Neutron-Proton Interactions in the Mass-80 Region

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In a model study, yrast band-crossing structures are shown to be qualitatively changed by residual neutron-proton interactions. This could explain some anomalies recently observed for light Kr isotopes. Large effects on the yrast-band g factors are predicted.

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Much data now exist (e.g., Refs. 1 and 2) on rotational states of deformed nuclei in the neutron-deficient mass-80 region where neutrons and protons occupy the same shells. Alignments occur which correspond to unpaired neutrons and protons in the same ($g_{9/2}$) orbitals and thus the neutron-proton interaction H_{np} should play an important role. Cranked-shell-model (CSM) calculations generally take the same or very similar deformations for neutrons and protons but otherwise regard them as independent particles so no attempt is made to include explicitly the short-range n-p interactions which, in the identical-nucleon case, lead to the pairing fields. Despite this, a reasonable description of properties in this region has often been obtained (see, e.g., Ref. 3). Some recent experimental observations[2] of crossing frequencies, however, have proved difficult to explain within the conventional CSM approach and the aim of this Letter is to investigate possible effects of H_{np} in a single ($g_{9/2}$) shell.

For a pair of identical nucleons with $j=\frac{9}{2}$ (isospin $T=1$) the allowed spin states are $J=0$, 2, 4, 6, 8. An attractive δ-function force (or any other reasonable residual interaction) depresses the $J=0$ state considerably below the others [see Fig. 1(a)] and this is the justification for the simplifying monopole pairing force.[4] The

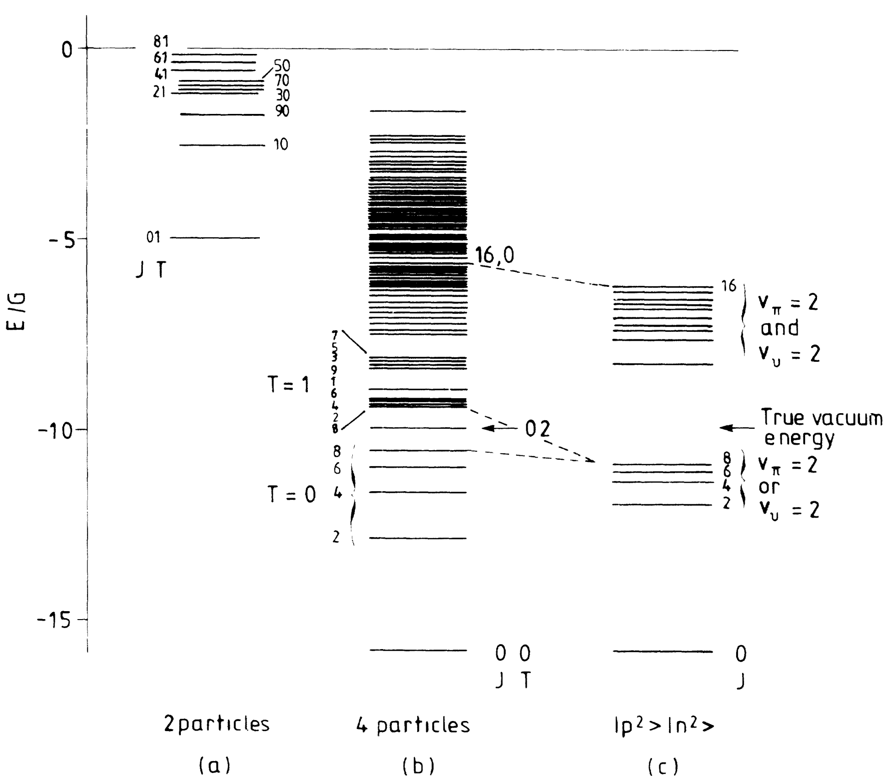

FIG. 1. (a) The spectrum of two $g_{9/2}$ nucleons interacting through a charge-independent δ-function force is shown in units of G [see Eq. (2)]. The spin J and isospin T of the states are indicated. The same force leads to the four-nucleon spectrum shown in (b). The lowest state allowed for four identical nucleons ($T=2$) is marked by the arrow. The states below this all have $T=0$ and are allowed only in the $2n$-$2p$ system. (c) The much simpler $2n$-$2p$ spectrum obtained if H_{np} is set to zero. [The vacuum energy has been set equal to that in (b).]

