

Selective Equilibration among the Current-Carrying States in the Quantum Hall Regime

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The Hall resistance of a two-dimensional electron gas is measured with gated probes to determine the extent of equilibration among the N current-carrying states in the quantum Hall regime. After traveling macroscopic distances ($\sim 80 \mu\text{m}$), current injected into the first state is equilibrated among the $N-1$ lowest states but equilibration into the highest state varies strongly across the Hall plateau. This is attributed to a change in the N th state from being localized within a magnetic length of the edge to substantially extending into the sample.

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The recently developed Büttiker formalism¹ has proved useful in understanding two-dimensional electronic systems in the quantum Hall regime. It is based on previous theories² in which the conducting properties are determined by states at the Fermi level, E_F , located spatially near the edge of the sample. Within the conductor, electrons occupy N equally spaced Landau levels below E_F , but near the edge these energy levels rapidly increase and intersect E_F (Fig. 1). This produces N orthogonal, quasi-one-dimensional "edge channels" at E_F on either side of the sample through which dissipationless current flows. Edge channels on opposite sides of the sample carry current in opposite directions. A net current is established if there is a difference in the magnitudes of these opposite flowing currents.

Under equilibrium conditions, each edge channel carries an equal fraction of the current on a given side of the sample. Recent experiments have demonstrated that electrostatically defined "nonideal" probes can be used to establish an unequal current distribution among the

channels.³ As this current travels along an edge it "equilibrates" by which we mean it tends to redistribute so that an equal fraction of the current is carried by each edge channel. Current equilibration takes place via electron-scattering processes, the required potential being provided by disorder or phonons. By using one nonideal probe as a current injector, and a second nonideal probe as a current detector, the extent of equilibration that occurs between the nonideal probes can be determined. If the spatial separation between the edge channels is not much greater than the magnetic length, equilibration is expected to occur after current travels a distance on the order of the zero-field inelastic and elastic lengths ($\sim 10 \mu\text{m}$). Recently, van Wees *et al.* have shown that for high magnetic fields and over a distance of $\sim 1 \mu\text{m}$ no equilibration occurs.³ Other experiments^{4,5} have been interpreted as implying the surprising result that no equilibration occurs for distances of 100 μm or more.

Contrary to this interpretation, our experiments indicate that with N available edge channels the current is always equilibrated among the $N-1$ lowest⁶ channels after traveling distances $\sim 80 \mu\text{m}$. We find further that the fraction of current that redistributes into the N th channel decreases dramatically as the center of the corresponding Landau level approaches E_F . It is the decoupling of this channel, rather than a general effect among all edge channels, that has been observed in previous experiments.^{4,5} We believe that this provides a new understanding of the current-carrying states. Current equilibrates among the $N-1$ channels existing near the edge of the sample. However, the N th channel spreads into the sample as the N th Landau level approaches E_F , causing a decrease in scattering between it and the true edge channels.

Our device [Fig. 2(a)] is formed by etching an AlGaAs/GaAs heterostructure (electron density $3.4 \times 10^{11} \text{ cm}^{-2}$ and mobility $500000 \text{ cm}^2/\text{Vsec}$ at 4.2 K) into a Hall bar. Nonideal probes are made by metallic gates with narrow openings of lithographic width 500 nm and length 300 nm on the probe leads. A negative volt-

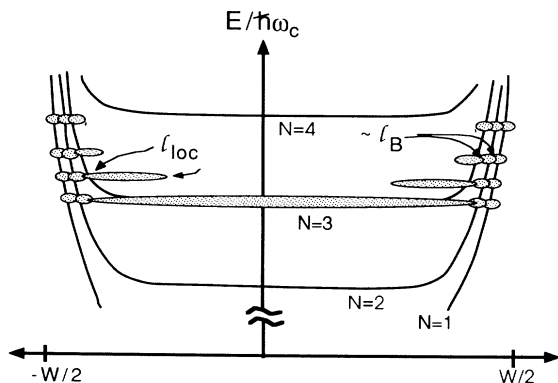


FIG. 1. Electron energy levels of a two-dimensional conductor of width W in a magnetic field in the absence of disorder. The shaded regions represent the spatial extent of the current-carrying channels in the presence of disorder, at different Fermi energies. Although not shown, the disorder also broadens the Landau levels and produces localized states between these levels.

