion coefficient increased with plasma temperature. The tokamak overcame the limits of Bohm-like diffusion by the addition of a plasma current and brought the improved transport (with the exception of electron thermal conductivity) to near neoclassical theory. A new problem arose, however: the well-known decrease in tokamaks and stellarators of the confinement time with the addition of auxiliary heating.

In two-body collisional theory, the confinement time increases with plasma temperature and decreases with plasma density, resulting in an increased confinement time with additional input power (except for cases involving trapped-particle effects).[2] This is the opposite of what is observed experimentally. It has often been pointed out that higher-order collisions are important, being responsible for the Langmuir paradox[3] and the anomalous cross-field diffusion. However, many efforts to clarify this problem, including pseudoclassical and neoclassical theory, remain within the framework of two-body collisional theory. Another approach, following Bohm's treatment of the many-body problem in terms of collective-wave turbulence,[4] has been to try to solve the confinement-time problem using the wave-turbulence formulation.[5] For example, attempts are often made to explain anomalous electron thermal conductivity and improved confinement in terms of the drift-wave turbulence[6] and trapped-particle instability,[7] respectively. However, the predictions of these theoretical treatments are still not confirmed experimentally. In particular, the plasma-current dependence of the energy confinement time cannot be explained by the drift-wave turbulence model.[8]

In this paper, we abandon the wave-turbulence and trapped-particle-instability approaches, and instead present a simple physical picture that explains the observed deterioration of confinement with additional heating, and its improvement with plasma current.

Empirical scaling relations (e.g., Kaye-Goldston scaling) indicate that the energy confinement time τ_E has a power (P_{in}) dependence given approximately by $\tau_E \propto P_{\text{in}}^{-0.58}$. This is the strongest dependence among such parameters as magnetic field, plasma current, etc. An example of this degradation of confinement with additional heating power can be seen in data from JET,[9] and this result is a serious hindrance to the development of a design for a fusion reactor. A direct way to overcome this problem is to increase the plasma minor radius a_p or plasma current I_p since the Kaye-Goldston empirical relation predicts $\tau_E \propto a_p^{1.16} I_p^{1.24}$. It is a matter of simple physics that more time is necessary for plasma to escape from a machine with larger a_p, and the larger devices have been constructed to take advantage of this fact. The invention of the tokamak was intended to improve the poor confinement by the addition of plasma current, although some people had pointed out the possibility of deterioration of confinement due to current-induced fluctuations. Figure 1(a) shows τ_E as a function of I_p in JT-60, in which the additional heating power is supplied by neutral-beam injection (NBI).[10] The I_p dependence is somewhat weaker than Kaye-Goldston scaling, but the confinement does increase with I_p. If we set $\tau_E \propto I_p^{\alpha}$, the value of α has a dependence on power and decreases with P_{in} as shown in Fig. 1(b).

On the other hand, it is reported that lower-hybrid current drive (LHCD) improves τ_E to near its Joule-heated value in the relatively low-density region, despite the additional heating power. In ASDEX[11] and Alcator-C,[12] τ_E with LHCD is equal to or larger than that for Joule-heated plasmas at average plasma densities of $\bar{n}_e < 6\times 10^{12}$ cm^{-3} and $\bar{n}_e < 2.5\times 10^{13}$ cm^{-3}, respectively. We also observe improved confinement during LHCD in JT-60.[13] Figure 2 shows the power dependence of τ_E for the combined experiment of LHCD and NBI in JT-60, where the lower-hybrid power is 3 MW and the parallel central refractive index n_{zc} is 1.7.[13] The value of τ_E is estimated from the variation of Shafranov Λ, assuming constant internal inductance l_i. Since l_i always decreases with NBI heating, neglecting changes in l_i gives an underestimate of τ_E. Not only is the absolute value of τ_E larger than what Kaye-Goldston scaling predicts,

