Fractional Giant Shapiro Steps and Spatially Correlated Phase Motion in 2D Josephson Arrays

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(Received 19 October 1989)

We have applied radio-frequency signals to 1000 by 1000 arrays of Josephson junctions and observed giant Shapiro steps at voltages 1000 times that expected for a single junction. When a perpendicular magnetic field is applied, *fractional* giant steps appear at voltages directly related to the number of flux quanta per unit cell. A phenomenological model is proposed that utilizes the spatial arrangement of the vortices in a superlattice relative to the underlying array to explain these novel steps.

PACS numbers: 74.50.+r, 74.40.+k, 74.60.Ge, 85.25.Dq

When a radio-frequency (rf) current, $i_{rf}\sin(2\pi\nu t)$, is applied to a single Josephson junction, Shapiro steps[1] occur in the dc I-V characteristics at voltages $V_n = nh\nu/2e$, where n is an integer. We have observed *giant* Shapiro steps[2] in large square arrays of superconducting-normal-superconducting (SNS) junctions, and novel *fractional* giant steps when a magnetic field is applied. In zero field, the giant steps occur at voltages

$$V_n = n\left(\frac{Nh\nu}{2e}\right), \quad n=0,1,2,\ldots, \tag{1}$$

where our arrays consist of N by N junctions with $N=1000$. This indicates that substantially all of the junctions are on the nth step at the same time and that the junctions are tending to phase-lock to the rf current. In the presence of a uniform perpendicular magnetic field, these giant Shapiro steps are drastically altered when the vortex superlattice created by the field is commensurate with the underlying junction lattice. The most prominent commensurate states in a square array occur when the applied magnetic field is such that the number of flux quanta per unit cell, $f=p/q$ (where p and q are integers), is $f=\frac{1}{4},\frac{1}{3},\frac{1}{2},\frac{2}{3},\ldots$. When the vortex superlattice is in one of these states and an rf current is applied, we observe fractional giant steps at voltages

$$V_n = n\left(\frac{Nh\nu}{q2e}\right), \quad n=0,1,2,\ldots. \tag{2}$$

In this Letter we will describe the behavior of these giant Shapiro steps and show that the novel *fractional* giant steps of Eq. (2) are the result of the broken spatial symmetry caused by fluxoid quantization.[3]

The arrays are similar to arrays reported earlier,[4] but are made with 0.2-μm-thick niobium islands on 0.3-μm-thick copper films deposited on a sapphire substrate. The cross-shaped niobium islands are formed by reactive ion etching and have a 2-μm separation defining the junction length and a lattice constant of 10 μm. Two single junctions with the same geometry as the junctions in the array are made concurrently on the same substrate. Both the single junctions and the square arrays have the same normal-state resistance ($\sim$2 mΩ), determined primarily by the copper. Using a four-point measurement circuit with a lock-in amplifier, the dynamic resistance, dV/dI, of the array was measured versus dc voltage for different external magnetic fields, rf frequencies, rf amplitudes, and temperatures. The rf power coupled into the sample is limited by its low impedance and by high-frequency noise filtering. The critical current of the array at a given temperature, $I_c(T)$, is taken as the current where the dynamic resistance is a maximum from the measured dV/dI vs I curve.

Two-dimensional (2D) arrays have been extensively studied as model systems for the Kosterlitz-Thouless (KT) transition[5,6] (f=integer) and the fully frustrated XY model[7] ($f=\frac{1}{2}$). At other values of f, the commensurability of the magnetic-field-induced vortex superlattices with the underlying array geometry causes the transition temperature[8] and critical current[9,10] to be complicated functions of magnetic field that are periodic with integer f and symmetric about $f=\frac{1}{2}$. These quantities have sharp maxima when the vortex superlattice is strongly commensurate with the underlying junction array. We show below that the rf effects in the presence of commensuration have a similar dependence on magnetic

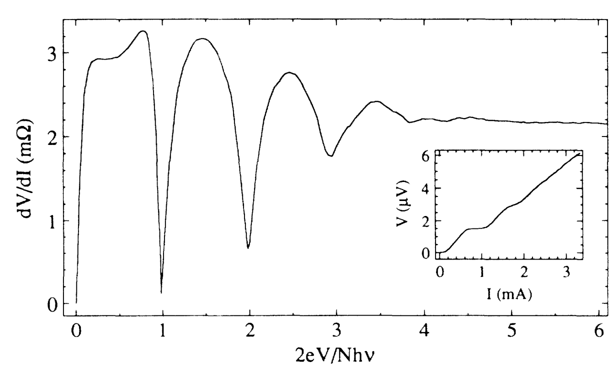

FIG. 1. A typical dynamic resistance vs normalized voltage curve in zero field for a 1000 by 1000 array. The data were taken at $T=3.0$ K, where $I_c=0.79$ mA, with $\nu=0.73$ MHz ($\Omega=Nh\nu/2eI_cR=1.0$) and $I_{rf}\sim I_c$. Inset: I-V curve measured with the same parameters.

field.

