SU(15) Grand Unification

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A grand unified model based on SU(15) is described. Baryon-number-violating processes such as proton decay are absent in both the gauge and Higgs sectors. A number of new gauge bosons are predicted at Superconducting Super Collider energy, including doubly charged gauge bosons coupling only to leptons.

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Grand unified theories (GUT's) have had a major impact on both cosmology and astrophysics; for cosmology they led to the inflationary scenario, while for astrophysics supernova neutrinos were first observed in proton-decay detectors. It remains for GUT's to impact directly on particle physics itself.

All of the known experimental data on elementary-particle interactions are impressively described by the standard model which includes the $SU(3)_c\times SU(2)_L\times U(1)_Y$ gauge group for strong and electroweak interactions. The simplest GUT based on SU(5) with a minimal field content is excluded by the present lower limit on the partial lifetime for the proton decay $p\to e^+\pi^0$. With more complicated GUT's based on, e.g., O(10) and E(6), predictions are less definite, but proton decay mediated by gauge bosons of the simple unifying group always occurs at lowest order. In general, the gauge group at accelerator energies in such theories may still be taken to be $SU(3)_c\times SU(2)_L\times U(1)_Y$, so that unless and until the rare process of proton decay or some other signal for the large scale, such as nonzero neutrino mass or superheavy monopoles, is observed experimentally, it is difficult to confirm or refute such models.

In this Letter, we examine a new type of GUT where proton decay mediated by gauge bosons is simply absent, and where consequently one may have both a much lower GUT energy scale and likely new gauge forces at energies accessible to accelerator experiments. The one drawback compared to SU(5) will be that anomaly cancellation is incomplete without mirror fermions, although the latter could be, in principle, replaced by other compensating fermions at higher mass scales in a more complete theory based on, e.g., superstrings. If the chiral anomalies are canceled by mirror fermions, it is assumed that such mirror fermions have masses of order $O(M_W)$. If they appear as mass-degenerate sets, they do not affect the renormalization-group determinations of the unification scales discussed below, and more generally need not affect the low-energy physics. Such mirror fermions are not attractive (an alternative anomaly free representation would be preferable), but are not excluded.

The model we propose is based on the gauge group SU(15). This will be an exact symmetry at energies well above 10^7 GeV and will be broken at some mass scale M_G of order 10^7 GeV to the subgroup $SU(12)_q\times SU(3)_l$. The **15** of SU(15) decomposes to $(\mathbf{12},\mathbf{1})+(\mathbf{1},\mathbf{3})$ under the subgroup. At a mass scale M_B only slightly below M_G (detailed estimates of all the relevant mass scales will be made below) the group $SU(12)_q$ is broken to $SU(6)_L\times SU(6)_R\times U(1)'_h$ with $\mathbf{12}=(\mathbf{6},\mathbf{1})_{+h}+(\mathbf{1},\bar{\mathbf{6}})_{-h}$. At a much lower mass scale M_A, of order 1 TeV, we break to the gauge group of the standard model. In summary, the gauge-symmetry-breaking pattern is

$$SU(15)\xrightarrow{M_G}SU(12)_q\times SU(3)_l \tag{1a}$$

$$\xrightarrow{M_B}SU(6)_L\times SU(6)_R\times U(1)_h'\times SU(3)_l \tag{1b}$$

$$\xrightarrow{M_A}SU(3)_c\times SU(2)_L\times U(1)_Y \tag{1c}$$

$$\xrightarrow{M_W}SU(3)_c\times U(1)_Q\,. \tag{1d}$$

In the breaking (1c), color $SU(3)_c$ is embedded in $SU(6)_L\times SU(6)_R$ as $(\mathbf{3}+\mathbf{3},\mathbf{1})+(\mathbf{1},\bar{\mathbf{3}}+\bar{\mathbf{3}})$, $SU(2)_L$ is embedded in $SU(6)_L\times SU(3)_l$ with $\mathbf{6}_L=3(\mathbf{2})_L$ and $\mathbf{3}_l=\mathbf{2}_L+\mathbf{1}_L$; finally $U(1)_Y$ is contained in $SU(6)_R\times U(1)'_h\times SU(3)_l$ according to

