Evidence for Organized Small-Scale Structure in Fully Developed Turbulence

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The intermittent fine structure of fully developed turbulence is studied by means of a visualization experiment. Characteristic locally ordered flow patterns, tentatively identified as layered vortex sheets, are observed.

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The theory of fully developed turbulence invokes a cascade of dynamical processes by which large-scale energy of motion is converted into sequentially smaller scales until viscous dissipation becomes dominant.[1] An important refinement of this picture follows from the observation[2] that by the time the energy has trickled down to the dissipative scales, it is distributed very nonuniformly in space. It is thought that this *intermittency* of the small-scale motion is a profoundly significant feature of fully developed turbulence, which should be at the core of any theory. This view has given rise to various scaling models of turbulence[3-8] in which the cascade is assumed to be non-space-filling, leading to a fractal or multifractal description of the flow field. It has also inspired a number of attempts to depict the turbulence as a superposition of isolated flow structures such as vortex tubes or sheets.[9-13]

Given the enormous amount of theoretical attention that has been devoted to the phenomenon of fine-scale intermittency, remarkably little is known about the actual nature of the small-scale flow fields. Measurements of the probability distribution of various velocity derivatives[2,14] provide indirect evidence of intermittency, but tell one nothing about what kind of dynamical structures are involved. Other data,[15-18] consisting primarily of observations that signals from hot-wire probes show intermittent behavior when subjected to high-pass filtering, have given only limited information about the geometry of the small-scale activity. Kuo and Corrsin,[15] in the most ambitious experiment of this type, have observed that the extent of the typical active region is large compared with the fine-structure scale itself, and suggest that the active regions are more nearly rodlike in shape than spherical or slablike.

The present paper reports the results of an attempt to learn more about this fascinating problem by means of a very simple visualization experiment. Turbulence is generated in a tank of water 26 cm on a side and 54 cm high by two horizontal grids 25.5 cm on a side and spaced 31.5 cm apart in the vertical direction. The grid assemblage is oscillated up and down a distance of 4.4 cm to agitate the fluid. Oscillating-grid turbulence is of interest as a model of certain geophysical and industrial mixing processes, and has been well studied.[19-23] It is an efficient way of generating random motion, and even with our modest setup it is not difficult to reach integral-scale Reynolds numbers of 10^3, and microscale Reynolds numbers of 10^2, implying a moderately well-developed inertial cascade.[24] In contrast to duct turbulence, there is almost no mean flow, so that the visualization of small-scale structure becomes much easier.

To observe the turbulence field, the water is seeded with tiny crystalline platelets,[25] approximately 10^{-5} cm thick and 10^{-3} cm across.[26] Contrary to common belief, the motion of such platelets in a uniform shear is quite irregular. However, the theory of Savas[27] shows that, when subjected to a shearing field du_x/dz, a random distribution of such flakes will, in a characteristic time $(du_x/dz)^{-1}$, develop a substantial alignment, such that their normals point nearly in the $\hat{\mathbf{z}}$ direction. The seeded fluid is illuminated by a 1.5-cm-thick vertical sheet of incoherent, parallel light, which is then viewed or photographed at right angles. The interpretation of images produced by reflecting flakes is quite complicated, but flakes have the two important advantages of responding to the instantaneous state of the fluid, especially in regions of strong shear, and of reflecting small-scale variations in the flow field with maximal sensitivity. Local fluctuations in the velocity field will give rise to changes in the flake orientation on the same scale, leading to sharp local variations in the amount of light reflected to the observer. Consequently, one expects the observed scale of intensity variations to be at least qualitatively indicative of the smallest scale on which the fluid is dynamically active.

The main result obtained from this experiment is the striking and unexpected feature that much of the fine-scale structure appears to possess a substantial degree of local order. As seen in Fig. 1, this shows up in the form of isolated regions containing several reflecting bands in parallel, a phenomenon which can only arise as the result of a characteristic localized flow pattern which occurs over and over again throughout the fluid. The parallel lines often appear to form tubelike structures, and are observed to appear and disappear locally in the flow at a rate determined by the turbulent activity, and to fade away smoothly rather than be disrupted by still-smaller-scale motion. This, together with the fact that they generally represent the smallest scales present, suggests that they occur near the dissipative end of the cascade.

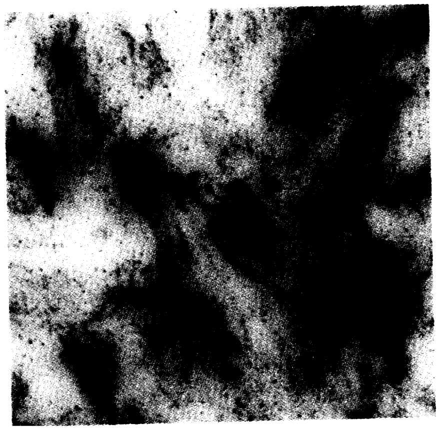

FIG. 1. Photographic negative of reflecting-flake pattern in turbulent flow. Dark regions are areas where flakes are preferentially oriented to reflect light coming from the left of the figure towards the observer. The image corresponds to a 5-cm by 5-cm area of the fluid, and was photographed from 70 cm away.