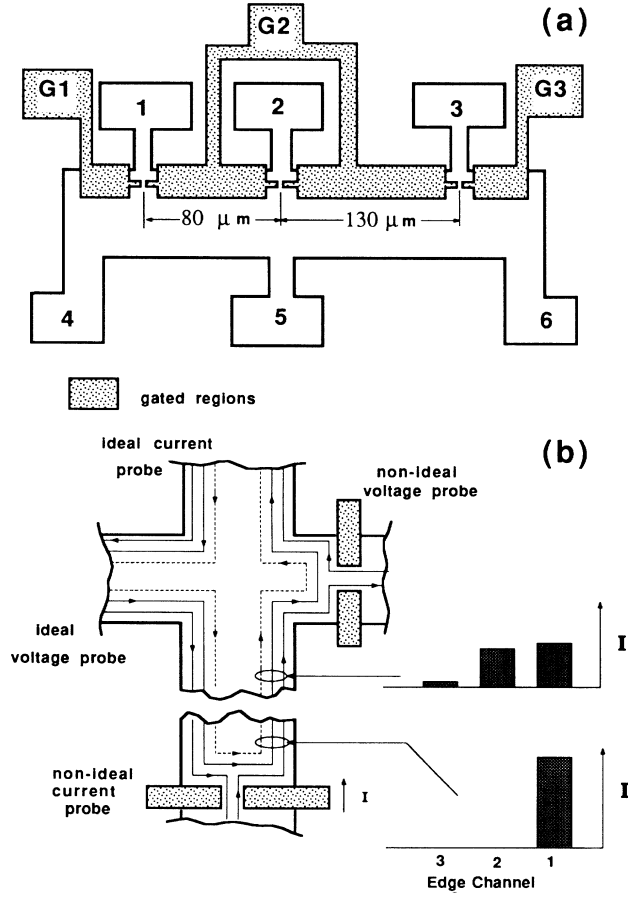


FIG. 2. (a) Schematic representation of the device used in these experiments. The properties of the nonideal probes, [1], [2], and [3], are controlled by voltages on the gates $G1$, $G2$, and $G3$. (b) Hall measurement with nonideal probes for $N=3$ Landau levels. The currents shown correspond to the values determined from the data in Fig. 3.

age V_G applied to the gate depletes the underlying electron gas and creates a constriction⁷ whose width and carrier density are controlled by V_G . The number of edge channels that allow transport between the probe and the sample is thus controlled by V_G and can be determined by measuring the two-terminal resistance of the constriction on the current (voltage) lead:⁷

$$R_{I(V)} = \frac{h}{2e^2} \left/ \sum_{k=1}^N T_{I_k}(V_k) \right.$$

$T_{I_k}(V_k)$ is the transmission coefficient for edge channel k through the constriction of the nonideal current (voltage) probe. $R_{I(V)}$ is $(h/2e^2)N^{-1}$ for zero voltage on the gate and increases with decreasing V_G . Resistance plateaus occur at $(h/2e^2)(N-1)^{-1}, \dots, h/2e^2$ corresponding to a decrease in the number of channels in the constriction.⁸

Figure 2(b) is a schematic drawing of an idealized device^{1,3} in the quantum Hall regime with three edge channels. Current is injected into the sample from the

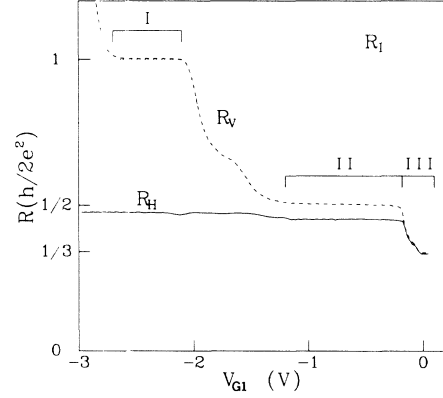


FIG. 3. Hall resistance $R_{24,15}$ as a function of V_{G1} at $B=2.3$ T. V_{G2} is fixed at -3 V so that current is injected into only the first channel. Included are the two-point resistances of the nonideal probes.