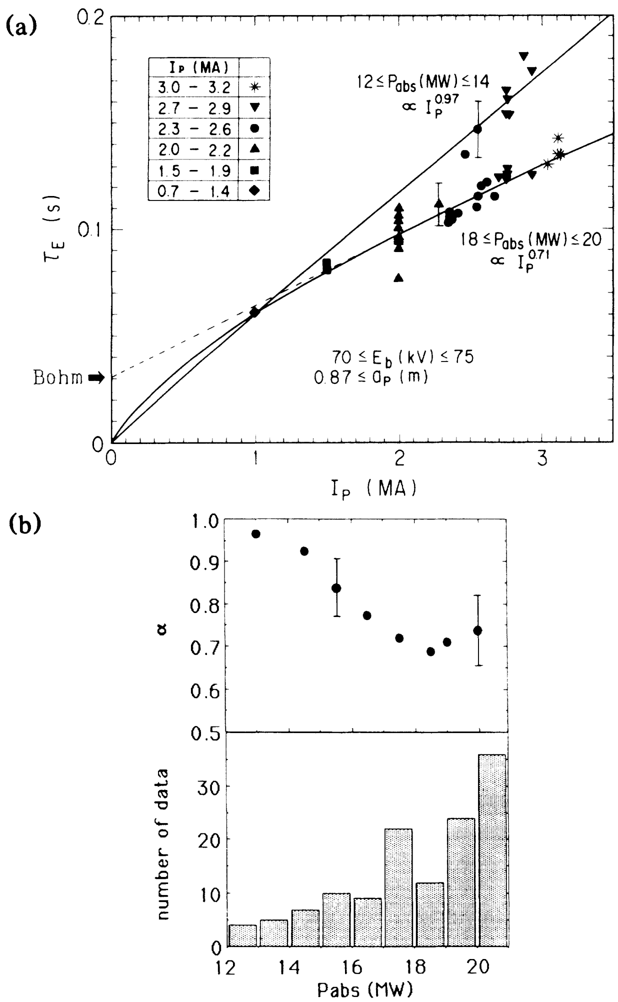

FIG. 1. (a) Experimental data for the dependence of τ_E on the plasma current I_p in JT-60, where E_b is the beam voltage of NBI. (b) The parameter α (from $\tau_E \propto I_p^\alpha$) as a function of the input power $P_{\rm in}$ ($=P_{\rm abs}$). The error bars represent 1 standard deviation.

but τ_E also does not degrade with increasing NBI power $P_{\rm NB}$.

To get a simple physical picture of confinement characteristics, we do an initial zeroth-order approximation. The temperature achieved during auxiliary heating is obtained from balancing the input and loss powers, expressed by

$$\frac{d}{dt}\int \tfrac{3}{2}n(T_e+T_i)dV = P_{\rm in} - P_{\rm RX} - \frac{1}{\tau_E}\int \tfrac{3}{2}n(T_e+T_i)dV\,, \quad (1)$$

where n is the plasma density, T_e and T_i are the electron and ion temperatures, and $P_{\rm RX}$ is the sum of the radiative and charge-exchange losses. In the stationary state ($d/dt=0$),

$$\int \tfrac{3}{2}n(T_e+T_i)dV = (P_{\rm in} - P_{\rm RX})\tau_E\,. \quad (2)$$

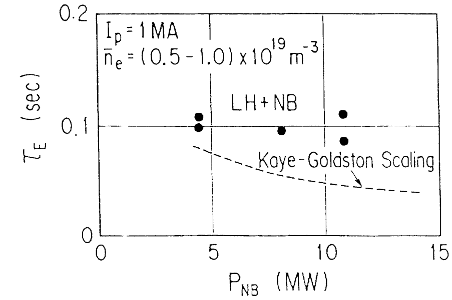

FIG. 2. τ_E vs $P_{\rm NB}$ for combined NBI heating and LHCD in JT-60.