$$Y=\sqrt{3}\Lambda+\sqrt{\tfrac{2}{3}}\,h+\sqrt{3}\,\mathcal{Y}\,, \tag{2}$$

with Λ, h, and $\mathcal{Y}$ generators of $SU(6)_R$, $U(1)'_h$, and $SU(3)_l$, respectively, normalized as 15×15 SU(15) matrices with

$$\mathrm{Tr}(\Lambda^a\Lambda^b)=2\delta^{ab}\,, \tag{3}$$

$$\Lambda=\frac{1}{\sqrt{3}}\,\mathrm{diag}(000000,-1\ -1\ -1\ 1\ 1\ 1,000)\,, \tag{4a}$$

$$h=\sqrt{\tfrac{3}{2}}\,\mathrm{diag}(\tfrac13\ \tfrac13\ \tfrac13\ \tfrac13\ \tfrac13\ \tfrac13,\ -\tfrac13\ -\tfrac13\ -\tfrac13\ -\tfrac13\ -\tfrac13\ -\tfrac13,000)\,, \tag{4b}$$

$$\mathcal{Y}=\frac{1}{\sqrt{3}}\,\mathrm{diag}(000000,000000,2\ -1\ -1)\,. \tag{4c}$$

The fermion families are not unified in SU(15) but instead each corresponds to one **15**; for example, in the first family

$$\mathbf{15}_L = (u_1 u_2 u_3 d_1 d_2 d_3, \bar{u}_1 \bar{u}_2 \bar{u}_3 \bar{d}_1 \bar{d}_2 \bar{d}_3, e^+ \nu_e e^-)_L \quad (5)$$

and similarly for the second, third, and any higher families. By analogy with the old idea[1] of embedding SU(5) in a larger SU(N), one could unify families in SU(15) by going to an N larger than 15.

The symmetry-breaking pattern indicated in Eqs. (1a)-(1d) can be realized by two **224**'s and a **15** of Higgs fields, for symmetry breaking at scales M_G and M_B, and by a further nine **15**'s at M_A. To give masses to fermions and to break symmetry at M_W we need a **120** of Higgs fields giving a symmetric mass matrix, and have the option of adding a **105** of Higgs fields to give an antisymmetric contribution; this is analogous to the freedom of adding the **45** of Higgs fields in SU(5); we hope to analyze the fermion mass matrix for SU(15) in detail elsewhere.

In order to analyze the values of the mass scales M_G, M_B, and M_A we use a renormalization-group (RG) analysis with the normalization conventions for coupling constants that all generators are normalized[2] for SU(15) as in Eq. (3). We take as input the phenomenological values of $\alpha_{3c}(M_W)$, $\alpha_{2L}(M_W)$, and $\alpha_{1Y}(M_W)$. At M_A the appropriate matching conditions are

$$\alpha_{3c}^{-1}(M_A) = \tfrac{1}{2}\alpha_{6L}^{-1}(M_A) + \tfrac{1}{2}\alpha_{6R}^{-1}(M_A), \quad (6a)$$

$$\alpha_{2L}^{-1}(M_A) = \tfrac{3}{4}\alpha_{6L}^{-1}(M_A) + \tfrac{1}{4}\alpha_{3l}^{-1}(M_A), \quad (6b)$$

$$\alpha_{1Y}^{-1}(M_A) = \tfrac{9}{20}\alpha_{6R}^{-1}(M_A) + \tfrac{1}{10}\alpha_{1'h}^{-1}(M_A) + \tfrac{9}{10}\alpha_l^{-1}(M_A). \quad (6c)$$

In the present case, we shall assume $\alpha_{6L}(\mu) = \alpha_{6R}(\mu)$ for $M_A \le \mu \le M_B$.