nonideal-current-probe reservoir through only the lowest of these edge channels. As this current flows towards the nonideal voltage probe, it redistributes among the available channels. The nonideal voltage probe can be adjusted to allow the detection of current through one, two, or all three edge channels. The average normalized current per channel entering the nonideal voltage probe is measured by the four-point Hall resistance, R_H . For N available edge channels^{1,3}

$$R_H = \frac{h}{2e^2} \frac{\sum_{k=1}^N \alpha_k T_{V_k}}{\sum_{k=1}^N T_{V_k}}. \quad (1)$$

T_{V_k} is the transmission coefficient between the nonideal voltage probe and the sample for edge channel k . α_k is the fraction of the total current in edge channel k as it enters the nonideal voltage probe. If the incoming current is completely equilibrated, $\alpha_k = N^{-1}$ independent of k and $R_H = N^{-1}h/2e^2$, the normal quantum Hall result. For a nonequilibrated current distribution, R_H depends upon which channels are sampled, and can be used to determine α_k for each channel.^{1,3}

Figure 3 shows the four-terminal resistance, $R_H = R_{24,15}$,⁹ measured at $T=0.45$ K and with $B=2.3$ T, so that three spin-degenerate edge channels exist in the sample. Nonideal probes 1 (voltage) and 2 (current) are separated by a distance of $80 \mu\text{m}$.¹⁰ The two-terminal resistances $R_I = R_{24,24}$ and $R_V = R_{15,15}$ determine which edge channels allow transport through the constrictions during the Hall resistance measurement. V_{G2} is fixed at -3 V so that $R_I = h/2e^2$ and current is injected into only the first channel. V_{G1} is varied from -3 V to 0 V so that R_V decreases corresponding to the detection of an increasing number of edge channels. We note three regimes: (I) $R_V = h/2e^2$ corresponding to $T_{V_1}=1$ and $T_{V_2}=T_{V_3}=0$; (II) $R_V = h/4e^2$, corresponding to $T_{V_1}=T_{V_2}=1$ and $T_{V_3}=0$; and (III) $R_V = (h/2e^2)[1/(2+T_{V_3})]$, corresponding to $T_{V_1}=T_{V_2}=1$ and $T_{V_3} \neq 0$. (Following the first regime is a transition region in which

T_{V_2} increases from 0 to 1.)

In the first regime, where the voltage probe samples only the first channel, $R_H = 0.48h/2e^2$. The normalized current in the first channel is then [using Eq. (1)] $\alpha_1 = 0.48$. In the second regime, the nonideal voltage probe samples the first two channels. This has little effect on R_H , which only decreases to $0.46h/2e^2$, indicating that $\alpha_2 = 0.44$. From the identity $\sum_{k=1}^N \alpha_k = 1$ we then find $\alpha_3 = 0.08$. Using these values, Eq. (1) predicts that the Hall resistance in the third regime is

$$R_H = \frac{h}{2e^2} \left[\frac{0.92 + 0.08T_{V_3}}{2 + T_{V_3}} \right] \approx \frac{h}{2e^2} \left[\frac{1}{2 + T_{V_3}} \right],$$

which is just the resistance R_V given above for the third regime.¹¹ This is confirmed by the data in Fig. 3. These measurements show that 92% of the injected current occupies the first two channels, between which it is almost equally divided. Only 8% of the injected current moves to the third channel over the 80- μm distance. Similar measurements done at $B = 1.75$ T, where four spin-degenerate edge channels are available, yield the following results: $\alpha_1 = 0.31$, $\alpha_2 = 0.28$, $\alpha_3 = 0.27$, and $\alpha_4 = 0.14$.

