Nonuniversality in Helical and Canted-Spin Systems

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Vector-spin models with competing interactions have in general nontrivial ground states which completely break rotational invariance. We study the prototypical stacked triangular Heisenberg antiferromagnet by means of a $O(3)\times O(2)/O(2)$ nonlinear σ model in a $2+\epsilon$ expansion. We find a dynamically generated O(4) symmetry. We propose that such systems in three dimensions have a first-order transition or a second-order one with either $N=4$ or tricritical mean-field exponents. We argue that this view is supported by experimental and numerical data.

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Vector-spin models with competing exchange interactions have in general canted ground states. As a consequence, in the low-temperature phase the O(3) group of spatial rotations is completely broken. The relevant order parameter is thus no longer a vector but a rotation matrix. There is still a controversy about the nature of the phase transition that occurs in such systems when $D=3$. This is the case of stacked triangular antiferromagnets[1] (STA) and body-centered-tetragonal[2] (BCT) lattices with Heisenberg spins where results of renormalization-group calculations in $4-\epsilon$ have been interpreted as evidence of a first-order transition,[3,4] while recent Monte Carlo (MC) simulations[1,2] in $D=3$ conclude in favor of a continuous transition. The question that naturally arises if the transition is continuous is whether or not canted-spin systems belong to a new universality class.

The purpose of this Letter is to shed light on the critical behavior of STA and related models in dimension 3 by means of $2+\epsilon$ renormalization-group calculations. Canted magnets are frequently encountered in nature and detailed experimental studies have been performed. Examples are the helical magnets Ho, Dy, and Tb. In addition, the dipole-locked A phase of helium-3 shares the same symmetry-breaking pattern.[4] Many theoretical Heisenberg models such as STA, BCT, stacked Villain lattices, and helical models, among others, are expected to be relevant to the experimental situation. These models are described by the following Landau-Ginzburg-Wilson effective action:[1,3,4]

$$A=\int d^Dx\,\tfrac{1}{2}[(\nabla\mathbf{\Phi}_1)^2+(\nabla\mathbf{\Phi}_2)^2]-\tfrac{1}{2}m^2(\mathbf{\Phi}_1^2+\mathbf{\Phi}_2^2)+\tfrac{1}{4}u(\mathbf{\Phi}_1^2+\mathbf{\Phi}_2^2)^2+\tfrac{1}{2}v(\mathbf{\Phi}_1\times\mathbf{\Phi}_2)^2,\qquad(1)$$

where the vectors have three components. This action has been intensively studied by means of renormalization-group (RG) techniques. In these works, a c-axis anisotropy is usually considered as irrelevant; we shall use the same hypothesis in this paper. It has been shown that no stable fixed point occurs up to order ϵ^2 in an ϵ expansion in the neighborhood of dimension 4. This fact has been interpreted as the signature of the occurrence of a first-order transition in three dimensions.[3-5] However, MC data suggest that a stable fixed point manifests itself at a finite distance from $D=4$. To study this intriguing phenomenon there is another general strategy available which is the perturbative expansion near two dimensions of a nonlinear σ model[6] (NLσ). We shall thus investigate the critical behavior of the NLσ model suited to the symmetry-breaking scheme of the above-mentioned models, by means of field-theoretic RG techniques[7] in a $2+\epsilon$ expansion.

The effective theory describing the long-distance properties of the Goldstone modes of these models is obtained in a standard way by letting the mass of the massive fields in the action (1) go to infinity. In this limit, the remaining fluctuating fields can be parametrized in terms of the elements of the coset space G/H, where G is the symmetry group which is broken down to H. The NLσ model is, in fact, completely characterized by the geometry of the space in which the fields exist. It has been studied in the general case of an arbitrary Riemannian manifold.[8] Here we are interested in a coset space which is compact, homogeneous, but nonsymmetric. This is to be contrasted with the case of the usual $O(N)$ NLσ model build on the symmetric space $O(N)/O(N-1)$. The action (1) has an $O(3)\times O(2)$ symmetry. The ground state $\mathbf{\Phi}_{10},\mathbf{\Phi}_{20}$ relevant for canted magnets is obtained when $v<0$. In this case, $\mathbf{\Phi}_{10}\cdot\mathbf{\Phi}_{20}=0$ and $\mathbf{\Phi}_{10}^2=\mathbf{\Phi}_{20}^2$. The symmetry group H leaving the ground state invariant is then found to be a O(2) group which operates on the spatial components of the order parameter and also on the indices 1,2 of the fields. This particular subgroup of $O(3)\times O(2)$ will be denoted by $O(2)_{\rm diag}$. The NLσ model is thus defined by $G/H=O(3)\times O(2)/O(2)_{\rm diag}$ and has three Goldstone modes, the dynamics of which is described by the following action:

