Quantum Fluctuations and the Single-Junction Coulomb Blockade

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We investigate the effect of quantum fluctuations on the Coulomb blockade in a single tunnel junction coupled to its environment by a transmission line of arbitrary impedance $Z(\omega)$. The quantized oscillating modes of the transmission line are suddenly displaced when an electron tunnels through the junction. For small Z (relative to the quantum of resistance), a weak power-law zero-bias anomaly occurs associated with the infrared-divergent shakeup of low-frequency transmission-line modes. For large Z, the full blockade is recovered. Comparison with recent experiments is made.

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Recent technological developments have allowed the study of tunnel junctions with capacitance so low that the charging energy associated with a single electron can be several meV.[1] This has interesting consequences at low temperatures, including the Coulomb blockade,[2] the Coulomb staircase,[3,4] and various oscillatory and dynamic effects.[5] Because of the difficulties associated with stray capacitance, the clearest observations of these effects have been in multijunction arrays. Recently, however, several groups[6-8] have reported the observation of a partial blockade in a single junction fed by a transmission line. The strength and line shape of the blockade appear to be controlled by the (frequency-dependent) impedance of the transmission line. The purpose of this paper is to show how the high-frequency environment of the junction affects the tunneling. Such questions have been extensively studied in connection with macroscopic quantum tunneling in superconductors,[9,10] but very few studies of normal junctions exist.[11-14]

The essential physics in the semiclassical theory[1,4,15] of the Coulomb blockade is that the tunneling electron gains energy eV at bias voltage V, but must pay a charging cost of $E_c=e^2/C_0$, where C_0 is the junction capacitance. If the net energy gain is not positive, then (at zero temperature) there is no empty final state available and Pauli blocking forbids tunneling. In this paper we go beyond the semiclassical theory to a full quantum-mechanical treatment of the charging energy by considering the fluctuations in the electromagnetic modes coupled to the junction.

A line of impedance Z leading to the tunnel junction allows the junction to discharge and dissipate its energy in a finite time $\tau_Z \equiv C_0 Z$. If the energy uncertainty h/τ_Z associated with this time exceeds E_c, then the blockade will be weakened. This sets the characteristic impedance scale for the classical-to-quantum crossover to be the quantum resistance $R_H \equiv h/e^2$. (Note the difference of a factor of 2 in our definition of R_H relative to that in Ref. 14.)

Following the seminal work of Caldeira and Leggett,[16] there has been considerable interest in the question of tunneling in the presence of dissipation.[9,10] Part of the focus of this work has been on the question of the response of the dissipative medium *during* the time the system is under the barrier. Büttiker and Landauer[17] have defined a traversal time τ_T which provides an estimate of the duration of the tunneling event. This time can be long for macroscopic quantum tunneling in small-gap systems like superconductors, but is very short for single-electron tunneling through an oxide barrier.[17] We therefore make the ansatz that the low-frequency transmission-line collective modes can be treated as harmonic oscillators which are displaced *suddenly*. An additional assumption is that the junction impedance is very high ($R \gg R_H$) so that the time between tunnel events, e/I, is long and coherence effects between separate events can be neglected.[18] That is, as a result of the inequality $\tau_T \ll \tau_Z \ll e/I$, we can neglect correlations between tunnel events and the only significant time scale in our model is the discharge time τ_Z.

Here, we outline a method for treating tunneling into a quantum transmission line and present some results on simple models of physical systems. First, we consider ideal transmission lines and show that the blockade behavior depends strongly on the impedance of the line.

Then, we consider the more realistic case of dissipative lines and lines containing impedance discontinuities which produce reflected waves.

We first model a transmission line in a lumped-circuit description as a collection of N identical capacitors $C_j = c$, inductors l, and resistors r (see Fig. 1). For the moment we take the dissipationless case $r=0$. The canonical coordinates are the charges of the capacitors $q_0, q_1, \ldots, q_N$, where q_0 refers to the charge on the tunnel-junction capacitor C_0 and the remaining variables refer to the transmission line. The Lagrangian for the circuit may be expressed in the form

$$\mathcal{L} = \frac{l}{2} \sum_{j=0}^{N} \sum_{k=0}^{N} \dot{\varphi}_j M_{jk} \dot{\varphi}_k - \frac{1}{2c} \sum_{j=0}^{N} \varphi_j^2 , \quad (1)$$

where M is some symmetric matrix (whose specific form will not be required) and $\varphi_j \equiv (c/C_j)^{1/2} q_j$. This is diagonalized by defining new coordinates Q_α by $\varphi_j \equiv \sum_\alpha \psi_\alpha(j) \times Q_\alpha$, where ψ_α is an orthonormal basis function satisfying

$$\sum_k M_{jk} \psi_\alpha(k) = \frac{1}{lc} \omega_\alpha^{-2} \psi_\alpha(j) , \quad (2)$$

with ω_α the frequency eigenvalue of the normal mode.