We consider first the effect of rf currents when f is an integer and there are no field-induced vortices in the array. Figure 1 shows a typical dV/dI vs V curve for $f=0$ at a temperature below the KT transition ($T_{KT} \sim 3.5$ K for this sample). The voltage axis has been normalized to $Nh\nu/2e$, so that $2eV/Nh\nu = n$ when the array is on the nth giant step. The measured I-V curve is included as an inset. These data are representative, and clearly show minima in the dynamic resistance corresponding to giant steps at voltages in agreement with Eq. (1). This behavior indicates that the N junctions in each row across the array are all on the same step and attempting to lock to the rf current. Similar data are found for other integer f, thereby establishing the periodicity of this effect.

Figure 2 shows the dynamic resistance versus normalized voltage when $f=0$, $\frac{1}{2}$, and $\frac{1}{3}$ at $T=2.1$ K, all for the same rf frequency and amplitude. The minima in the dynamic resistance occur at the voltages given by Eq. (2). For example, when $f=\frac{1}{3}$, steps are observed when $2eV/Nh\nu = \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \ldots$. We have also observed fractional giant steps for $f=\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{2}{5}$, $\frac{2}{3}$, and $\frac{3}{4}$, confirming the q dependence of the fractional steps. The fractional steps at $f=\frac{2}{3}$ are at voltages identical to those for $f=\frac{1}{3}$, thereby establishing the symmetry about $f=\frac{1}{2}$ of the fractional giant steps.

The voltages of these novel field-induced fractional giant steps can be explained by a simple picture incorporating the motion of vortices arranged in superlattices with $q \times q$ unit cells on a periodic "egg-carton" pinning potential[6,11] due to the junction array. When a magnetic field is applied, the vortices arrange themselves into these $q \times q$ unit cells so as to be commensurate with the pinning potential and to satisfy fluxoid quantization and current conservation. Our model for the fractional steps does not attempt to describe the detailed motion of the vortices, but only to present a qualitative picture that would result in a total phase slip across the array of $2\pi Nn/q$ per rf drive cycle and thus give the experimentally discovered voltages. Dynamical simulations on arrays of resistively shunted junctions are required to determine the vortex motion in more detail. We will use the $f=\frac{1}{2}$ case to describe the model and then generalize for other magnetic fields.

In the zero-temperature *ground state* for $f=\frac{1}{2}$ ($q=2$), the vortices are distributed in a checkerboard superlattice[9] [see Fig. 3(a)]. The dc and rf bias currents exert a Lorentz force on these vortices; the force on the clockwise vortices is opposite to the force on the counterclockwise vortices. These forces cause the ground-state configuration to deform. Our picture is that the Lorentz force due to the external currents causes pairs of adjacent clockwise and counterclockwise vortices to interchange positions. When all the clockwise and counterclockwise vortices have interchanged positions, the vortex superlattice has, effectively, moved one array unit cell, which causes a phase slip of πN across the array. When this happens once every rf cycle, then the voltage is given by Eq. (2), with $n=1$ and $q=2$. A giant step occurs at this voltage because the slipping motion of the superlattice locks to the periodic pinning potential of the junction array. If the dc current is increased sufficiently, then two successive interchanges occur per rf cycle, effectively moving the vortex superlattice through two array unit cells, a distance of $2a$. This causes a total phase slip of $2\pi N$ per rf cycle and gives the $n=2$ giant step for $q=2$. For this second case, the configuration of the $f=\frac{1}{2}$ vortex superlattice is also periodic in time, repeating itself once every rf cycle. This will give the same voltage as the $n=1$ giant step in zero field because all the junctions in the array are phase slipping by 2π per rf cycle. Although it is conceptually simple to think of the entire superlattice as literally moving, such a picture is ambiguous in the $f=\frac{1}{2}$ case because of the equal number of clockwise and counterclockwise vortices: Half of the vortices move one way, the other half the other way. For all other magnetic fields, there is no such ambiguity.