The results of these measurements suggest a simple method to determine α_N as a continuous function of magnetic field. We make the assumption that equilibration among the $N-1$ lower channels is complete and constant with field, i.e., $\alpha_1 = \alpha_2 = \dots = \alpha_{N-1}$. The normalized current in the N th channel, α_N , is then given by $\alpha_N = 1 - \sum_{k=1}^{N-1} \alpha_k = 1 - (N-1)\alpha_1$. By detecting only the lowest edge channel, α_1 is determined from the Hall resistance and hence α_N is deduced. The results of this measurement are seen in Fig. 4. Here, $R_H = R_{34,25}$ measured with probes 3 and 2 contacting only the lowest edge channel¹² (nonideal probes) is compared with $R_{34,25}$ measured while contacting all N channels (ideal probes). The inset is a detailed view of the low-field region, where spin splitting can be neglected. Consider the N th quantum Hall plateau as measured by the ideal R_H . At the low-field end of the plateau (where E_F is well above the N th Landau level) the nonideal-probe R_H is $N^{-1}h/2e^2$, indicating [from Eq. (1)] that $\alpha_N = N^{-1}$. At the high-field end of the plateau (where E_F is just above the N th Landau level) the nonideal-probe R_H is approaching $(N-1)^{-1}h/2e^2$, indicating that $\alpha_N \sim 0$. We note that R_H never rises above $(N-1)^{-1}h/2e^2$, in agreement with our assumption of complete equilibration among the $N-1$ lower edge channels. These results imply that scattering between the N th edge channel and the $N-1$ edge channels decreases as the N th Landau level approaches E_F , i.e., the N th channel "decouples" from the remaining current-carrying channels. Further measurements indicate that for a fixed magnetic field value the fraction of current filling the N th channel increases (as indicated by a decrease in R_H) as temperature or nonideal-probe separation distance, l , increases. For $B = 2.3$ T ($N = 3$) and $T = 0.45$ K, $R_H = (1/2.1)h/$

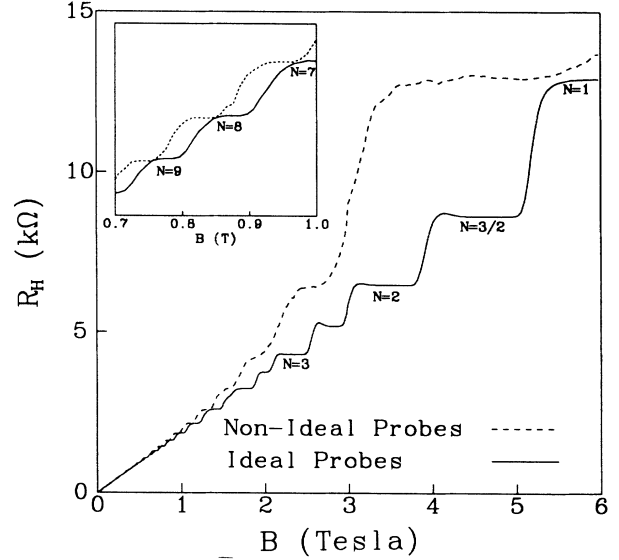


FIG. 4. Hall resistance $R_{34,25}$ measured with ideal probes and with probes that sample only the lowest channel. Inset: Expanded view of the low-field regime.

$2e^2$ for $l = 130$ μm and $R_H = (1/2.3)h/2e^2$ for $l = 210$ μm . For $l = 80$ μm , $R_H = (1/2.3)h/2e^2$ for $T = 1.0$ K and $R_H = (1/2.9)h/2e^2$ for $T = 4.2$ K.

When the quantum Hall effect breaks down, states in the center corresponding to the N th Landau level exist at E_F , allowing electrons to scatter across the width of the sample.¹³ In Fig. 4, the ideal-probe R_H increases, but a relatively stable plateau is observed in the nonideal-probe R_H at $(h/2e^2)(N-1)^{-1}$. This demonstrates that outside of the quantum Hall regime the $N-1$ edge channels fully equilibrate with each other, but, as was first shown by van Wees *et al.*,⁴ they are decoupled from the N th channel.

