Algebraic Scattering Theory for Heavy Ions

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Algebraic potentials from SO(3,1) and SO(3,2) representations of scattering functions are deduced by matching to scattering functions obtained by fitting ^{12}C-^{12}C elastic-scattering differential cross sections. Their variations with energy suggest a simple mapping between algebraic and coordinate-space interactions.

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Recent studies[1,2] have introduced techniques for defining S functions for elastic scattering from consideration of dynamical symmetry groups and their asymptotic properties. In particular the groups SO(3,N) for $N=1$ and 2 yield S functions that equate to scattering from modified Coulomb potentials,[2] and therefore these algebraic models are appropriate for use in the analysis of collisions between heavy ions. The models are valid even at high energies where the strong absorption model[3] has had success since most data reflect properties of the interaction in a sensitive radial region in the proximity of the strong-absorption radius,[4] and where the potential strengths are characteristically small in comparison with the Coulomb field strength. It has been shown[2] that the presence of a dynamical symmetry implies a particular functional form for the S matrix, defining thereby "algebraic potentials" $v_l(k)$ and $w_l(k)$ by

$$S_l(k)=\frac{\Gamma(l+1+iv_l(k))}{\Gamma(l+1-iv_l(k))},\ \text{for SO(3,1)},\tag{1}$$

$$S_l(k)=\frac{\Gamma([l+2+w_l(k)+if(k)]/2)\Gamma([l+1-w_l(k)+if(k)]/2)}{\Gamma([l+2+w_l(k)-if(k)]/2)\Gamma([l+1-w_l(k)-if(k)]/2)},\ \text{for SO(3,2)},\tag{2}$$

where $f(k)$ is the Coulomb parameter, and v,w are functions of l and k. It is hoped that the above functional forms for the elastic scattering can be generalized through choice of v and w to encompass strong-interaction and -absorptive scattering processes when, nevertheless, those processes remain dominated by the Coulomb force. Although developed for unitary S functions, application to absorptive interaction problems, as in the case of heavy-ion scattering, can be approximated[1] by simply allowing the algebraic potentials to be complex[2] with the conditions $\mathrm{Im}[v_l^2(k)]<0$ and $\mathrm{Im}[w_l^2(k)]<0$.

As has been noted,[2] it is not clear just what would be the underlying physical model in the Schrödinger picture as the algebraic approach is formulated at the level of the S functions. In order to find a potential that, inserted into a Schrödinger equation, produces the same S functions (and so cross sections) one needs to solve the inverse-scattering problem at fixed energy. For heavy-ion interactions, that inverse problem has been studied recently,[5-9] with varying degrees of success and validity. In some cases,[6,7] the potentials obtained by inversion showed strong fluctuations with range which might cast doubts upon their physical relevance. Such fluctuations can be a consequence of the ill-posedness of the inverse-scattering problem.[10] For energies 360–2400 MeV with ^{12}C on ^{12}C, however, the WKB approximation method has been found[5] to be both stable and accurate in defining potentials to small radii well inside the sensitive radial region. No unphysical fluctuations resulted and in all cases the inverted and best-fit optical-model potentials were very similar in the sensitive region. Semiclassical studies of the inverse-scattering problem have also been made[8,9] using an eikonal approximation to relate forms of the algebraic potentials in l space to asymptotic forms of the interaction in coordinate space. Both studies, however, used parametrized forms for $v_l(k)$ and $w_l(k)$ that are based only upon the assumption of a similarity in shape between the strong-absorption S function[3] and the algebraic potentials.

In this study we first consider strong-interaction S functions[9] obtained from a "best-fit" optical-potential study of the 1016-MeV ^{12}C-on-^{12}C elastic-scattering data.[11] We then fit the SO(3,2) S-function form to those best-fit results and obtain the algebraic potential $w_l(k)$. The process is repeated using a family of optical potentials obtained by scaling the overall strengths of the real and imaginary parts of that best-fit optical potential to assess the sensitivity of w_l to features of the optical potential.