We need to compute the shakeup spectrum generated by the sudden displacement of the coordinate φ_0 by the amount $\hbar\lambda$ [where $\lambda = (c/C_0)^{1/2} e/\hbar$] due to the tunneling of a single charge e. At zero temperature, the initial state of the oscillator system just after the tunneling event is $|\Psi(0)\rangle = \exp(-i\lambda p_0)|0\rangle$, where the generator of displacements p_0 is simply the momentum conjugate to φ_0 and $|0\rangle$ is the ground state. The boson shakeup spectrum is computed from the Green's function

$$\begin{aligned} \mathcal{G}(t) &= -i\theta(t) \langle \Psi(t) | \Psi(0) \rangle \\ &= -i\theta(t) \exp\{\lambda^2 [f(t) - f(0)]\} , \end{aligned} \quad (3)$$

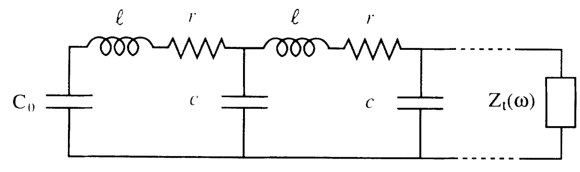

FIG. 1. A schematic model of a tunnel junction C_0 connected to an RLC transmission line terminated by an impedance $Z_t(\omega)$ after a length d.

where $f(t) \equiv \langle 0 | p_0(t) p_0 | 0 \rangle$. Using $p_0(t) = \sum_\alpha \psi_\alpha(0) \times P_\alpha(t)$ (where P_α is conjugate to Q_α), and from the Hamiltonian obtaining $\langle 0 | P_\alpha^2 | 0 \rangle = \hbar / 2c\omega_\alpha$, we have

$$f(t) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\hbar}{2c\omega} a(\omega) e^{-i\omega t} , \quad (4)$$

where the spectral density is

$$a(\omega) \equiv 2\pi \sum_\alpha |\psi_\alpha(0)|^2 \delta(\omega - \omega_\alpha) .$$

We can bypass formal solution of the eigenmode problem by noting that $f(t)$ is determined by precisely the same physics that controls the *classical* discharge of the junction. The classical time dependence of the charge decay is identical (for a harmonic system) to the expectation value of the quantum result,

$$\begin{aligned} \bar{\varphi}_0(t) &= \langle 0 | \exp(i\lambda p_0) \varphi_0(t) \exp(-i\lambda p_0) | 0 \rangle \\ &= \hbar\lambda \int_0^\infty \frac{d\omega}{2\pi} a(\omega) \cos(\omega t) . \end{aligned} \quad (5)$$

Thus the spectral density can be obtained directly from the Fourier transform of the classical time dependence,

$$a(\omega) = -4 \,\mathrm{Re} \frac{1}{i\omega - 1/C_0 Z^*(\omega)} . \quad (6)$$

The final expression for the Green's function is

$$\mathcal{G}(t) = -i\theta(t) \exp\left\{ \frac{e^2}{2C_0} \int_0^\infty \frac{d\omega}{2\pi} \frac{1}{\hbar\omega} \mathrm{Re}\left[\frac{-4}{i\omega - 1/C_0 Z^*(\omega)} \right] (e^{-i\omega t} - 1) \right\} . \quad (7)$$

Readers familiar with the macroscopic quantum tunneling[9,10,16] or x-ray photoemission[19] literature will recognize that the shakeup spectrum, $A(\omega) = -2\,\mathrm{Im}\tilde{\mathcal{G}}(\omega + i\delta)$, has a characteristic infrared divergence, $A(\omega) \sim \omega^{g-1}$, where $g \equiv 2Z(0)/R_H$. We use the integral-equation method of Minnhagen[20] to solve for this spectral density.