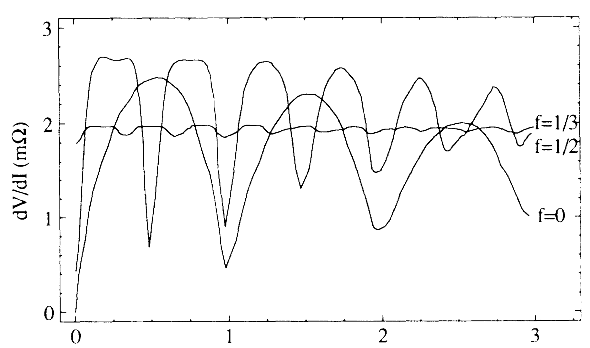

FIG. 2. Dynamic resistance vs normalized voltage for different magnetic fields, corresponding to $f=0$, $\frac{1}{2}$, and $\frac{1}{3}$. The data were taken at $T=2.1$ K, where $I_c=7.9$ mA, and with the same rf frequency, $\nu=0.73$ MHz ($\Omega = Nh\nu/2eI_cR=0.1$), and amplitude, $I_{rf} \sim 0.75 I_c$, for each curve.

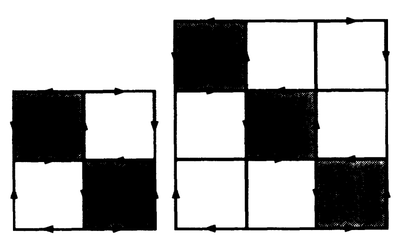

FIG. 3. Unit cells for vortex superlattice ground-state ($T=0$) configurations with zero transport current. Arrows indicate the direction of current flow. Shaded unit cells are regions of positive vorticity. (a) $f=\frac{1}{2}$, $q=2$; currents equal $i_c \sin(\pi/4)$; (b) $f=\frac{1}{3}$, $q=3$; currents equal $i_c \sin(\pi/3)$.

In general for $f=p/q$, the vortices lie in $q\times q$ superlattice cells (see Fig. 3), so that the ratio of the number of array unit cells moved per rf cycle, n, relative to the vortex superlattice unit cell size, q, determines the voltage at each step. For example, if the superlattice moves one array unit cell per rf cycle, the voltage is given by Eq. (2) with $n=1$. When the number of array unit cells moved is not a multiple of the vortex superlattice cell size, then n/q is a fraction and the voltage across the array will be a *fractional* giant step. It is the commensurability of the vortex superlattice with the periodic pinning potential of the array of junctions that is essential for producing the fractional giant steps. When n/q is an integer, the vortex superlattice returns to a configuration which is identical to its starting configuration in each rf cycle. In this latter case, the Josephson frequency $\nu_J=2eV/Nh$ of each junction is a harmonic of the drive frequency, that is, $\nu_J/\nu=n/q$ is an integer, and all the junctions phase slip by $2\pi n/q$ per rf cycle. These "harmonic" giant steps occur at the same voltage as the giant steps in zero field.

The behavior of the zero-field giant Shapiro steps is primarily determined by the characteristics of the single junctions in the array. The characteristic frequency of a *single* overdamped junction[12] is $\nu_c=2ei_c(T)R/h$, where R is its normal state resistance and $i_c(T)$ is its temperature-dependent critical current (without fluctuations). ν_c marks the crossover between the effectively ac voltage-biased ($\nu>\nu_c$) and ac current-biased ($\nu<\nu_c$) regimes, and is used to define the reduced drive frequency, $\Omega=\nu/\nu_c$. We measured the I-V curves of our *single* junctions and found the rf frequency and amplitude dependence of the step widths to be in excellent agreement with the simulations of Russer.[13]

We wish to emphasize that no "subharmonic" steps have been observed in the single junctions. This agrees with theory, since it has been proven[14] that the resistively shunted junction equation without capacitance and with a purely sinusoidal current-phase relation does not produce subharmonic steps. Since no subharmonic giant steps have been observed in the array at zero field, we conclude that the junctions in the array also behave like resistively shunted junctions. The fractional giant steps must then be entirely due to the collective behavior of the junctions.