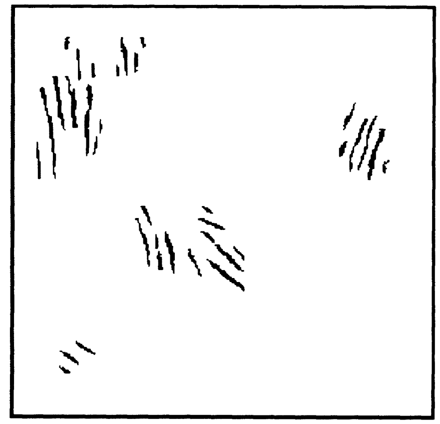

FIG. 2. Image shown in Fig. 1, signal processed to isolate locally ordered structure.

The observed structures must reflect an evanescent but pronounced local oscillation in the intensity or the direction of the shear. Such an effect could be produced, for example, by rows of vortices arising from the Kelvin-Helmholtz instability in transient vortex sheets, by Taylor-Görtler cells originating in curved shear layers, or by spiral vortices. Although all of these have been put forth as possibly important in turbulence modeling,[9-13] an interpretation in terms of spiral vortices or, more generally, layered vortex sheets seems to be favored. Two-dimensional turbulence calculations show spiral vortices being generated naturally during vortex coalescence and rollup, and other layered-vortex-sheet structures arising from the deformation and squeezing together of regions of high vorticity by larger-scale fields.[28,29] Similar effects can be expected to occur to some extent in three dimensions. It is also interesting to note that the most recent and sophisticated structural model of turbulence invokes spiral vortices as its basic element.[12] In contrast to this, the Kelvin-Helmholtz instability at short wavelengths competes with the viscous spreading of the shear layer. On the basis of earlier theoretical studies[30] we conclude that to produce the observed structures through this process would require turbulent velocities at least an order of magnitude larger than those actually present. Similarly, the fact that the banding is observed to lie along the axis of a tube, rather than across it, seems inconsistent with a Taylor-Görtler or Taylor-Couette mechanism.

Additional support for the layered-vortex-sheet hypothesis is provided by a subsidiary experiment in which vortex rings are created in the tank and visualized in the same manner. The spiral vortex that makes up the core of such a ring soon after it is formed[31] produces a pattern very similar to those seen in Fig. 1. Observed by eye, the dynamical behavior of the ring-visualization pattern closely resembles that of the turbulence fine structures. Similar effects can be observed by moving a straightedge through the water to create starting and stopping vortices.

The organized fine structure varies with the turbulence parameters. Although our experiment is primarily qualitative in character, we have essayed to quantify this relationship by estimating the characteristic smallest spacing between the bright bands, the average extent of the structures perpendicular to the direction of the bands, and the fraction of the image area occupied by the organized structures. Given the large variability from structure to structure and image to image, as well as the difficulty of deciding what in fact constitutes a locally organized pattern, these numbers should be taken as indicative only. Images were selected from a sizable data base of photographs, digitized, and then analyzed in two ways. The first method was simply to take measurements from the original image, making subjective judgments as to what constituted an ordered structure. The second approach was to apply a series of image-processing steps, including local Fourier filtering and thresholding to isolate local spatial order, various point operations, and the subsequent removal of obvious artifacts. With this somewhat more objective procedure, the image of Fig. 1 is converted to that shown in Fig. 2. Numbers are then read off the processed image. As can be seen from Fig. 3, the two procedures yield rather good agreement.

Oscillating-grid turbulence can be modeled[21,22] by considering the grid as a plane source of turbulence with a given integral scale l and rms velocity μ. Away from the grid region, the turbulence is assumed to be undergoing decay with time such that $l \propto t^{1/2}$ and $\mu \propto t^{-1/2}$, with the integral-scale Reynolds number $R_l = \mu l/v$ remaining constant.[1] Unlike the situation in a typical duct experiment, here the turbulence *diffuses* away from the grid.

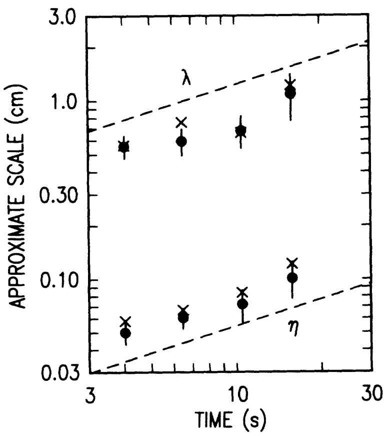

FIG. 3. Structure dimensions compared with estimated turbulence scales λ and η. The lower points represent half the measured band-to-band spacing, the upper points the perpendicular extent of the structures. Dots and error bars refer to values obtained from the actual images, crosses to values obtained from the image-processed versions.