At M_B, we have

$$\alpha_{6L}(M_B) = \alpha_{6R}(M_B) = \alpha_{1'h}(M_B) = \alpha_{12q}(M_B), \quad (7)$$

and finally, at M_G,

$$\alpha_{12q}(M_G) = \alpha_{3l}(M_G) = \alpha_{15}(M_G). \quad (8)$$

The RG equations, at one-loop order, have the form

$$\mu\, d\alpha_i(\mu)/d\mu = B_i \alpha_i^2(\mu), \quad (9)$$

and for the different subgroups, assuming n_f families and neglecting scalars,

$$B_{3c} = -\frac{1}{2\pi}(11 - \tfrac{4}{3}n_f), \quad (10a)$$

$$B_{2L} = -\frac{1}{2\pi}(\tfrac{22}{3} - \tfrac{4}{3}n_f), \quad (10b)$$

$$B_{1Y} = \frac{1}{2\pi}(\tfrac{4}{3}n_f), \quad (10c)$$

$$B_{6L} = B_{6R} = -\frac{4}{2\pi}(22 - \tfrac{1}{3}n_f), \quad (10d)$$

$$B_{1'h} = \frac{4}{2\pi}(\tfrac{1}{3}n_f), \quad (10e)$$

$$B_{3l} = -\frac{4}{2\pi}(11 - \tfrac{1}{3}n_f), \quad (10f)$$

$$B_{12q} = -\frac{4}{2\pi}(44 - \tfrac{1}{3}n_f). \quad (10g)$$

There is only one free mass scale in our model and we take this to be M_A. Given M_A, we may compute M_B, by substituting Eqs. (9) and (10) into Eq. (7); then we may compute M_G by Eq. (8). Typical results are given in Table I. The values of the standard-model coupling constants used in the estimate of Table I are[3] at $M_W = 81$ GeV, $\alpha_{3c}^{-1}(M_W) = 9.35$, $\alpha_{2L}^{-1}(M_W) = 29.1$, and $\alpha_{1Y}^{-1}(M_W) = 59.2$; the errors on these phenomenological values are approximately 10%, 2%, and 1%, respectively. Because these values have been used as input, our theory incorporates the correct value of $\sin^2\theta_W = 0.228$; also the matching conditions, Eq. (6) at M_A, Eq. (7) at M_B, and Eq. (8) at M_G, are fulfilled *precisely*. In SU(5), by contrast, there is the problem[3] that the extrapolations of α_{3c}, α_{2L}, and α_{1Y} do not fulfill the matching condition at M_G when one inputs the most recent data from low-energy experiment.

Now we turn to some phenomenological implications of our SU(15) grand unification. As already mentioned, proton decay is absent in the gauge sector which conserves B exactly. To prove this, note that representing a **15** by a single line with an arrow and a gauge boson by two lines with opposing arrows, every diagram has the property that any such line entering the diagram also leaves the diagram. The same argument extends immediately to the **224**'s and **15**'s of the Higgs sector. More interesting is that even with **120**'s and **105**'s of Higgs fields, as introduced above to give fermion masses, B conservation still holds because the fermions are in the fundamental representation. To check this, note that each component in these Higgs representations has a well-defined B assignment. Thus, the Yukawa couplings which provide quark and lepton masses do not contribute to proton decay.

As far as low-energy phenomenology is concerned, we first consider naturalness of the Glashow-Iliopoulos-Maiani (GIM) mechanism for suppression of the flavor-changing neutral currents (FCNC). In the present model there is a fully natural GIM suppression of FCNC be-

TABLE I. Some examples of results for M_B and M_G for different values of the input M_A.

M_A (GeV)	M_B (GeV)	M_G (GeV)
250	4.0×10^6	6.0×10^6
500	5.8×10^6	8.9×10^6
10^3	8.3×10^6	1.3×10^7
2×10^3	1.2×10^7	1.9×10^7

cause the quarks have been distributed between $SU(6)_L$ and $SU(6)_R$ precisely such that all q_L are in $SU(6)_L$ and all q_R ($\equiv\bar{q}_L$) are in $SU(6)_R$. This situation is quite similar to the generalization of natural GIM suppression operative in chiral color.[4]