The real and imaginary S functions obtained from those optical-model potential calculations are shown on the left and right of Fig. 1, respectively. These optical-potential calculations were made using the code ECIS88[12] and the scattering data at 1016 MeV were used to deter-

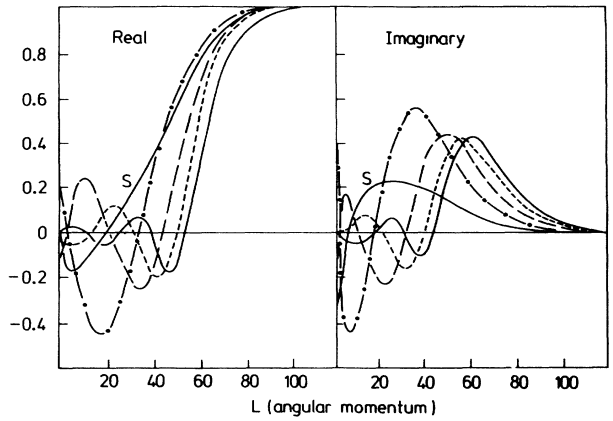

FIG. 1. S functions from optical-model potential calculations.

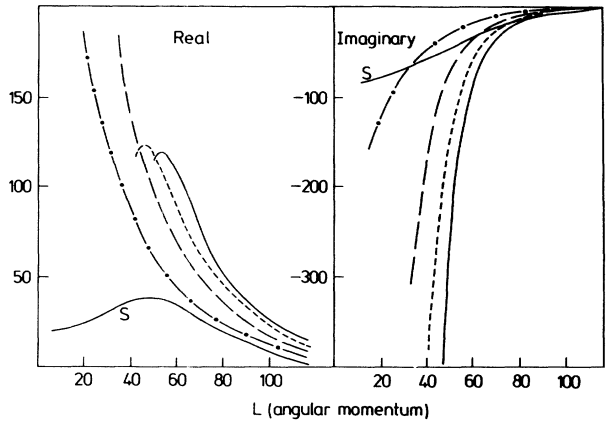

FIG. 2. Algebraic potential w functions.

mined (by a χ^2 minimization search) that the basic potential parameters V and W are 153 and 31 MeV, respectively. The associated S functions are displayed by the continuous curves in Fig. 1 that do not have the tag "S." The continuous curves with the tag S are the S-function values obtained using the "ersatz" parameter values of 16 MeV for both V and W. For comparison we have calculated S functions using (nuclear) potential strengths $\frac{3}{4}$ (short-dashed curves), $\frac{1}{2}$ (long-dashed curves), and $\frac{1}{4}$ (dot-dashed curves) of the best-fit values. Clearly, scaling the potential leads to a family of S functions that vary in form only for small values of l. It is characteristic of standard Woods-Saxon potentials for ^{12}C-on-^{12}C scattering that the S functions at low l oscillate and are small in magnitude. Those characteristics make it impossible to match those S functions to the algebraic, SO(3,2) form at such angular momenta. Consequently the $w_l(k)$ functions shown in Fig. 2 terminate at minimum l values that equate approximately to those at which the Re$[S_l(k)]$ vanish. Nevertheless, as the nuclear potentials are scaled down in strength, the S functions lead to a family of $w_l(k)$ curves. With a major change to the relative strengths of V and W (as given by the ersatz set) the $w_l(k)$ functions obtained vary quite markedly from the others. Thus the S functions as given by the data, by optical-potential analysis (or inversion), lead to a unique potential at least in the sensitive region and, for the current circumstances at least, to a unique $w_l(k)$ function in the algebraic theory. Clearly, data on this test reaction need to be obtained at a number of intermediate energies before one can draw any strong conclusion about a relationship between coordinate-space and algebraic potentials. It would seem, however, that the general shape of the $w_l(k)$ reflects the relative strengths of the real and imaginary parts of the scattering potential while the location of that shape with l reflects the overall strength of the potential in relationship to the projectile energy.

One need not start with an optical-model potential as the data at 1016 MeV, as well as at 360, 1449, and 2400 MeV from ^{12}C elastically scattering off of ^{12}C, are well represented by a strong-absorption model (SAM) of scattering, with associated S functions that not only are smoothly varying functions of l but also suitable for inversion. Those SAM S functions can be obtained from fits[13] to ^{12}C-^{12}C elastic-scattering cross sections using a McIntyre five-parameter function. With the possible exception of the 1449-MeV results, the associated cross sections are excellent fits to measured data. For the energies in increasing order we note that the S functions for l values less than 20, 30, 30, and 40 are of little or no significance in fitting the current measured data. Consequently any smooth continuation may be used, or their values simply set equal to zero.