In light of these results, we now consider previous experiments in which lack of equilibration over macroscopic distances was observed.^{4,5} Longitudinal resistance measurements made by van Wees *et al.*⁴ show a disconnection of the lower edge channels from the top edge channel, but provide no information concerning scattering among the lower edge channels. Resistance measurements made by Komiyama *et al.*⁵ are at a field value where only two spin-degenerate edge channels are available for transport. Both of these results are consistent with our experiment since scattering between the highest edge channel and the remaining lower edge channels can be negligible. We also show that this scattering rate changes drastically as the highest Landau level moves with respect to E_F and that scattering among the $N-1$ lower channels is not suppressed. The number of edge channels and the Landau-level energies relative to E_F were previously neglected, but are shown here to be the most important parameters for determining equilibration properties.

Finally, we consider the high-field region in Fig. 4.

All spin-resolved plateaus are absent from the nonideal-probe R_H and therefore the two spin-resolved levels must decouple simultaneously from the lower edge channels. This is not surprising since the spin-splitting energy is generally much less than $\hbar\omega_c$. There is an enhancement in the spin-splitting energy due to exchange interactions when E_F lies between the two levels, but here the lower spin level has already decoupled from the remaining channels. Current does not equilibrate into only one of the two spin-split channels, and consequently nonideal-probe spin-resolved plateaus are not observed.

If the spatial separation between edge channels is much larger than the magnetic length, l_B , the interchannel scattering rate is suppressed and nonequilibrium current distributions will persist over large distances.¹⁴ Using a parabolic model for the edge potential based on recent theoretical¹⁵ and experimental¹⁶ results, the spatial separation between edge channels is $\sim l_B$ at $B \sim 4$ T. This implies that the interchannel scattering rate is $\sim \frac{2}{3}$ the zero-field inelastic and elastic rates.¹⁴ This model thus cannot account for the suppression of scattering into the N th edge channel at low fields, nor can it explain the selective scattering among edge channels.

Given these considerations, Jain¹⁷ has proposed a qualitative model which considers the effect of disorder on the spatial extent of the current-carrying states (Fig. 1). Without disorder, all the states at E_F are localized within $\sim l_B$ of the sample edge (except when E_F is coincident with the Landau level). With moderate disorder, the $N-1$ outer states remain localized near the edge of the sample and easily equilibrate with each other. The N th edge state, however, hybridizes via the impurity potential with states associated with the N th Landau level near E_F . This increases the spatial extent, or "localization length"¹⁸ (l_{loc} in Fig. 1), of this state away from the edge of the sample. When the N th Landau level is closer to E_F , the N th current-carrying state at E_F has a longer localization length. It extends further into the sample, reducing its amplitude near the sample edge. This lessens the overlap between the N th state and the $N-1$ states and thus suppresses scattering between these states. The scattering is almost completely suppressed when the N th state extends across the sample and the quantum Hall effect breaks down.

In conclusion, we find that in the quantum Hall regime the lowest $N-1$ channels readily equilibrate over macroscopic distances ($\sim 80 \mu\text{m}$), but the uppermost channel becomes decoupled as the magnetic field is varied across a quantum Hall plateau. This is attributed to a change of the state at E_F associated with the N th edge channel from being localized within a magnetic length of the edge to substantially extending into the center of the sample.

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⁶" N th" or "highest" refers to the edge channel derived from the Landau level closest to E_F (see Fig. 1). "Lower" or " $N-1$ " refers to those channels derived from the $N-1$ Landau levels lower in energy than the N th Landau level.

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⁸The two-terminal resistance measured experimentally is a combination of the constriction resistance and the series resistance of the contacts and leads. The series resistance is determined by subtracting the two-terminal quantum Hall resistance $N^{-1}\hbar/2e^2$ from the resistance measured at $V_G = 0$.

⁹The standard notation for a four-point resistance measurement is $R_{ij,kl}$, where $[ij]$ are the current probes and $[kl]$ are the voltage probes.