At low energy (M_A) our model has a proliferation of gauge bosons at Superconducting Super Collider energy corresponding to the generators of $SU(6)_L\times SU(6)_R\times U(1)'_h\times SU(3)_l$. With respect to the standard model, these have quantum numbers as follows:

$$\mathbf{35}_L = \underset{g_L^{a\pm,0}}{(\mathbf{8},\mathbf{3})_0} + \underset{g_L^a}{(\mathbf{8},\mathbf{1})_0} + \underset{W_{L,q}^{\pm,0}}{(\mathbf{1},\mathbf{3})_0} \tag{11a}$$

$$\mathbf{35}_R = \underset{g_R^a,g_R^{\prime a}}{2(\mathbf{8},\mathbf{1})_0} + \underset{S_R^{a\pm}}{(\mathbf{8},\mathbf{1})_{\pm1}} + \underset{b_{R,q}}{(\mathbf{1},\mathbf{1})_0} + \underset{C_{R,q}^{\pm}}{(\mathbf{1},\mathbf{1})_{\pm1}} \tag{11b}$$

$$\mathbf{1}' = \underset{b_q'}{(\mathbf{1},\mathbf{1})_0} \tag{11c}$$

$$\mathbf{8}_l = \underset{W_l^{\pm,0}}{(\mathbf{1},\mathbf{3})_0} + \underset{b_l}{(\mathbf{1},\mathbf{1})_0} + \underset{X_l^{++},X_l^{+},\bar{X}_l^{-},\bar{X}_l^{--}}{(\mathbf{1},\mathbf{2})_{\pm3/2}} \tag{11d}$$

Let us *define* $g_R^{\prime a}$ to be orthogonal to the gluon g^a ($a=1$–8); then g^a is the diagonal sum of g_L^a and g_R^a. The usual $W^\pm$ are mixtures of $W_{L,q}^\pm$ and $W_l^\pm$. The neutral γ,Z^0 are the usual mixtures of W^0 [for $SU(2)_L$] and B [for $U(1)_Y$] and these in turn are combinations of $W_{L,q}^0$ and W_l^0 (for W^0) and $b_{R,q}$, b_q', and b_l (for B).

Beyond these established gauge bosons, our model predicts a variety of new forces at the scale M_A which could be as low as 250–2000 GeV. There are seven new color octets: one is the combination (g_5^a) of g_L^a and g_R^a orthogonal to g^a and is an octet of axigluons; the other six color octets are $g_L^{a\pm,0}$, $g_R^{\prime a}$, and $S_R^{a\pm}$. The mixture of $W_{L,q}^i$ and W_l^i orthogonal to W^i will be a new $SU(2)_L$ triplet $W^{\prime i}$. The charged color singlets $C_{R,q}^\pm$ couple only to quarks; the charged and doubly charged X_l^{++}, X_l^+, $\bar{X}_l^-$, and $\bar{X}_l^{--}$ couple only to leptons. One mixture of $b_{R,q}$ and b_q', orthogonal to B, forms B_q coupling only to quarks; the final new gauge boson is a neutral singlet B, a combination of $b_{R,q}$, b_q', and b_l.

At low energy this theory thus appears as a doubled chiral color theory[4] where the quantum numbers of the gauge bosons of $SU(3)_L\times SU(3)_R\times SU(2)_L\times U(1)_Y$ are "doubled"; there are in addition four charged gluon octets ($g^{a\pm},g_5^{a\pm}$), three singlets $C_q^\pm,B_q$ coupling only to quarks, and two $SU(2)_L$ doublets of gauge bosons (X_l^{++},X_l^+), $(\bar{X}_l^{--},\bar{X}_l^-)$ coupling only to leptons.

Some of the phenomenology overlaps with that of chiral color and hence we know that the new "gluons" all have masses above 50 GeV.[5] The most interesting new low-energy phenomena peculiar to the present theory may be the leptonic processes mediated by the X_l particles. In Møller scattering $e^-e^-\rightarrow e^-e^-$ at the proposed SLAC TeV Linear Collider, for example, this doubly charged gauge boson will appear as an exotic resonance.

The present work has important differences compared to Ref. 6 which was one inspiration of the present work. In Ref. 6, integrally charged quarks are initially emphasized but fractionally charged quarks are also discussed; it is for the latter case that the SU(15) and $SU(12)_q\times SU(3)_l$ assignments of quarks and leptons coincide with ours. Our physics is quite different because of the low-energy symmetry-breaking scale M_A, and the much lower unification scales M_B,M_G. Also, Ref. 6 does not note the exact baryon-number conservation of the gauge sector.

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