Simple and Predictive Model for Quark and Lepton Masses

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A model is proposed which predicts that m_t is large, that one family (e, u, and d) has only radiatively generated—and hence small—masses, and that at the unification scale $m_\tau^0 \simeq m_b^0$ but $m_\mu^0 \neq m_s^0$ and $m_e^0 \neq m_d^0$. An essential feature of the model is a close connection between the pattern of breaking of unified gauge symmetry at large scales and the pattern of fermion masses at low scales, which gives a group-theoretical understanding of many features of the quark and lepton masses.

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The pattern of quark and lepton masses and mixings is a long-standing puzzle. There has been no shortage of interesting and plausible speculations on this problem, but all seem to fall short in one way or another. The list of promising theoretical ideas includes "radiative mass hierarchy,"[1] the "Fritzsch form,"[2] the proportionality of up- and down-quark masses,[3] "factorization" of tree-level mass matrices[4] (i.e., $m_{ij}^{\text{tree}} \propto a_i b_j$), and unification mass relations.[5] Unfortunately, virtually all models based on these ideas are liable to one or more of the following criticisms: (1) lack of predictions, (2) bad predictions, or (3) complicated or *ad hock* assumptions.

In this Letter we propose a model which, though incorporating many of the ideas mentioned above, largely escapes these types of criticisms. It is based on the group E_6. However, for ease of exposition we present an SO(10) version first as the group theory of SO(10) may be more familiar to readers. The extension to E_6 is trivial and amounts to introducing one new parameter. The model is to be regarded as a model of the *tree-level masses of the charged quarks and leptons.* We show how radiative fermion masses might arise simply, but we do not commit ourselves to a particular scheme. We divide the fields of the theory, then, for purposes of discussion, into those that are directly relevant to the origin of the tree-level masses of charged quarks and leptons, and those which are not but which may be relevant to other issues such as right-handed neutrino masses, radiative fermion masses, and the breaking of E_6 [or SO(10)] down to the standard model. The latter we call simply the "additional fields." The Yukawa and fermion mass terms L_{Yuk} we divide into $L_0 + L_{\text{AF}}$, where AF stands for additional fields. The fermion content of L_0 consists of three ordinary families in spinor representations, denoted $\mathbf{16}_i$, $i=1,2,3$, and an extra "mirror" or "vectorlike" pair of families, denoted $\mathbf{16}+\overline{\mathbf{16}}$ (without subscripts). These mirror families are the only nonminimal feature of our model. The Higgs-field content of L_0 consists of an adjoint representation, denoted $\mathbf{45}_H$, and a complex vector representation denoted $\mathbf{10}_H$. The $\mathbf{10}_H$ contains two neutral components that acquire SU(2)×U(1)-breaking vacuum expectation values (VEV's) denoted v [in the 5 of SU(5)] and v' [in the $\bar{5}$ of SU(5)]. Because the $\mathbf{10}_H$ is complex, $|v| \neq |v'|$ in general. Our whole discussion of tree-level fermion masses and our predictions follow just from these three terms,

$$L_0 = M\,\mathbf{16}\,\overline{\mathbf{16}} + \sum_{i=1}^{3} b_i\,\mathbf{16}_i\,\overline{\mathbf{16}}\,\mathbf{45}_H + \sum_{i=1}^{3} a_i\,\mathbf{16}_i\,\mathbf{16}\,\mathbf{10}_H + \text{H.c.} \tag{1}$$

This is the most general form allowed by SO(10)×K, where K is a symmetry under which the fields transform by the following phases: $\mathbf{16} \to \alpha\,\mathbf{16}$, $\overline{\mathbf{16}} \to \alpha^*\,\overline{\mathbf{16}}$, $\mathbf{45}_H \to \alpha\,\mathbf{45}_H$, $\mathbf{10}_H \to \alpha^*\,\mathbf{10}_H$, and $\mathbf{16}_i \to \mathbf{16}_i$. (In the E_6 model we will take K to be Z_2, i.e., $\alpha = \alpha^* = -1$. Here that particular choice would allow a $\mathbf{16}_i\,\mathbf{16}\,\mathbf{10}_H^*$ coupling unless it were forbidden by, say, a Peccei-Quinn symmetry.) This symmetry prevents terms like $\sum_{i,j} f_{ij}\,\mathbf{16}_i\,\mathbf{16}_j \times \mathbf{10}_H$ which would render the entire quark and lepton mass matrices "uncalculable" free parameters. The fact that the Yukawa couplings a_i and b_i are vectors and not matrices in the three-dimensional family space of the $\mathbf{16}_i$ (which is akin to the idea of factorization[4] mentioned above) leads directly to the tree-level masslessness of one generation as may be easily seen as follows. Obviously, without loss of generality, one may orient the axes in family space so that b_i points in the "3 direction" and a_i in the "2-3 plane;" thus $b_i = (0,0,b)$ and $a_i = (0, s_\theta, c_\theta)a$, with $s_\theta \equiv \sin\theta$, $c_\theta \equiv \cos\theta$. Clearly the family $\mathbf{16}_1$ does not then appear in Eq. (1) and so remains massless at tree level.