¹⁰The magnetic field points into the page so that electron edge current flows from the nonideal current probe into the nonideal voltage probe. Additional Hall resistance measurements made with the roles of the nonideal current and voltage probes reversed ($R_H = R_{15,24}$) produced identical results as Hall resistance measurements made with ideal probes. This is expected for this configuration since the current equilibrates in ideal probes 4 and 5 before it reaches the nonideal voltage probe.

¹¹In the third regime, the electron gas underneath gate G1 is not completely depleted and the voltage probe in this regime does not consist of a short, narrow channel. This does not affect our results, however, since the region between the current and voltage probes is unchanged.

¹² V_{G2} and V_{G1} are first set so that $R_V = R_I = \hbar/2e^2$ when $B = 0$. As the field is increased, R_I and R_V are checked and the gate voltages are adjusted to insure good contact with only the lowest edge channel.

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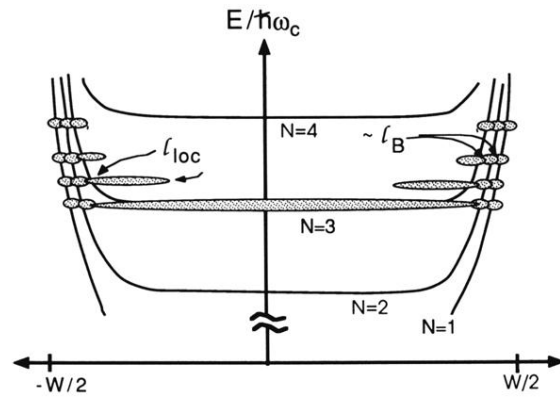


FIG. 1. Electron energy levels of a two-dimensional conductor of width W in a magnetic field in the absence of disorder. The shaded regions represent the spatial extent of the current-carrying channels in the presence of disorder, at different Fermi energies. Although not shown, the disorder also broadens the Landau levels and produces localized states between these levels.

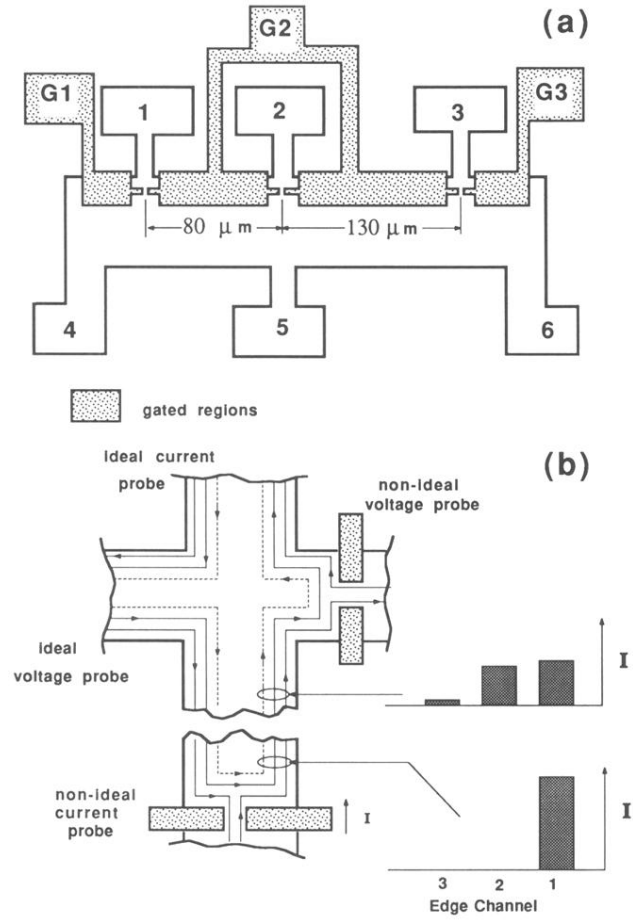


FIG. 2. (a) Schematic representation of the device used in these experiments. The properties of the nonideal probes, [1], [2], and [3], are controlled by voltages on the gates $G1$, $G2$, and $G3$. (b) Hall measurement with nonideal probes for $N=3$ Landau levels. The currents shown correspond to the values determined from the data in Fig. 3.