$$S=\frac{1}{2}\int d^Dx\,{\rm Tr}[P(R^{-1}\nabla_\mu R)^2]\,, \qquad (2)$$

where R is an element of O(3) and $P={\rm diag}[g_1,g_1,g_2]$. This field theory has two coupling constants g_1 and g_2. The action (2) is right [left] invariant under O(2) [O(3)] since it is invariant under the transformation $R\to URV$, where $U\in O(3)$ and V belongs to the O(2) group that commutes with the matrix P. As pointed out above, the order parameter for canted magnets belongs to O(3). It describes all possible orientations of a triple of canted spins leaving their relative angles fixed. The action (2) has been derived microscopically by Dombre and Read[9] for the isotropic triangular antiferromagnet. They found that $g_2=0$. It is worth pointing out that this condition is not stable under renormalization and that one has to study the most general case compatible with the symmetries of the problem.

Generally speaking, the symmetry-breaking patterns compatible with an order parameter in O(3), $O(3)\times O(p)\to O(p)_{\rm diag}$, $p=1,2,3$, can be described by the action (2) where the diagonal matrix P is chosen to commute with $O(p)$. All these models possess three massless spin waves with as many different velocities as there are different coupling constants in the diagonal matrix P. The renormalization properties of NLσ models depend only on the geometry of the coset space. In particular, the β function is given up to two loops in terms of the Riemann and Ricci tensors of the manifold G/H viewed as a metric space.[8] Computational details will be given in a forthcoming publication.[10] We have obtained the complete two-loop recursion relations for the general coupling matrix P of the model (2). In what follows, we shall discuss the case $P={\rm diag}[g_1,g_1,g_2]$. Since the expressions are quite lengthy, we quote here simply the one-loop formulas:

$$dT/dl=-\epsilon T+\tfrac{1}{4}(1+\eta)^2T^2\,,$$
$$d\eta/dl=-\tfrac{1}{2}\eta(1+\eta)^2T\,, \qquad (3)$$

where $\eta=(g_1-g_2)/(g_1+g_2)$ and $T=g_1^{-1}$ is a temperature scale. Apart from the trivial fixed point $T=0$, $\eta=0$, there is only one fixed point which occurs for $(T^*)^{-1}=g_1^*=g_2^*$, i.e., $\eta^*=0$. We have obtained T^* at two-loop order: $T^*=4\epsilon-2\epsilon^2$. This fixed point has only one direction of instability; hence it describes a simple second-order transition. As is readily seen in Eq. (2), the line $g_1=g_2$ has a larger symmetry which is $O(3)\times O(3)/O(3)_{\rm diag}=O(4)/O(3)$. Since the fixed point is on this line, this means that it is the Wilson-Fisher $N=4$ fixed point in $D=2+\epsilon$. Using the recursion equation (3), we find $1/\nu=\epsilon+\frac{1}{2}\epsilon^2$. This is exactly the result found in $2+\epsilon$ for the $O(N)$-vector model when $N=4$, as expected. We thus find no new fixed point for the $O(3)\times O(2)/O(2)_{\rm diag}$ model but we find the general phenomenon of increased symmetry at a critical point in agreement with common belief. More generally, we find that all $O(3)\times O(p)/O(p)_{\rm diag}$, $p=1,2,3$, models are governed by the O(4) fixed point. This dynamically generated O(4) symmetry means that in the critical domain the three Goldstone modes become completely equivalent. In particular, the associated spin-wave velocities are renormalized to a single value. It is a new phenomenon to find such an O(4) symmetry in a Heisenberg system. Note that this is completely different from the O(4) proposed in the case of XY spins.[11]