n-p system, however, has the extra $(T=0)$ $J=1$, 3, 5, 7, 9 states of which the $J=1$ and 9 are lowest, and so we have chosen to do our calculations with *no truncation* of the two-body matrix elements. Since we shall also be discussing the relatively small effects of differences of crossing frequencies between calculations with and without H_{np}, we do not wish to introduce any errors through the use of the Hartree-Fock-Bogoliubov (HFB) approximation[5] and we shall, therefore, as far as possible, perform exact diagonalizations of our Hamiltonian. However, the number of states increases rapidly with particle number if both neutrons and protons are admitted [e.g., see Fig. 1(b)] and considerable mixing between these states occurs when a deformed mean field and cranking are applied, i.e., for the Hamiltonian[6]

$$H' = -g\sum_{i<j}\delta(\mathbf{r}_i-\mathbf{r}_j) - \sum_i [4(4\pi/5)^{1/2}\kappa Y_{20}(\hat{\mathbf{r}}_i)+\omega j_i^x]\,. \qquad (1)$$

In this case the only good quantum numbers are the total isospin T (the sum over i includes both neutrons and protons and we ignore the Coulomb interaction) and the signature α of the many-particle state, which is defined[5] by the rotation $R_x(\pi)\psi=\exp(-i\pi J_x)\psi=\exp(-i\pi\alpha)\psi$. We shall, therefore, initially discuss the cranked spherical problem in which J and M_x are also good quantum numbers.

The matrix elements of the δ-function interaction may be expressed in terms of the energy

$$G=\frac{g}{4\pi}\int_0^\infty R^4(r)r^2dr\,, \qquad (2)$$

where R is the single-particle radial wave function. The two-particle δ-function energies are shown in Fig. 1(a) in units of G. They lead to the complicated four-particle spectrum shown in Fig. 1(b). All these states are allowed for the $2n+2p$ system $(T_3=0)$. The lowest state available to four identical nucleons $(T=2)$ is indicated by the arrow. With $H_{np}=0$ the lowest $2n$-$2p$ state would simply have twice the energy of the two-particle $T=1$ ground state. This energy is $-10G$ compared with $-15.93G$ obtained with $H_{np}\neq 0$.

Without the *n-p* force all the states of the $2n$-$2p$ system are products of the $T=1$ states whose energies are shown in Fig. 1(a). We show in Fig. 1(c) the resulting spectrum $E(v,J)=E(v_\pi,J_\pi)+E(v_\nu,J_\nu)$, where $v=v_\pi+v_\nu$ is the seniority[7] and J is the angular momentum of the states $(\mathbf{J}=\mathbf{J}_\pi+\mathbf{J}_\nu)$. (Note that isospin is not a good quantum number for the product state.) In order to compare with the energies of the four-particle system *with* an *n-p* interaction [shown in Fig. 1(b)], we have taken the same vacuum energy for the two sets of states. The product spectrum is highly degenerate if both neutron and proton states have seniority 2. The neutron-proton forces lift most of these degeneracies and spread the spectrum out as seen in Fig. 1(b). A more interesting degeneracy exists in the $v=2$ part of the product spectrum. Each state here is doubly degenerate since the broken pair may be either two neutrons or two protons. (The antisymmetric combination is a pure $T=1$ state, while the symmetric combination is mainly $T=0$ with some $T=2$ admixture.) H_{np} again lifts this degeneracy so that the analogous spectrum in Fig. 1(b) has both a $T=0$ and a $T=1$ level for spins $J=2$, 4, 6, 8 corresponding to the degenerate $v=2$ doublets in Fig. 1(c). This is indicated by the dashed lines for the $v=2$ states with $J=8$. The $T=0$ states are spread out by H_{np} while those with $T=1$ have been squeezed together at a somewhat higher energy, as have the odd-J states with $T=1$.