The first two terms in Eq. (1) make a mirror pair of families superheavy. (Both M and $\langle \mathbf{45}_H \rangle$ are assumed to be of the grand unification scale.) If $\langle \mathbf{45}_H \rangle$ were zero then the superheavy mirror pair of families would be just $\mathbf{16}+\overline{\mathbf{16}}$. But since $\langle \mathbf{45}_H \rangle \neq 0$ the second term in Eq. (1) leads to mixing of the $\mathbf{16}_i$ and the $\mathbf{16}$. Thus three families which are mixtures of particles in the $\mathbf{16}_i$ and the $\mathbf{16}$ remain without superheavy masses—these are the known light families—and only acquire tree-level masses as a result of SU(2)×U(1) breaking *via* the third term of Eq. (1). Thus our computation of the quark and lepton mass matrices is in two steps. First, we identify which quarks and leptons have superlarge masses, and which ones constitute the three light families. Next, from the third term of Eq. (1) we find the $SU(2)\times U(1)$-breaking masses of those light families.

A very important point is that the $\langle \mathbf{45}_H\rangle$, which breaks SO(10), couples directly to the fermions and thus SO(10)-breaking effects enter our tree-level mass relations. This does not happen in models with only the minimal set of fermions since $\mathbf{16}\times\mathbf{16}$ does not contain the adjoint (**45**).

Since the $\mathbf{45}_H$ is in the adjoint representation of SO(10) its VEV is a linear combination of SO(10) generators: $\langle \mathbf{45}_H\rangle = \sum_a C_a\lambda^a$. The group SO(10) contains the subgroups $SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_X$. There is, consequently, a two-dimensional space of directions that $\langle \mathbf{45}_H\rangle$ may point in [without breaking $SU(3)_c\times SU(2)_L$] spanned by the generators of $U(1)_Y\times U(1)_X$. We define Ω and z implicitly by

$$\langle \mathbf{45}_H\rangle = \tfrac{1}{5}\,\Omega(X+6zY/2)\,. \tag{2}$$

$Y/2$ is the conventionally normalized weak hypercharge, and X is normalized so that under $SU(5)\times U(1)_X$ the $\mathbf{16}\rightarrow \mathbf{10}^1+\bar{\mathbf{5}}^{-3}+\mathbf{1}^5$. Denoting generators normalized by $\mathrm{tr}_{16}\lambda^2=\frac{1}{2}$ by a tilde one has

$$\langle \mathbf{45}_H\rangle = 4\Omega[(\tfrac{2}{5})^{1/2}\tilde{X}+z(\tfrac{3}{5})^{1/2}\tilde{Y}/2]$$
$$= -4\Omega[(\tfrac{2}{5}-\tfrac{3}{5}z)\tilde{I}_{3R}+(\tfrac{6}{5})^{1/2}(1+z)(B-L)\tilde{}\,]\,,$$

where I_{3R} is the third generator of $SU(2)_R$. Further, we define $T\equiv b\Omega/M$. Both Ω and M are of unification scale so $T\sim 1$ naturally.