These properties of the S functions are true irrespective of any choice of model calculation, but in the context of our studies they simply set lower limits to our ability to match to the algebraic-model SO(3,2) form of

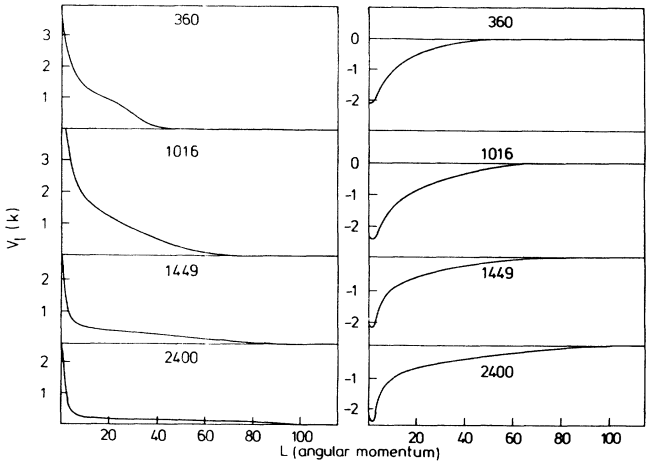

FIG. 3. Algebraic potential v functions for ^{12}C-^{12}C scattering.

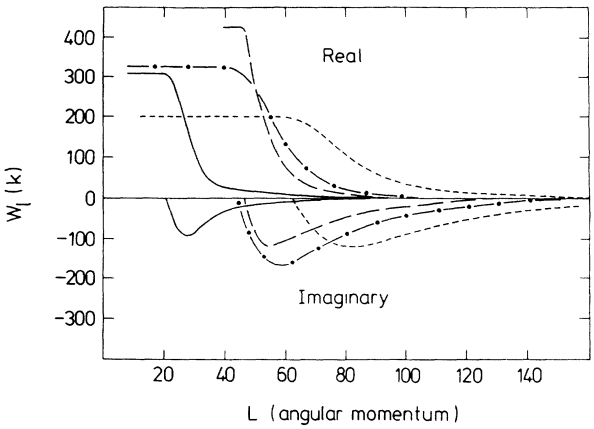

FIG. 4. Algebraic potential w functions for ^{12}C-^{12}C scattering.

the S functions and so extract values of $w_l(k)$. No such problems were found in matching the SO(3,1) form to get values for $v_l(k)$. The results of that SO(3,1) form matching at each energy, using the S functions of Fig. 1, are shown in Fig. 3. The real and imaginary parts of the $v_l(k)$ are shown on the left and right of this figure, respectively. Clearly there is a characteristic form for these functions. The real parts are positive definite and monotonically decreasing with l (rapidly so for small l), while the imaginary parts are all negative definite, being largest at small l values and monotonically diminishing toward zero at high l. The decline to zero of $v_l(k)$ with high l values slows with increasing energy. The 1016-MeV results are atypical to the extent that the real and imaginary parts of $v_l(k)$ at that energy for most l values are larger in size than any of those at the other three energies.

The real and imaginary parts of the $w_l(k)$ functions extracted by matching the SO(3,2) algebraic S-function forms to those of the strong-absorption model are displayed in Fig. 4. The 360-, 1016-, 1449-, and 2400-MeV results are identified by the continuous, long-dashed, dash-dotted, and short-dashed curves, respectively. Again a family form is evident, albeit now the results do not extend to very small l values. To map the SO(3,2) form to an S function of the order of 0.1 in magnitude places $w_l(k)$ near the negative real axis with the real part approaching a large (negative) integer, for which arguments Γ functions diverge. Recall, however, the S functions for such small values of l are of no consequence so far as current data are concerned. Nevertheless, the range of l values for which the $w_l(k)$ can be defined is sufficient to make trends obvious. From essentially a constant value at small l (both real and imaginary parts) the real part decreases monotonically and sharply before approximating an exponential at high l values. The imaginary part has a "well-like" shape with an exponential tail at high l values. There is coincidentally more nonlinear variation with energy in these $w_l(k)$ functions than in the $v_l(k)$ set. The real part of the 1016-MeV and the imaginary part of the 1449-MeV algebraic potentials $w_l(k)$ seem anomalous. Semilogarithmic plots of both $v_l(k)$ and $w_l(k)$ reveal that they decrease exponentially and asymptotically with l as

$$v_l(k) \text{ [and } w_l(k)] \rightarrow \exp[-\alpha(k)l] . \tag{3}$$

TABLE I. The exponent coefficients $\alpha(k)$.

E (MeV)	Re$v_l(k)$	Im$v_l(k)$	Re$w_l(k)$	Im$w_l(k)$
360	0.160	0.230	0.068	0.070
1016	0.140	0.092	0.100	0.038
1449	0.092	0.085	0.051	0.036
2400	0.074	0.066	0.030	0.027

The values of the exponent coefficient $\alpha(k)$ are listed in Table I from which it is evident that, with the 1016-MeV values excepted, there seems to be a smooth monotonic trend to the character of the asymptotic behavior of the algebraic potentials with energy. The current data are too sparse as yet to be useful in defining that trend, or even to claim with certainty that the 1016-MeV values are anomalous.