First, we assume for simplicity that the energy confinement time is replaceable by the particle confinement time,

$$\tau_E = a_p^2/D\,. \quad (3)$$

Equations (2) and (3) can now be solved for τ_E as a function of $P_{\rm in}$ if the nature of the diffusion coefficient D is known. As a first example, using the classical diffusion coefficient $D=\rho^2\nu \propto n/B^2\sqrt{T}$ leads to the following form of the confinement scaling:

$$\tau_E = \tau_T \propto P_{\rm in} a_p^2 B^4 n^{-3} R^{-1}\,, \quad (4)$$

where B is the magnetic field and R is the major radius and we have assumed $T_e = T_i$, $\int dV = \pi a^2 2\pi R = 2\pi^2 a^2 R$, and $P_{\rm RX}=0$. The strong dependence of $P_{\rm in}$ and n does not change for pseudoclassical or neoclassical diffusion. As a second example, Bohm diffusion, $D=D_B \propto T/B$, leads to

$$\tau_E = \tau_B \propto P_{\rm in}^{-0.5} a_p^2 B^{0.5} n^{0.5} R^{-1}\,. \quad (5)$$

The parameter dependence of Eq. (5) is closer to experiment than that of Eq. (4).

Explanations of the improvement of τ_E with I_p in tokamaks are often based on turbulence theory or on decreasing trapped-particle banana width due to increasing I_p. The simple mechanism of the Z pinch is apt to be forgotten when considering tokamaks; however, it must actually occur. The $\mathbf{E}_\Omega \times \mathbf{B}_\theta$ Z pinch was considered for linear machines as early as the beginning of magnetic fusion research;[14] E_Ω is the inductive electric field and B_θ is the poloidal magnetic field. In tokamak discharges, all electrons experience a force $F = eE_\Omega$, an antireaction to the external momentum transfer responsible for the flow of plasma current. Is it is not reasonable that the $\mathbf{F}\times\mathbf{B}/eB^2$ drift acts to improve confinement, reducing the anomolous cross-field diffusion? This Z pinch has a realistic physical background, in contrast to the Ware pinch or additional inward fluxes[15] which have not been confirmed experimentally. Including the Z pinch, the total flux is

$$\Gamma = -D_B\,\partial n/\partial r - nE_\Omega B_\theta/B^2 = -D_{\rm eff}\,\partial n/\partial r\,, \quad (6)$$

where $B=(B_t^2+B_\theta^2)^{1/2}$, B_t is the torodial magnetic field, and we use the Bohm diffusion coefficient in Eq. (6) because we consider the basic transport mechanisms to be the same with and without plasma current. Equation (6) shows that D in Eq. (3) can be replaced by

$$D_{\text{eff}} = D_{\text{Oh}} = D_B - (nE_{\Omega} B_\theta / B^2)(\partial n/\partial r)^{-1},$$

and we get

$$\tau_E = \tau_Z = \frac{a_p^2}{D_{\text{Oh}}} = \tau_B \left\{ 1 - \frac{nE_{\Omega} B_\theta}{B^2} \left(\frac{\partial n}{\partial r} \right)^{-1} \right\}^{-1} \propto P_{\text{in}}^{-0.5}(1+\beta_0 I_p), \tag{7}$$

where β_0 is a numerical factor. If the term $(nE_{\Omega}B_\theta/B^2)(\partial n/\partial r)^{-1}$ in Eq. (7) cannot be neglected, then the Z pinch may play a significant role in the improved confinement. Although this term is small at the plasma boundary, it cannot be neglected at all plasma minor radii since the Bohm diffusion coefficient becomes small for small T_e and large B, and because $\partial n/\partial r$ is small at the central region of plasma. We estimate the global energy confinement time by substituting D_{eff} into the expression for the electron thermal conductivity χ_{Ee} ($=\beta_1 D_{\text{eff}}$) combined with the neoclassical prediction for the ion thermal conductivity χ_{Ei}:

$$\tau_E = [(a_p^2/\chi_{Ee})^{-1} + (a_p^2/\chi_{Ei})^{-1}]^{-1}. \tag{8}$$

The radiative and charge-exchange power-loss term is given by

$$P_{\text{RX}} = \int_0^a \beta_2 (1.31\times 10^{-38}) Z_{\text{eff}}^2 T_e^{1/2}\, dV;$$