Let F denote one of the $SU(3)\times SU(2)\times U(1)$ multiplets in a **16**, namely either Q, u^c, d^c, L, l^+, or ν^c. F_i will denote such a multiplet from the $\mathbf{16}_i$, F without an index will denote one from the mirror **16**, and F^c will denote one from the $\overline{\mathbf{16}}$ ($F^c=Q^c$, u, d, L^c, l^-, or ν). Then the superheavy mass terms in Eq. (1) can be written as $M[F+(T/5)\alpha(F)F_3]F^c$, where $\alpha(F)$ is the charge $(X+6zY/2)$ for the multiplet F. $\alpha(Q)=1+z$, $\alpha(u^c)=1-4z$, $\alpha(d^c)=-3+2z$, $\alpha(L)=-3-3z$, and $\alpha(l^+)=1+6z$. Thus F^c and $[F-(T/5)\alpha(F)F_3]/N_{\alpha(F)}$ become superheavy and the orthogonal combinations $[-(T/5)\alpha(F)F+F_3]/N_{\alpha(F)}\equiv F_{3'}$ as well as F_1 and F_2 remain light, i.e., do not get superheavy masses. $N_{\alpha(F)}$ is a normalization factor $N_{\alpha(F)}=[1+\alpha(F)^2T^2/25]^{1/2}$.

We are now in a position to compute the tree-level mass matrix of the light quarks and leptons. These arise from the term $\sum_i a_i\,\mathbf{16}_i\,\mathbf{16}\,\mathbf{10}_H$ in Eq. (1). For the up quarks this term gives

$$a\upsilon(s_\theta\mathbf{16}_2+c_\theta\mathbf{16}_3)\mathbf{16}\supset a\upsilon\{(s_\theta Q_2+c_\theta Q_3)u^c+(s_\theta u_2^c+c_\theta u_3^c)Q\}$$
$$=a\upsilon(\{s_\theta Q_2+c_\theta[Q_{3'}+\tfrac{1}{5}T(1+z)Q_h]/N_{1+z}\}[-\tfrac{1}{5}T(1-4z)u_{3'}^c+u_h^c]/N_{1-4z}$$
$$+\{s_\theta u_2^c+c_\theta[u_{3'}^c+\tfrac{1}{5}T(1-4z)u_h^c]/N_{1-4z}\}[-\tfrac{1}{5}T(1+z)Q_{3'}+Q_h]/N_{1+z})\,.$$

In extracting the light-fermion mass matrix we may neglect terms involving superheavy fermions (denoted by the subscript h), which only produce mixings of order $M_W/M_{\rm GUT}$. One obtains directly

$$\sum_{i,j}u_iM_{ij}^{\rm up}u_j^c=(u_1\,u_2\,u_{3'})\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & s_\theta(1-4z)/N_{1-4z}\\ 0 & s_\theta(1+z)/N_{1+z} & c_\theta(2-3z)/N_{1+z}N_{1-4z}\end{pmatrix}\begin{pmatrix}u_1^c\\ u_2^c\\ u_{3'}^c\end{pmatrix}\left(\frac{T}{5}a\upsilon\right). \tag{3}$$

In the same way one obtains

$$\sum_{i,j}d_iM_{ij}^{\rm down}d_j^c=(d_1\,d_2\,d_{3'})\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & s_\theta(-3+2z)/N_{-3+2z}\\ 0 & s_\theta(1+z)/N_{1+z} & c_\theta(-2+3z)/N_{1+z}N_{-3+2z}\end{pmatrix}\begin{pmatrix}d_1^c\\ d_2^c\\ d_{3'}^c\end{pmatrix}\left(\frac{T}{5}a\upsilon'\right) \tag{4}$$

and

$$\sum_{i,j}l_i^-M_{ij}^{\rm lepton}l_j^+=(l_1\,l_2\,l_{3'})\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & s_\theta(1+6z)/N_{1+6z}\\ 0 & s_\theta(-3-3z)/N_{-3-3z} & c_\theta(-2+3z)/N_{-3-3z}N_{1+6z}\end{pmatrix}\begin{pmatrix}l_1^+\\ l_2^+\\ l_{3'}^+\end{pmatrix}\left(\frac{T}{5}a\upsilon'\right). \tag{5}$$

All mass ratios and mixing angles depend then on four parameters, θ, z, T, and υ/υ'. It is easy to see from Eq. (1) that all complex phases in the couplings can be absorbed by field redefinitions. Altogether then only four parameters count: θ, z, T, and $|\upsilon/\upsilon'|$, and for $|T|<1$ the dependence on T is weak. One finds from Eqs. (3)-(5), neglecting T for the present (i.e., setting $N_a = 1$),

$$(m_b/m_\tau)|_{\mu_{\rm GUT}} \simeq 1\,, \tag{6a}$$

$$\left.\frac{m_s}{m_\mu}\right|_{\mu_{\rm GUT}} \simeq \frac{(1+z)(-3+2z)}{(-3-3z)(1+6z)} = \frac{1}{3}\left[\frac{3-2z}{1+6z}\right], \tag{6b}$$