The Shapiro steps in our single junctions appear flat at the lowest attainable temperatures (1.38 K), indicating that the junctions are then truly phase locked to the rf current. The giant steps of the array could not be quantitatively compared with those of the single junctions, because the giant steps are more rounded and dV/dI does not always go completely to zero at each step. The giant steps become flatter at lower temperatures, but the steps are not completely flat even at the lowest measurable temperature (2.1 K). (The single junctions cannot be measured at the same temperature and characteristic frequency as the arrays because the single-junction steps are completely washed out by thermal fluctuations at 2.1 K. The arrays cannot be measured effectively below 2.1 K because the available I_{rf} is insufficient to induce observable steps as a result of the larger critical currents at lower temperatures.) This experimental evidence suggests that the rounded giant steps in the 2D array are primarily a result of thermally induced vortices and thermally activated phase slip. This view is consistent with theory, since the ratio of the energy barrier to the temperature, central to the Ambegaokar-Halperin theory[15] for noise rounding due to thermally activated phase slip, is ~ 85 times larger for one of our junctions at 1.38 K than at 2.1 K. Nonuniform current flow due to the sample size being larger than the perpendicular penetration depth, or due to differences in single-junction critical currents throughout the array, may also contribute to rounding.

The rf frequency and amplitude dependence of the giant step widths in the array at zero field qualitatively follows the behavior of the single junctions for the same reduced drive frequency $\Omega=\nu/\nu_c$, if we use the critical current of a single junction in the array, $i_c(T)=I_c(T)/N$, in the definition of ν_c. Since i_c is strongly temperature dependent, ν_c is also, so that we can explore a wide range of frequencies [note the different values of Ω for the different $I_c(T)$ in Figs. 1 and 2]. Despite the rounding effects, we have observed giant steps in our 2D arrays in the frequency range 90 kHz–50 MHz, and up to 1 GHz in a 1000-junction *series* array. The observation of the giant steps in the 2D arrays at higher frequencies appears to be limited only by the larger $I_{rf}=Ni_{rf}$ necessary to achieve sizable steps. The sample size (1 cm$\times$1 cm) is much smaller than the shortest wavelength of applied rf radiation (for 1 GHz).

The dependence of the field-induced *fractional* steps on rf frequency and amplitude is still under investigation, but some interesting qualitative features have been observed. The step widths have a complicated dependence on rf amplitude, particularly for $I_{rf}\sim I_c$. The frequency dependence of the step widths, in general, qualitatively follows the behavior for single overdamped junctions,[13] in that they get smaller as Ω gets smaller. The field-dependent critical currents, $I_c(f)$, for commensurate vortex superlattices, appear to influence the absolute step width for these states; experimentally we find that $I_c(f=\frac{1}{2})>I_c(f=\frac{1}{3})$ and that the giant step widths for $f=\frac{1}{2}$ are greater than those for $f=\frac{1}{3}$. We propose that the fractional step widths may scale with $I_c(f)$, and that the characteristic frequency describing the response of the array in the presence of a commensurate field is scaled by $I_c(f)$ instead of $I_c(f=0)$.

These novel phenomena in large overdamped arrays are relevant to other fields. Superradiance,[16] the coherent phase locking of multiple junctions to their own radiation, has been sought by many authors.[17] Hadley[18] has shown that critically damped series and 2D arrays could possibly emit coherent radiation. Even though the impedance and characteristic frequency of our arrays are much too low for use in practical devices, these results show that phase locking to externally applied rf fields is possible in very large 2D arrays.

We wish to thank M. G. Forrester for designing the masks and for helpful discussions, H. Rogalla for invaluable advice and guidance in sample fabrication, and M. Octavio for helpful discussions. This research was supported in part by National Science Foundation Grants No. DMR-86-14003 and No. DMR-89-12927, Office of Naval Research Grant No. N00014-89-J-1565, and Joint Services Electronics Program Grant No. N00014-89-J-1023.

Note added.— After this work was submitted, we performed dynamical simulations[19] that confirm the model proposed above. Independent simulations, stimulated by these experiments, have been carried out by Lee, Stroud, and Chung,[20] and also reproduce the fractional giant Shapiro steps.

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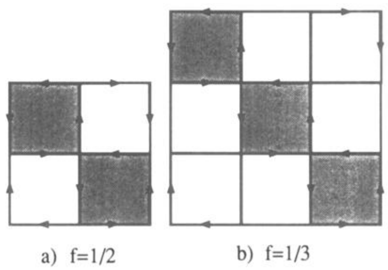

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