If an electron from the Fermi level tunnels, it arrives with excess energy eV above the Fermi level on the other side. The Pauli principle limits the allowed boson shakeup to this amount. Hence (at $T=0$) the differential conductance obeys

$$\frac{dI}{dV} = \frac{1}{R} \int_0^{eV/\hbar} \frac{d\omega}{2\pi} A(\omega) , \quad (8)$$

where R is the junction resistance. The spectral density sum rules $\int_0^\infty (d\omega/2\pi) A(\omega) = 1$ and $\int_0^\infty (d\omega/2\pi) \hbar\omega \times A(\omega) = e^2/2C_0$ guarantee that, in the limit of large voltage, the conductance will obey the usual Coulomb offset,

$$I(V) = \frac{V - e/2C_0}{R} . \quad (9)$$

That is, the mean shakeup energy is exactly the classical charging energy. In the opposite limit of small bias, the infrared divergence in $A(\omega)$ implies a power-law zero-bias anomaly for the conductance,

$$dI/dV \approx V^g . \quad (10)$$

It is clear from Eq. (7) that the discharge time τ_Z, rather than the traversal time τ_T, sets the important ultraviolet energy scale. Within the framework of this model the traversal time τ_T corresponds to a much higher frequency which is largely irrelevant. We can, however, improve on the sudden-approximation aspect of the model by noting that boson modes at frequencies higher than the inverse traversal time will be displaced *adiabatically* rather than suddenly and hence will suffer no shakeup. The only effect is to slightly renormalize C_0 upwards due to the distributed capacitance in the transmission line within a distance $c\tau_T$, where c is the speed of waves in the line. However, this is not directly relevant to the low-energy physics, as is evident from Eq. (10), where the exponent g is independent of C_0. It does, however, reduce the asymptotic offset given by Eq. (9).

For large g, the shakeup spectrum is peaked around the classical energy and, as shown in Fig. 2, we recover the full Coulomb blockade (smeared out by quantum fluctuations). For small g, we see the characteristic power-law zero-bias anomaly. The shakeup spectrum has, in addition to the ω^{g-1} divergence at low energy, a long tail out to energies of order E_c/g (for small g). The origin of this Lorentzian tail is the energy uncertainty h/τ_Z associated with the rapid discharge of the tunneled electron into the transmission line. The existence of this effect means that the I-V characteristic has an extensive nonlinear regime and does not achieve the sum-rule form given in Eq. (9) until rather large voltages $V \approx e/2C_0 g$. We emphasize that all curves have the *same* asymptotic offset.

Our results (applicable only to high-resistance junctions) are in qualitative agreement with experiment.[6-8] For low-impedance transmission lines, one sees a weak zero-bias anomaly with a rather broad nonlinear region before saturation at large voltages. This behavior is particularly clear in the inset of the upper panel in Fig. 1 of Cleland, Schmidt, and Clarke.[8]

Ideally, the present model could be tested by making transmission lines with a wide range of specific inductances and capacitances, but it is rather difficult to produce lossless transmission lines with impedances which differ significantly from the free-space impedance of 377 Ω. It *is* possible, however, to increase the impedance by including dissipative elements in the transmission line, making the long-time low-frequency decay diffusive. In the spirit of Caldeira and Leggett,[16] we include extra boson degrees of freedom to model the dissipation. If we take these to couple in the Lagrangian in Eq. (1) to all of the φ's and $\dot\varphi$'s (including $\dot\varphi_0$ but *not* φ_0), then all of the formal results above follow without modification by simply recomputing $Z(\omega)$ with the dissipation included.