β_1 and β_2 are numerical factors. We solve Eqs. (1), (2), and (7) as simultaneous equations on T_e with τ_E with the aid of a computer.

An example of this calculation is shown in Fig. 3, where we assume current and density profiles of the form $[1-(r/a)^2]^{\alpha_j}$ ($j = J, n$) and use a diffusion coefficient $D_B = eT_e/16\delta kB$, $\delta = r^4 + (1 - a_p^4 - \delta_1) r^2/a_p^2 + \delta_1$. This

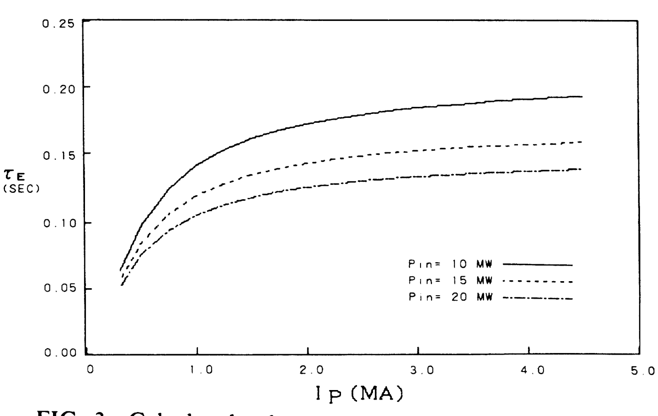

FIG. 3. Calculated values of τ_E vs I_p for several different neutral-beam-heating powers P_{NB}. This calculation uses the parameter values $B_t = 4.5$ T, $n_{e0} = 4.5\times 10^{19}$ m^{-3}, $\alpha_n = 2.6$, $\delta_1 = 15$, $\beta_1 = 1.8$, $\beta_2 = 6.4$, and $Z_{\text{eff}} = 2.5$, appropriate for JT-60.

diffusion coefficient is equal to Bohm diffusion at the plasma boundary ($r = a_p$) and is δ_1 times smaller than the Bohm value at the plasma center ($r = 0$). The value of E_{Ω} is expressed as $E_{\Omega} = \eta j_{\Omega}$, where η is the Spitzer resistivity. When the plasma temperature increases with additional heating, η decreases at constant j_{Ω}. As a result, E_{Ω} decreases with increasing temperature. This reduction of E_{Ω} in Eq. (7) may lead to a weakened pinch effect, giving a dependence of τ_E on P_{in} close to $\tau_E \propto P_{\text{in}}^{-0.58}$ for $I_p \neq 0$, and $\tau_E \propto P_{\text{in}}^{-0.5}$ for $I_p = 0$. From the experimental data (Fig. 1), τ_E increases with I_p and the value of α in $\tau_E \propto I_p^\alpha$ is less than 1.2 and decreases with P_{in}, as in Fig. 3. The enhancement of the radiation loss, which is caused by the increase of the electron temperature, equivalently leads to improvement of τ_E as is seen in Eq. (2). These results are consistent with the deterioration of τ_E with additional heating being brought about by the increase in plasma temperature, and the improvement with I_p being due to a simple Z pinch.