Hence, instead of the usual $l \propto d^{1/2}$, $\mu \propto d^{-1/2}$ variation with distance d from the grid, a $l \propto d$, $\mu \propto d^{-1}$ dependence is observed.[20] We have extended this model to the two-grid case and to the case where the grid oscillation is turned off suddenly. In the latter, the turbulence becomes quite homogeneous after a short interval and decays in the universal fashion, with an effective time origin close to the time when the grid oscillation is turned off. The proportionality constants which determine the exact values of l and μ are determined by the grid geometry as well as the frequency and amplitude of the stroke. We derive these constants from the results of Hopfinger and Toly,[20] to whose grid geometry and running conditions the present experiment is closely matched. Given l and μ (or equivalently R_l), the Taylor microscale λ and the Kolmogorov scale η can be estimated from the standard relations $\lambda = (15/A)^{1/2} l/R_l^{1/2}$ and $\eta = A^{-1/4} l/R_l^{3/4}$, where A is an unknown constant of order 1.[1]

Figure 3 shows measurements made as a function of time in decaying turbulence with $R_l \simeq 1220$ and $R_\lambda \simeq 130$. The dashed lines are the various turbulence scales evaluated as discussed above, on the assumption that $A = 1$. Several important qualitative conclusions can be drawn from this figure. The dimensions of the organized structures are seen to increase in a manner similar to that of the turbulence scales. The smallest dynamically active scale, which one might reasonably estimate as, say, half of the band-to-band distance, is in fact close to the estimated Kolmogorov length. The overall extent of the active regions is seen (at least at these Reynolds numbers) to be comparable to the Taylor microscale. All of these findings support the conclusion that a dynamically significant feature of small scale turbulence is indeed being observed.

Are these locally organized structures the source of the fine-scale intermittency detected in previous experiments? One notes that their geometry seems quite consistent with the observations of Kuo and Corrsin, and that a high-pass probe passing through one of the images would, in fact, experience "bursts" of activity similar to those observed in hot-wire experiments. An estimate of how much space these structured regions occupy is, of course, important in deciding this question, but such an estimate is subject to considerable uncertainties. The structures typically take up of order 10%–20% of the images that were analyzed, but the images themselves were chosen with a bias towards maximizing this number. On the other hand, the visualization technique is orientation sensitive, and will at best see only about a third of the structures present. There also may well be atypical, unresolved, or below-threshold structure which is not adequately visualized. Given all this, we estimate that somewhere between 5% and 50% of the fluid volume is occupied by organized small-scale structure. Past experiments to determine the fraction of the fluid volume occupied by intermittent fine structure have been reviewed and reinterpreted by Sreenivasan.[18]

For $R_\lambda \simeq 100$ such experiments typically find values of order 20%. Thus the organized structures observed here seem at the least to be an important component of previously observed fine-scale intermittency.

In summary, we have obtained visual evidence for the existence in fully developed turbulence of characteristic small-scale flow patterns which are tentatively identified as layered vortex sheets. The pattern geometry is consistent with previously observed properties of intermittent bursts. The structures are found to scale with the turbulence scales and to exhibit a spatial variation limited by the Kolmogorov length. At microscale Reynolds numbers of order 100, this organized fine-scale structure is found to occupy an appreciable fraction of the total fluid volume.

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[1]See, for example, J. O. Hinze, *Turbulence* (McGraw-Hill, New York, 1975); H. Tennekes and J. L. Lumley, *A First Course in Turbulence* (MIT, Cambridge, 1972). Our notation follows that of the latter.

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[24]In characterizing the turbulent state it is desirable to use a Reynolds number defined in terms of the properties of the turbulence itself, rather than in terms of the generating mechanism. Thus, it is customary to use the rms value of the random velocity component $\mu = \langle u^2 \rangle^{1/2}$ for the characteristic velocity which enters the Reynolds number. The characteristic length is conventionally chosen to be either the integral scale $l = \int_0^\infty \rho(r) dr$, or the microscale λ, defined by $\rho(r) = 1 - r^2/\lambda^2$, where $\rho(r)$ is the correlation function $\langle u(0)u(r) \rangle / \langle u^2 \rangle$. For further discussion, see Refs. 1.

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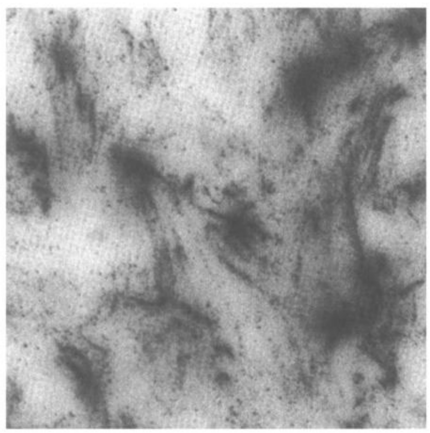

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