For a cranked system, certain states are more important than others. Those of greatest interest will be those which will become lowest in energy at higher rotational frequencies and thus become yrast or near yrast. In the absence of a deformation, the cranking term $-\omega J_x$ in our Hamiltonian may be replaced by $-\omega M_x$ with $-J\leq M_x\leq J$. Thus the most aligning substate from a

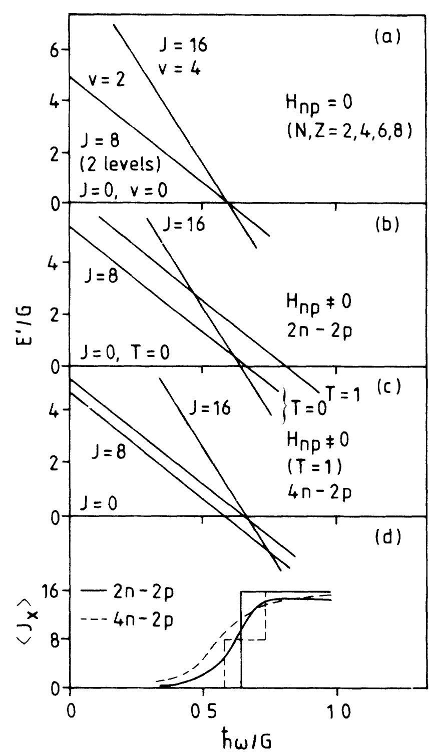

FIG. 2. The routhians of the cranked spherical problem $(\kappa=0)$ are shown as a function of the cranking frequency for the most aligning states with one broken pair $(J=8)$ and two broken pairs $(J=16)$. (a) The results for $H_{np}=0$. These results are the same for any even value of N and Z. If $H_{np}\neq 0$, the results become number dependent and (b) and (c) show results for the $2n$-$2p$ and $4n$-$2p$ systems, respectively. (d) The alignments corresponding to the yrast states of (b) and (c) are shown by the solid and dashed step functions. The smooth curves are the same quantities for $\kappa=2.4$.

given level will have a routhian $E'=E^*-\omega J$ and will cross the spin-zero vacuum at $\omega=E^*/J$. The most aligning states with one broken pair have $J=8$ and that with two broken pairs has $J=16$. (Note that four *identical* nucleons give a maximum $J=12$.) In Fig. 2(a) we show these cranked levels for $H_{np}=0$. In this case the two $J=8$ states ($v_\pi=2$ or $v_\nu=2$) are degenerate ($E^*=4.8G$) and cross the $J=0$ vacuum at the same frequency $\omega=0.6G$. More interestingly the $v=4$ state with $J=16$ has just twice that excitation energy and thus crosses at precisely the same frequency. These results are independent of the (even) neutron and proton numbers since in the case of $H_{np}=0$, the δ-function interaction separately conserves neutron and proton seniorities.[7]

In a calculation including a deformed field the states of different J (but the same T) will mix but we still find $J=8$ to be the dominant component in the configuration with two aligned particles and $J=16$ to be dominant for four aligned particles. It is therefore important to see how much their excitation energies are shifted when H_{np} is switched on. We see from Figs. 1(b) and 1(c) that the shifts in E^* are relatively small compared with the large increase in binding energy. These small shifts can, however, lead to significant changes in the crossing structures as we shall now see.

In Fig. 2(b) we show the cranked $2n$-$2p$ results with $H_{np}\neq 0$. The two 8^+ levels now have different energies; furthermore, the 16^+ level no longer has twice the energy of the lowest 8^+ and so does not cross the vacuum at the same frequency. Indeed, it crosses the vacuum *first* so the four-aligned-particle state becomes yrast *before* the two-aligned-particle state. Figure 2(c) shows the analogous results for the $4n$-$2p$ system (or vice versa). We see that in this case one of the 8^+ states crosses the vacuum before the 16^+ state. Thus we now have a strong particle-number dependence in the spectra. This arises because H_{np} does not conserve seniority.

In Fig. 2(d) the step functions show the alignment $\langle J_x\rangle$ of the yrast bands for the above two cases. For the $4n$-$2p$ system (dashed line) there are two alignment changes of $8\hbar$ each separated in frequency, whereas for the $2n$-$2p$ system (solid line) all the $16\hbar$ increase in alignment occurs at a single frequency. This behavior is similar to that observed in the light Kr isotopes,[2] where for ^{76}Kr a double alignment is seen, but in ^{78}Kr two separate alignments occur separated in $\hbar\omega$ by 0.35 MeV. Although the details of this effect will depend on the inclusion of other shells and the deformation, etc. (see below), we believe that the reason for this behavior could be this lowering of the configuration containing four aligned nucleons ($2n+2p$) for some particle numbers.