A Z-pinch effect must be present for rf current drive (RFCD) with the electric field parallel to the toroidal direction, since the wave transfers momentum to the plasma in a manner similar to Joule heating. For this case, the effective diffusion coefficient in Eq. (6) is

$$D_{\text{eff}} = D_{\text{RFCD}} = D_B - \frac{n_{\text{res}} E_{\text{rf}} B_\theta}{B^2} \left(\frac{\partial n}{\partial r} \right)^{-1}, \tag{9}$$

where n_{res} is the density of resonant electrons, E_{rf} is the time-averaged rf electric field, and $n_{\text{res}}E_{\text{rf}}$ is proportional to E_{rf}^2 as in the pondermotive force.[16] It has been shown using a Dawson-like treatment that the $\mathbf{E}_{\text{rf}} \times \mathbf{B}_\theta$ term in Eq. (9) does not cancel when the wave is traveling.[17] In the RFCD case, only resonant electrons are pinched while for Joule heating all of the electrons are affected. The Z pinch in LHCD is expected to be of the same order as or somewhat larger than for Joule-heated plasmas since n_{res} is reported to be more than several percent of the total density in PLT LHCD experiments,[18] and E_{rf} is expected to be about 10^3 times larger than E_{Ω}. An example of τ_E vs NBI power calculated including the LHCD Z-pinch effect is shown in Fig. 4, in analogy to Fig. 2. In this calculation, χ_{Ee} is reduced by the pinch effect ($\chi_{Ee} = \beta_3 D_{\text{RFCD}}/\sqrt{q}$) and D_B in Eq. (9) is replaced by the value predicted by Kaye-Goldston scaling. The improvement of τ_E in Fig. 4 comes from the fact that n_{res} becomes maximum at a certain electron temperature, where $T_{e0} = (4.6P_{\text{NB}} + 23.3)/(3\bar{n}_e + 14)$ and $T_{i0} = (9P_{\text{NB}} + 15)/(3.5\bar{n}_e + 17.5)$, with T_{e0} and T_{i0} the central electron and ion temperatures in keV, power is in MW, density is in 10^{13} cm^{-3}, and the profiles are assumed to have the form $[1-(r/a)^2]^{\alpha_T}$. These values are extrapolated from JT-60 data.[19] The numerical factors α_n, α_T, β_1, β_2, and β_3 are determined so as to fit the experimental values.

A higher bulk temperature inevitably causes an enhanced cross-field diffusion, which can be explained by

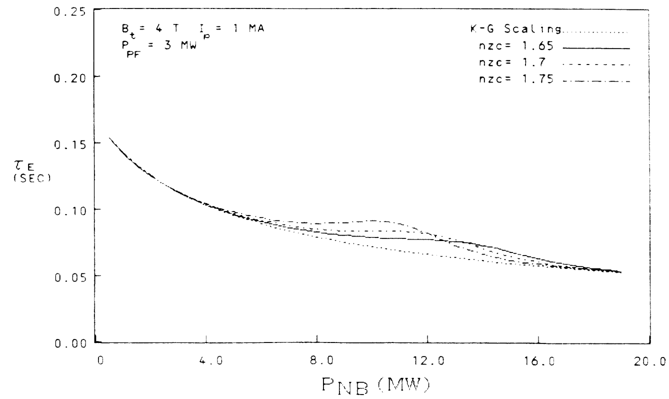

FIG. 4. Calculated value of τ_E vs neutral-beam-heating power P_{NB} for a case with combined NBI and LHCD, at several values of the parallel central index of refraction n_{zc}. The parameters used are $B_t = 4$ T, $I_p = 1$ MA, $P_{rf} = 3$ MW, $\bar{n}_e = 1 \times 10^{19}$ m^{-3}, $\alpha_n = 1.3$, $\alpha_T = 2$, $\beta_3 = 4$, and $Z_{eff} = 1$.

