Comment on "Soap Froth Revisited: Dynamical Scaling in the Two-Dimensional Froth"

In a recent Letter,[1] Stavans and Glazier have presented various experimental results for the statistics of the evolution of the two-dimensional soap froth which, in conjunction with their earlier work,[2] have settled some important issues. At the same time, however, they have raised a fresh problem in the observation that the vertex angles of the soap-froth network depart significantly from the value of 120°, which has hitherto been regarded as inevitable, and is a feature of all idealized models to date. The discrepancy is of the order of 10°. It correlates with n, the number of sides of the cell within which the angle is contained.

Various possible sources of this discrepancy were indicated[1] without any conclusion. We wish to suggest that this anomalous result is a consequence of the existence of finite Plateau borders, together with the presumed method of measurement of the angles, and explicable in terms of the usual idealized model of the soap froth. In this the films which connect the Plateau borders are treated as lines, without finite thickness. We also adopt the usual assumption of constant surface tension throughout, and cell pressure differences which determine film curvatures according to $\Delta p = 2Tr^{-1}$.

A lemma,[3] which is very helpful in considering the effect of the finite Plateau border, is as follows: The lines which adjoin a triangular Plateau border (which are circular arcs) may be continued *with the same curvature* to meet in a single point. Furthermore, the lines must meet at 120°.

Armed with this lemma, we can see that the model predicts 120° angles for the intersection of films, even for the case of finite Plateau borders, provided that the films are extended into the Plateau border with constant curvature. If, however, the directions of the films at the edges of the Plateau border are used, as might be more natural in experiment, it is clear that the result no longer holds.

The discrepancy can be roughly estimated. As a first approximation, we shall treat cell side lengths as being the same. This is a good approximation[4] only for $n \geq 5$, since three- and four-sided cells tend to have much smaller sides: An allowance for this could easily be incorporated, but without knowledge of the precise sampling procedure used in Ref. 1 it is not worthwhile here. We also attribute an equal size to all of the Plateau borders, and define f to be the fraction of the *extended* film, constituting a cell side, which is "buried" in the Plateau border at one end. With the stated assumption, and bearing in mind that, on average, one (extended) side of a n-sided cell must turn through the angle,

$$\phi = 60 - 360n^{-1} \text{ deg}, \quad (1)$$

we see that, at each vertex, each of the two intersecting lines turns by an angle $f\phi$ within the Plateau border. With no allowance for this, the internal angle of the two films at the vertex will be measured as

$$\Theta = 120 + 2f\phi \text{ deg}. \quad (2)$$

If, for example, we put $f = 0.1$, we obtain

$$\Theta = 132 - 72n^{-1} \text{ deg}, \quad (3)$$

which is in quite good agreement with the data presented in Ref. 1. This value of f does not seem unreasonable upon examination of a magnified version of Fig. 5 of Ref. 1.

Clearly this matter merits a careful reexamination to establish whether the standard idealized model may be vindicated in the manner proposed here.

The lemma that we have indicated has other obvious uses. Any equilibrium structure without finite Plateau borders can be "decorated" with them, preserving equilibrium—but this is only valid provided the Plateau borders of neighboring vertices do not overlap. This has obvious implications for the mechanical properties of foams with (small) Plateau borders.

In conclusion, we might also mention that finite Plateau borders may also provide the resolution of another anomaly noted in Refs. 1 and 2—the observation of discrepancies in relation to Von Neumann's law. Again, the above lemma is useful in estimating the effect. Details of such an analysis will be presented in a future paper.[5]

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[1]J. Stavans and J. A. Glazier, Phys. Rev. Lett. **62**, 1318 (1989).

[2]J. A. Glazier, S. P. Gross, and J. Stavans, Phys. Rev. A **36**, 306 (1987).

[3]Such a result was indicated to us by H. Princen (private communication), but we know of no published proof. Our own will be published separately [D. Weaire and F. Bolton (to be published)].

[4]J. A. Glazier, Ph.D. dissertation, University of Chicago, 1989 (unpublished).

[5]Weaire and Bolton, Ref. 3.