An important consequence of including H_{np} is that the two-aligned-particle state is a linear superposition of aligned neutrons and aligned protons, which means that the g factor will roughly be the average of the aligned $2n$ and $2p$ g factors. In a case like ^{78}Kr where two separate alignments occur and a few states may be unambiguously ascribed to the two-aligned-particle configuration, a measurement of the corresponding g factors would give a measure of the importance of the n-p correlations. A CSM calculation would of course predict a magnetic moment of a state of spin I given by

$$\mu/\mu_N=gI\approx g_R(I-\langle J_x\rangle)+g_{\text{val}}\langle J_x\rangle\,, \tag{3}$$

where $g_R=Z/A$ is the core g factor and g_{val} is the valence-nucleon g factor which would be purely a neutron or proton term. In our calculation g_{val} would have some average value. Specifically for the case we have considered above, we would have CSM values for aligned neutrons $g_{\text{val}}=g_n=-0.43$ and for protons $g_{\text{val}}=g_p=1.51$. These are, of course, just the Schmidt values. With n-p correlations we have[8] for a valence-particle state of total angular momentum J

$$g_{\text{val}}=\tfrac{1}{2}(g_n+g_p)+\tfrac{1}{2}(g_p-g_n)\sum_{J_n,J_p}c^2(J,J_n,J_p)\left[\frac{J_p(J_p+1)-J_n(J_n+1)}{J(J+1)}\right], \tag{4}$$

where c are the amplitudes of the state with neutron and proton spins J_n and J_p. (In the aligned configuration $\langle J_x\rangle=J$.) For the $2n$-$2p$ system this simplifies to $g_{\text{val}}=\frac{1}{2}(g_n+g_p)=\bar{g}=0.54$. This is a general result for $N=Z$ with a charge-independent Hamiltonian.[7] For the $4n$-$2p$ system our calculations yield $g_{\text{val}}=\bar{g}+0.046$ and for the $4p$-$2n$, $g_{\text{val}}=\bar{g}-0.046$. We see that these latter results differ little from $\bar{g}$. This should be true more generally so long as valence nucleons occupy the same shells, since the aligned angular momentum will then always be split to some extent between neutrons and protons. Thus the inclusion of n-p correlations gives a significant difference in the valence contribution to the g factors of physical states based on the two-aligned-particle intrinsic state. We are currently considering which nuclei might be most accessible to these difficult measurements. In the case of ^{82}Sr, the yrast 8^+ magnetic moment does indeed appear to be an average of the neutron and proton moments.[9] Unfortunately, data on states throughout the band-crossing region are not available.

We should now consider possible deformation effects. In Fig. 2(b) the vacuum, the lowest 8^+ state, and the 16^+ state all have $T=0$ and $\alpha=0$, and although a deformation will mix them with states of higher energy, the yrast structure will still be essentially determined by these most aligning states. We have calculated the cranked $T=0$, $\alpha=0$ spectrum for the $2n$-$2p$ system for a prolate deformation with $\kappa=2.4G$. In Fig. 2(d) we show (solid line) the corresponding alignment. The sudden jump has simply been smoothed out by the interband interactions. For the $4n$-$2p$ system we have performed a weak-coupling calculation[10] and show the resulting yrast alignment by the dashed curve in Fig. 2(d). Unfortunately the two separate alignments seen in the spherical case (and in most experimental data) have been washed out into a single broad alignment due to the artificially strong interactions in our model. These interactions increase with increasing κ (Ref. 11) but our κ value is reasonable, corresponding to $\beta_2 \approx 0.2$ if we take $G \approx 0.58$ MeV.[12] The problem arises since in a realistic treatment of the mass-80 region one should include the $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$ shells. These substantially reduce the $(g_{9/2})^n$ configurations in the vacuum compared with our model, which is purely such a configuration. This consequently reduces the coupling of the vacuum to the higher states with two aligned $g_{9/2}$ nucleons since the deformed single-particle field $(\sim Y_{20})$ can mix the states only through that component of the vacuum which already contains a $(g_{9/2})^2$ term. We hope to include effects of extra shells in a truncated basis.

In conclusion, we have shown that a number dependence of the crossing frequencies can arise solely as a result of n-p correlations, irrespective of any shape or other structural changes. Our model displays features qualitatively similar to those observed in the light Kr isotopes (though our basis is not sufficiently large to give a quantitative comparison). The predicted correlations should manifest themselves in the magnetic dipole moments of the yrast levels in a region dominated by configurations containing just two aligned nucleons.

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