$$V_{ts} \simeq \tan\theta\left[\frac{3-2z}{2-3z} - \frac{1-4z}{2-3z}\right], \tag{6c}$$

$$\frac{m_\mu}{m_\tau} \simeq \tan^2\theta\frac{(1+6z)(-3-3z)}{(-2+3z)^2}, \tag{6d}$$

$$\left.\frac{m_c/m_t}{m_s/m_b}\right|_{\mu_{\rm GUT}} \simeq \frac{1-4z}{-3+2z}, \tag{6e}$$

$$(m_t/m_b)|_{\mu_{\rm GUT}} \simeq |v/v'|\,. \tag{6f}$$

The most striking of these predictions is the first, that $(m_b/m_\tau)|_{\mu_{\rm GUT}} \simeq 1$, the old minimal-SU(5) prediction.[5,6] This arises from the circumstance that the 3'3' component of both $M^{\rm down}$ and $M^{\rm lepton}$ are proportional to $-2+3z$. This is not a coincidence but a consequence of the fact that l and d both get mass from the Higgs field ϕ' whose VEV is v', and so $\alpha(L)+\alpha(l^+) = -\alpha(\phi') = -2+3z$ and $\alpha(Q)+\alpha(d^c) = -\alpha(\phi') = -2+3z$. However, as we see from Eq. (6b), $(m_s/m_\mu)|_{\mu_{\rm GUT}}$ is *not* predicted to be near unity (unless z happens to vanish). One must use Eqs. (6b)-(6d) (three relations) to fix z and θ and then use Eq. (6e) to predict m_t. One can get a reasonable fit for Eqs. (6b)-(6d) with $z \approx -1$ which is an interesting value group theoretically. Equation (2) implies that for $z \approx -1$, $\langle \mathbf{45}_H \rangle = -4\Omega[\tilde{I}_{3R} + ({\rm small})(B-L)^{\sim}]$ which corresponds to an approximate Pati-Salam $SU(4)_c$ symmetry realized on the superheavy fermion masses. Unfortunately $z \approx -1$ gives the proportionality prediction, from Eq. (6e), $m_t \approx m_c m_b/m_s$ which is bad. We are therefore motivated to enlarge the group to E_6. The net effect of this is to introduce one more parameter.

The fermions are now in $\mathbf{27}_i$ $(i=1,2,3) + \mathbf{27} + \overline{\mathbf{27}}$, and the relevant Higgs fields in $\mathbf{78}_H + \mathbf{27}_H$. The generalization of Eq. (1) is

$$L_{\rm Yuk} = M\,\mathbf{27}\,\overline{\mathbf{27}} + \sum_{i=1}^{3} b_i\,\mathbf{27}_i\,\overline{\mathbf{27}}\,\mathbf{78}_H + \sum_{i=1}^{3} a_i\,\mathbf{27}_i\,\mathbf{27}\,\mathbf{27}_H + {\rm H.c.} \tag{7}$$

Most of the discussion of the SO(10) model carries over to the E_6 case. At tree level 2×27 fermions become superheavy and 3×27 remain light.

For purposes of discussion it is helpful to think in terms of an SO(10) subgroup of E_6 and to denote an SO(10) R representation contained within an S representation of E_6 by $R(S)$. We assume that the SO(10)-spinorial components of the $\mathbf{78}_H$ and $\mathbf{27}_H$ do not acquire superlarge VEV's [i.e., $\langle \mathbf{16}(\mathbf{78}_H)\rangle$, $\langle \overline{\mathbf{16}}(\mathbf{78}_H)\rangle$, and $\langle \mathbf{16}(\mathbf{27}_H)\rangle$]. The usual quarks and leptons are in $\mathbf{16}(\mathbf{27})$ representations, and the "extra" $\mathbf{10}(\mathbf{27})$ and $\mathbf{1}(\mathbf{27})$ fermions become superheavy—how will be discussed later. This means that the model effectively just reduces at low energies to the previous SO(10) model with one crucial difference. The $\langle \mathbf{78}_H \rangle$ now has a *three-dimensional* space of directions to point in corresponding to the U(1) generators in the decomposition

$$E_6 \supset SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X \times U(1)_R\,.$$

