Comment on "Exact Solution for Diffusion in a Random Potential"

Zeldovich *et al.*[1] have argued that, for diffusion on a lattice with random sources and sinks, the concentration at the origin, $P(0)$, scales as $\ln P(0) \sim \lambda^2 t^2$. At each lattice site there is a source of strength λV with V a Gaussian random variable with mean zero and variance unity. In the continuum limit, Tao[2] has shown that $\ln P(0) \sim \lambda^4 t^3/D$, where D is the diffusion coefficient. Tao's numerical simulations yield a slower growth law which is $\ln P(0) < t^{3/2}$. The purpose of this Comment is to show that all these results are explained by a Flory theory.

Diffusion with random sources and sinks is equivalent to a polymer in a random potential, cf. Tao[2] and Edwards and Muthukumar,[3] with $\ln P(0)$ equal to the free energy of a polymer constrained to return to the origin. Solutions to the polymer problem[4,5] yield the same results as Refs. 1 and 2.

The growth law of Ref. 1, $P(0) \sim \exp(\lambda^2 t^2/2)$, is due to the presence of extremely rare sources of strength $V = \lambda t$ that dominate behavior at time t. The probability that $V = \lambda t$ occurs at a site is roughly $\exp(-\lambda^2 t^2/2)$. For Tao's simulation this probability is 10^{-500}. Thus the asymptotic behavior cannot be seen in numerical simulation.

We have constructed a Flory theory for the equivalent polymer problem which takes into account the number of realizations, M. For a random walk of N ($=2Dt$) steps that visits s distinct sites in the presence of a Gaussian random potential the free energy is

$$F(t,s) = \pi^2 Dt/s^2 + s^2/4Dt - \pi - I_M(t,s) . \quad (1)$$

The first three terms describe a polymer on a 1D lattice[5] while $I_M(t,s)$ is an effective potential that arises from truncating the probability density for V in a way which reflects the fact that there are M realizations in the ensemble,[4,6]

$$I_M(t,s) = \ln\left[(2\pi)^{-1/2} \int_{-V^*}^{V^*} dV \exp(-\tfrac{1}{2} V^2 + \lambda t V s^{-1/2})\right] . \quad (2)$$

Here V^* is the expected largest value in a set of M realizations of V and is estimated from $M \operatorname{erf}(V^*/\sqrt{2}) \approx 1$. The partition function for the polymer is found from s^* that minimizes $F(t,s)$ at fixed t; $P(0) = \exp[-F(t,s^*)]/s^*$ is equivalent to the quantity studied by Tao. For an infinite number of realizations, $I_\infty(t,s) \sim t^2/s$ in agreement with Refs. 4. As t becomes large, s^* decreases as $1/t$ leading to Tao's result, $\ln P(0) \sim t^3$. If there is a lattice cutoff, s^* decreases to it after which $\ln P(0) \sim t^2$ in agreement with Zeldovich *et al.* For a finite number of realizations, $I_M(t,s) \sim t/\sqrt{s}$ as t becomes large and s^* becomes independent of t, $\ln P(0) \sim t$.

We have repeated Tao's numerical simulation. We show $\ln P(0)$ vs $t^{3/2}$ in Fig. 1 with crosses. Our numerical results are in excellent agreement with Tao's. The lower solid curve in Fig. 1 is found from Eqs. (1) and (2) with $V^* = 3$ and compares well with the simulation.

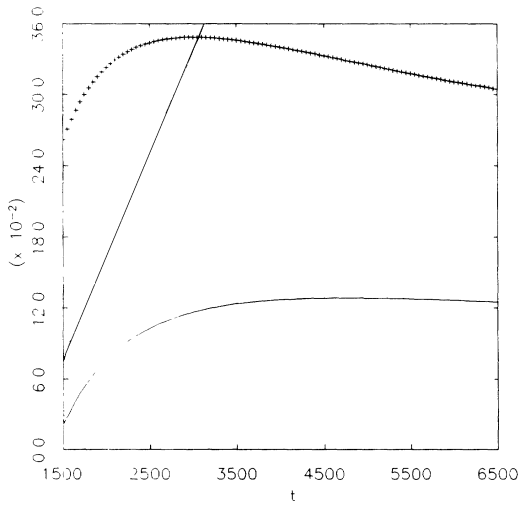

FIG. 1. The concentration at the origin vs time plotted in the form $\sqrt{D}[\ln P(0)]/\lambda^2 t^{3/2}$ vs t. The crosses are the numerical simulation using the same parameters as Tao ($M = 1000$, $D = 0.1$, $\lambda = 0.01$). The upper smooth curve is the result for an infinite number of realizations. The lower smooth curve is the Flory theory for the parameters of the simulation.

The disagreement between the numerical work and the analytic result is due to finite-realization effects. The Flory theory provides a useful framework for obtaining exact asymptotic results and understanding simulations and experiments.

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