An estimate of the appropriate asymptotic form (in l) of the algebraic potentials from the SO(3,1) model can be inferred by equivalence to a Schrödinger picture. With v_R, v_I designating the real and imaginary parts of each $v_l(k)$, Eq. (1) enables us to identify the real and imaginary parts of the phase shifts at each value of k as δ_l^R and δ_l^I, respectively, per

$$S_l(k) = \exp(2i\delta_l) = \frac{\Gamma(l+1-v_I+iv_R)}{\Gamma(l+1+v_I-iv_R)} , \tag{4}$$

giving

$$\delta_l^R = \tfrac{1}{2} v_R[\psi(l+1-v_I)+\psi(l+1+v_I)] + v_R \sum_{n=0}^{\infty} \frac{l+1+n}{(l+1+n)^2-v_I^2} - \sum_{n=0}^{\infty}\left[\Delta_n + \arctan\left(\frac{2(l+1+n)v_R}{(l+1+n)^2-v_R^2-v_I^2}\right)\right], \tag{5}$$

where ψ are digamma functions and Δ_n denote the $\pm\pi$ changes in phase which may exist for small values of l. For values of l very much greater than $|v_R|$ and $|v_I|$, there are no problems of discontinuities in δ_l^R nor do any resonances exist that make either summation in Eq. (5) diverge so that $\delta_l^R(k) \rightarrow v_R \ln(l+1)$. Thus, from the Schrödinger picture we may extract candidate forms for v_R by calculating phase shifts in the first Born approximation. For a Yukawa potential, v_R behaves asymptotically as the exponential $\exp[-(l+1)\mu/k]$, where μ is the range of the Yukawa. For a Gaussian interaction, an exponential asymptote is also predicted. With the SO(3,2) algebra, however, it is not possible

to extract asymptotic information on $w_l(k)$, as, in the large-l limit, only the Coulomb-interaction values remain. The very form of Eq. (2) implies Coulomb asymptotics. As a model *Ansatz*, however, and based upon our results derived using SAM, it seem reasonable to fit $v_l(k)$ and $w_l(k)$ with similar model forms, approximating the exponential part as $\exp[-\alpha(k)l]$, with $\alpha(k)$ a simple function of k ($\sqrt{E}$) such as

$$\alpha(k)=c_0+c_{-1}E^{-1/2}+c_1E^{1/2}. \tag{6}$$

Using the data in Table I (excluding the 1016-MeV values) gives (for E in MeV)

$$\alpha(k)=\begin{cases}0.05+2.29(E)^{-1/2}-4\times10^{-4}(E)^{1/2}, & \mathrm{Re}v_l(k),\\ -0.18+7.0(E)^{-1/2}+2\times10^{-3}(E)^{1/2}, & \mathrm{Im}v_l(k),\end{cases} \tag{7}$$

and

$$\alpha(k)=\begin{cases}0.18-1.2(E)^{-1/2}-2.6\times10^{-3}(E)^{1/2}, & \mathrm{Re}w_l(k),\\ -0.18+1.15(E)^{-1/2}-2\times10^{-4}(E)^{1/2}, & \mathrm{Im}w_l(k).\end{cases} \tag{8}$$

We stress, however, that the current data are too sparse for any strong confidence to be given to the actual coefficients c_n in the above, although it does seem clear that a simple functional form is appropriate and from that the long-range potential behavior in coordinate space may be inferred.

But there are characteristics of these algebraic potentials other than the asymptotic tails that have a simple functional l dependence. Of these the most relevant are the peak values in the imaginary parts of $w_l(k)$ which occur at values given by

$$l_p\approx -6+1.77\sqrt{E}\,, \tag{9}$$

as they are comparable to the classical grazing values, l_g, at which the real parts of the S matrices have values of 0.5. It is at and beyond those grazing values that asymptotic conditions may have significance. With the strong-absorption models the S functions for $l<l_g$ have only minor contributions in the calculated cross sections.

In summary, there appears to be a family characteristic between the optical-model potentials of heavy-ion scattering (in strong-absorption situations) and the algebraic potentials as determined by either the SO(3,1) or the SO(3,2) group representations of S functions. The "potential forms" required to fit data using the algebraic models exhibit complex behavior for small l ($<l_g$) but appear to be asymptotic to forms consistent with those predicted by simple potential models.

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