The net effect is to introduce one more parameter which will be denoted w. Then Eq. (2) is replaced by $\langle \mathbf{78}_H \rangle \equiv \frac{1}{5}\Omega[X + 6zY/2 + w(R)]$, where the R charge of the SO(10) **16** in the **27** is normalized to 1. This produces only one modification in the relations of the SO(10) model: $X+6zY/2$ is replaced everywhere by $X+6zY/2+w$. So in Eqs. (3)-(6) one replaces $1+z = \alpha(Q)$ by $1+z+w$, $-3+2z = \alpha(d^c)$ by $-3+2z+w$, and so forth. Note that $-2+3z = \alpha(Q)+\alpha(d^c)$ is replaced by $-2+3z+2w$, and $+2-3z = \alpha(Q)+\alpha(u^c)$ is replaced by $+2-3z+2w$. One still has, and for the same reason, $(m_b/m_t)|_{\mu_{\rm GUT}} \simeq 1$. One uses the analogs of Eqs. (6a)-(6d) to fit for θ, z, w, and T and uses the results in the analog of Eq. (6e) to predict m_t.

What we have actually done is to use the exact tree-level expressions to perform a fit to the known quark and lepton masses.[7] In running the fermion masses from the grand unification scale down to low energies we have used the one-loop β functions neglecting the effects of Yukawa interactions [and assuming unrealistically that E_6 breaks to $SU(3)\times SU(2)\times U(1)$ at a single scale for simplicity]. A more refined analysis is in progress.[8] We have taken $\alpha_s(M_W) = 0.107$. For m_b and m_c we have used the central values of Gasser and Leutwyler[7] [m_b(1 GeV) = 5.8 GeV (for our choice of α_s), and m_c(1 GeV) = 1.35 GeV]. Since the uncertainties in m_s and V_{ts} are so large we have made a contour plot in Fig. 1 of m_t(phys) versus these quantities with m_s(1 GeV) $= 175 \pm 55$ GeV (the range given in Ref. 7) and $0.3 \le V_{ts} \le 0.062$ (the Particle Data Group[9] value). One can see that the model definitely predicts values of m_t that are *large* in comparison to the usual SO(10) (or E_6) result $m_t = m_c m_b/m_s \approx 30$ GeV, even values that are above 100 GeV. We have also shown some contours for m_t with the values m_b(1 GeV) = 5.9 GeV and m_c(1 GeV) = 1.40 GeV (dashed lines) to give an idea of the uncertainty in m_t due to these parameters. One point should be emphasized; the value of "m_s" that is being discussed here is the "tree-level" value. The radiative corrections that produce m_e, m_u, and m_d will modify this, perhaps—judging from $\sin\theta_C \simeq 0.2$—by as much as 20%. This would introduce an uncertainty of 20% also into the value of m_t. This is the largest source of theoretical uncertainty in our model.

The numerical fits for the range of parameters of Fig. 1 yield the values for our model parameters $\theta = 0.38 \pm 0.05$ rad, $z = 0.83 \pm 0.06$, $w = 3.22 \pm 0.33$, and $T = 0.27 \pm 0.03$. Note that for T this small its effect

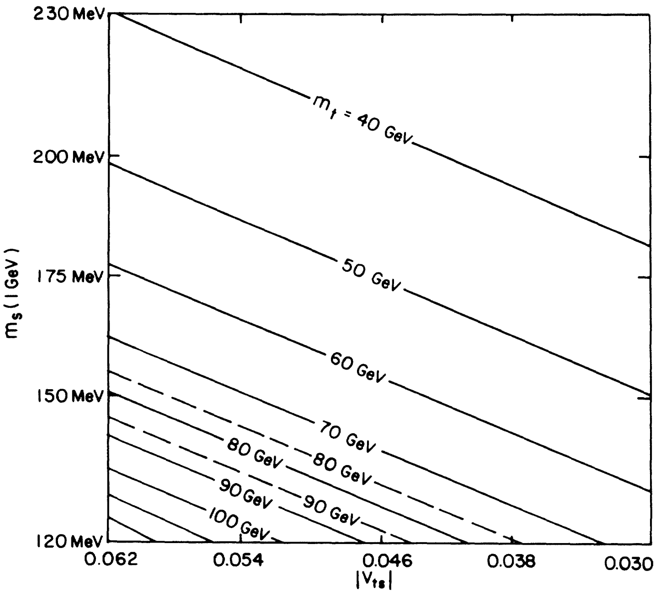

FIG. 1. A contour plot of m_t vs V_{ts} and m_s(1 GeV). We have taken $\alpha_s(M_W)=0.107$, m_b(1 GeV) = 5.8 GeV, and m_c(1 Gev) = 1.35 GeV. The dashed contours represent m_b(1 GeV) = 5.9 GeV, m_c(1 GeV) = 1.40 GeV.