In Fig. 3 we show results for the zero-bias anomaly for the case of an extremely resistive transmission line at a series of different temperatures.[21] Because of the strong frequency dependence of the transmission-line impedance (dominated by the specific resistance and capacitance), the zero-temperature result (curve a) is very different from any of those for the lossless transmission line shown in Fig. 2. In fact, it is not particularly useful to describe this curve by a single exponent. There is both a strong blockade at low voltage *and* a tail that extends to very high voltages. Because of a combination of these effects, the I-V curves initially appear to reach a blockade limit with a capacitance larger than the actual junction capacitance. The inset in Fig. 3 shows the ratio of dI/dV at zero voltage to the value of dI/dV at asymptotically large voltage (the "bare" junction impedance). Note that the semiclassical model (thermal

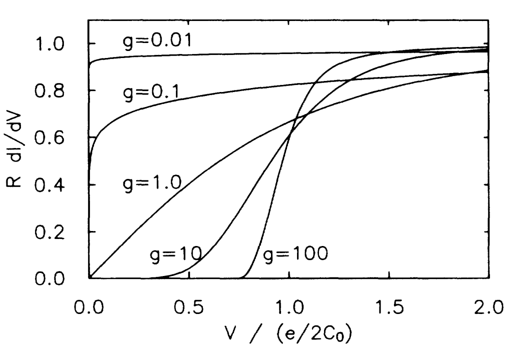

FIG. 2. The current-voltage characteristics of a tunnel junction connected to an infinite, lossless transmission line of impedance $Z = gR_H/2$. R is the asymptotic value of $(dI/dV)^{-1}$. The curves were calculated at zero temperature in the limit that the junction impedance R goes to infinity.

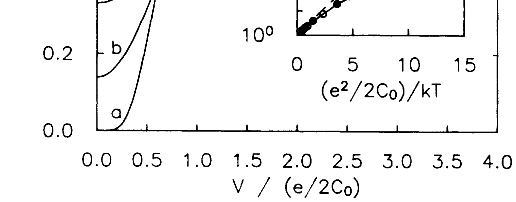

FIG. 3. The current-voltage characteristics of a tunnel junction connected to a resistive transmission line at finite temperature. The curves a–g are for temperatures 0, 0.5, 1.0, 2.0, 5.0, 10, and 20 K. The parameters are junction capacitance $C_0 = 0.2$ fF, specific resistance $r = 8000$ Ω/μm, specific capacitance $c = 0.02$ fF/μm, and specific inductance $l = 600$ fH/μm. Inset: The values of $R(0)/R$ plotted vs the charging energy over the temperature for these same curves (open circles). The dashed line gives the semiclassical result. The solid circles show $R(0)/R$ for a series of junctions at constant temperature $T = 1.3$ K as the junction capacitance is varied over $C_0 = 0.05$, 0.1, 0.2, 0.5, 1, 2, 5, 10, and 20 fF.

fluctuations only) gives a strong, essentially exponential (Arrhenius) temperature dependence while the quantum fluctuations yield a much weaker temperature variation (which can be quite slow for low Z). The quantum Langevin model used by Cleland, Schmidt, and Clarke[8] correctly captures some of the physics of quantum smearing of the blockade by zero-point fluctuations of the instantaneous charge on the junction. However, it incorrectly treats the dynamics and completely misses the slow divergence of $R(0)$ (which is rigorously present in the model) as the temperature is lowered. On the other hand, the quantum Langevin model has the advantage of being simple to extend to the case of finite current.

One of the interesting features of the data of Delsing *et al.*[6] is the appearance of small oscillations in dI/dV in the wings of the curve. Nazarov[22] has attributed these oscillations to random features in a universal conductance fluctuation type of model. We have considered the possibility that they are due to wave reflection from the discontinuity in the structure at the contact pad which is located[23] 1.5 mm away from the junction. The reflected waves produce periodic resonances[10] in $Z(\omega)$ and hence modulation in the conductance which by suitable choice of parameters can be made qualitatively consistent with the data. However, we find that the oscillations wash out very rapidly with temperature and the data (over the small range shown in their papers) do not seem to show as strong a long-range tail as we find in our results for any reasonable set of parameters. While this impedance-discontinuity model may not explain the oscillations seen by Delsing *et al.*, we propose that such oscillations *could* be observed in an experiment specifically designed to search for them.

In summary, we have investigated the Coulomb blockade in a single small tunnel junction coupled to its environment through a transmission line of arbitrary impedance. Quantum fluctuations of the transmission-line modes smear out the blockade at low impedance relative to the quantum impedance, leaving a power-law zero-bias anomaly. The full blockade is recovered only for high impedance, but even there the asymptotic offset of the I-V curve is not achieved until surprisingly large voltages relative to the width of the blockade.

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Note added.—After the original submission of this manuscript, a paper by Devoret *et al.*[14] appeared which treats the same model considered here.

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