the Bohm diffusion or by drift-wave turbulence. However, the I_p dependence cannot be explained by conventional theories such as trapped-particle models or drift turbulence which consider only the effect on the diffusion coefficient without the pinch term in Eq. (6). The discussion so far may have a discrepancy in that $\tau_E \propto I_p^a$ implies that $\tau_E = 0$ for $I_p = 0$. As shown in Fig. 1(a), we notice that $\tau_E \neq 0$ at $I_p = 0$ rather than $\tau_E \propto I_p^a$, just as predicted by Eq. (7). The absolute value of τ_E at $I_p = 0$ in Fig. 1(a) coincides with the Bohm time. This provides a smooth connection between tokamaks and stellarators.[20] As predicted by Kaye-Goldston scaling ($\tau_E \propto B_t^{-0.09}$), the B_t dependence is weak when I_p is large since both the Bohm flux and the pinch effect decrease with increasing B_t. But in this paper we interpret the improvement of τ_E with I_p to be due to an $\mathbf{E}_\Omega \times \mathbf{B}_\theta$ pinch rather than to a reduction in the diffusion coefficient itself. This is a very simple mechanism; however, this idea may give new insight to the understanding of transport theory.

When the pinch effect for inductive current drive weakens for high plasma temperatures, one avenue of improvement might be the injection of RFCD momentum. The pinch effect for RFCD would be more effective than that for inductive current drive since E_{rf} and n_{res} in Eq. (9) can be raised externally by increasing the rf power and $N_{parallel}$, and because the RFCD plasma current is carried by higher-energy electrons and is independent of the bulk temperature. However, any noninductive-current-drive method that does not impart momentum to the plasma would only create the rotational transform without the benefits of the pinch effect.

In conclusion, we present a simple physical picture in which transport for plasmas with additional heating is still due to Bohm diffusion. Instead of a reduction in the diffusion coefficient, improved confinement with plasma current is due to an inward $\mathbf{E} \times \mathbf{B}$ Z pinch. This suggests that rf current drive can produce this pinch, compensating for the deterioration of the confinement due to the additional heating.

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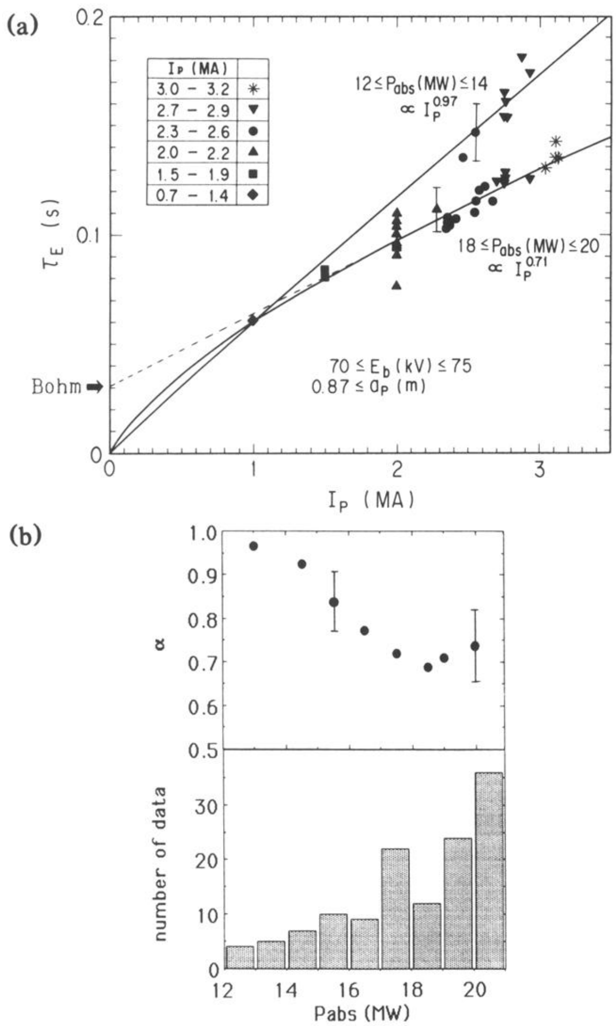

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