[through $N_a=(1+\frac{1}{25}a^2T^2)^{1/2}$] on the fit is truly negligible. The values of z and w have, again, a group-theoretical significance:

$$\langle \mathbf{78}_H\rangle = [X+6(0.83)Y/2+3.22R]$$
$$\propto [(B-L)^{\sim}+0.1\tilde{I}_{3R}+1.8\tilde{R}]\,,$$

which corresponds to an approximate $SU(2)_R$ realized on the superlarge fermion masses.

We now turn briefly to the additional fields of the theory and their couplings. There are three crucial tasks that these fields must perform. (a) They must render the right-handed neutrinos superheavy. (b) They must render the extra **10**(**27**) and **1**(**27**) fermions superheavy. And (c) they must generate radiative masses for the e, u, and d. All of these tasks can be accomplished in more than one way. As an illustration let the additional fields consist of several E_6-singlet fermions, denoted $\mathbf{1}_i$, and a fundamental representation of Higgs fields, denoted $\mathbf{27}_{H'}$. Let these be even under the above-mentioned Z_2. Let us impose another Z_2 under which the additional fields $\mathbf{1}_i$ and $\mathbf{27}_{H'}$ are odd and everything else even. Then $L_{\rm AF}$ consists of the terms

$$\sum_{i,j} c_{ij}\,\mathbf{27}_i\,\mathbf{1}_j\,\mathbf{27}_{H'}^{\dagger}+\sum_{i,j} M_{ij}\,\mathbf{1}_i\,\mathbf{1}_j\,.$$

Assume that in the $\mathbf{27}_{H'}$ the *only* superlarge VEV is in the **16**(**27**). (This and our other assumptions can be shown to be technically natural.[8]) One sees immediately that the $\langle \mathbf{16}(\mathbf{27}_{H'})\rangle$ gives superlarge masses to the right-handed neutrinos, and hence by the "seesaw" mechanism the left-handed neutrinos are very light. The radiative masses of e, u, and d arise from diagrams where the $\mathbf{27}_i$ emits a $\mathbf{27}_{H'}$ to become a virtual singlet fermion which then absorbs a $\mathbf{27}_{H'}^{\dagger}$ to become a $\mathbf{27}_j^{\dagger}$. The lowest-dimension resulting operators are of the form $\mathbf{27}_i\mathbf{27}_j\mathbf{27}_H\mathbf{78}_H$ or $\mathbf{27}_i\mathbf{27}_j\mathbf{27}_H^{\dagger}\mathbf{27}_H^{\dagger}$. In SO(10) terms these give $\mathbf{16}(\mathbf{27}_i)\,\mathbf{16}(\mathbf{27}_j)\langle\mathbf{10}(\mathbf{27}_H)\rangle\langle\mathbf{1}$ or $\mathbf{45}(\mathbf{78}_H)\rangle$, etc. These also give superlarge radiative masses to the exotic fermions in the **27**'s through terms like $\mathbf{10}(\mathbf{27}_i)\,\mathbf{10}(\mathbf{27}_j)\langle\mathbf{1}(\mathbf{27}_H)\rangle\langle\mathbf{1}(\mathbf{78}_H)\rangle$. There could be a problem with both $\langle\mathbf{10}(\mathbf{27}_H)\rangle$ and $\langle\mathbf{1}(\mathbf{27}_H)\rangle$ being nonzero as they are in a single multiplet. However, one can just as easily have several $\mathbf{27}_H$ fields. [For example, if the third term in Eq. (7) becomes $\sum_K\sum_{i=1}^3 a_i^K\,\mathbf{27}_i\times\mathbf{27}\,\mathbf{27}_H^K$, that does not affect the model as $a_i v$ becomes just $\sum_K a_i^K\langle\mathbf{10}(\mathbf{27}_H^K)\rangle$ which is still a vector in family space.] This and other technical details concerning the pattern of VEV's will be treated at length in Ref. 8.

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