How can we relate these findings with the previous studies of the linear theory given by Eq. (1)? In a $D=4-\epsilon$ calculation, one finds the O(6) fixed point on the u axis at a distance of order ϵ from the origin. The operator $(\mathbf{\Phi}_1\times\mathbf{\Phi}_2)^2$ opens a direction of instability as is expected from the general study of N-component models. The RG flow will thus drive the transition to first order. Certainly, the low-temperature expansion which is at the basis of the NLσ model must miss this kind of behavior since it forgets about exponentially small contributions from the massive modes. This mismatch between two and four dimensions is very different from the standard $O(N)$ case where one can follow smoothly the Wilson-Fisher fixed point from $4-\epsilon$ and $2+\epsilon$ expansions. Such a phenomenon also happens in the case of σ models built on Grassmannian spaces and used in the field-theoretic study of localization.[12]

Let us now discuss the $D=3$ physics. The simplest hypothesis is that all the $O(3)\times O(2)/O(2)_{\rm diag}$ models exhibit a first-order transition. However, there is a more intriguing possibility which is in agreement with the perturbative results: These models can undergo a first-order transition or a second-order O(4) transition according to their microscopic Hamiltonian. In addition, one expects to find some systems at the boundary between these two behaviors. The boundary of the basin of attraction for the O(4) fixed point is governed by the tricritical point which is at the origin when $D=3$. In this scenario, the systems characterized by the $O(3)\times O(2)/O(2)_{\rm diag}$ breaking can have a first-order transition or a second-order transition with O(4) exponents or a second-order transition with tricritical mean-field (TMF) exponents. Eventually, by tuning parameters it should be possible to observe O(6) critical behavior. In this scheme, we do not have to introduce any unknown fixed point, contrary to Kawamura.[13] It is the simplest possibility to account for second-order transitions in $D=3$. We can rephrase it in a more appealing manner: If we consider a definite model and vary its dimension, in the neighborhood of

$D=2$, it will be in the basin of attraction of the O(4) fixed point. If we increase the dimension, it will cross the boundary of the basin. For this peculiar nonuniversal D_c, it has tricritical behavior. Beyond this value, the transition becomes first order. We therefore think that the critical behavior of canted-Heisenberg-spin systems is nonuniversal. This is to be contrasted with the hypothesis of a new universality class for canted magnets. In what follows, we shall show that our simple hypothesis is likely to be true.

Let us now discuss results obtained from MC simulations on the STA [1] and BCT [2] lattices with Heisenberg spins. As mentioned above, both models undergo a continuous transition with critical exponents $\gamma=1.1\pm0.1$, $\nu=0.55\pm0.03$, $\beta=0.28\pm0.02$ and $\gamma=1.0\pm0.1$, $\nu=0.57\pm0.02$ for the STA and BCT models, respectively. These exponents are clearly different from those of the $N=4$ universality class, namely,[14] $\gamma=1.47$, $\nu=0.74$, $\alpha=-0.22$, $\beta=0.39$. We are thus left with the possibility that the transition is tricritical (or at least in the very neighborhood of the tricritical point) with classical exponents $\gamma=1$, $\nu=0.5$, $\beta=0.25$: The MC data support this hypothesis.

The helical-paramagnetic transition in heavy rare-earth-metal Tb, Dy, and Ho helimagnets has been studied extensively during the last decade.[15-23] The lack of universality in these results does not seem to support the existence of a new universality class for helimagnets. We shall show how our simple hypothesis enables us to understand the experimental data. (i) The β exponents found for Ho in Ref. 24, $\beta=0.39\pm0.03$, and for Dy in Ref. 25, $\beta=0.39\pm0.02$, are close to the O(4) values. (ii) Tindall and co-workers[15,16] found for the same elements a weakly first-order transition. (iii) The values of the critical exponents found by Gaulin, Hagen, and Child[17] for both Dy, $\nu=0.57\pm0.05$, $\gamma=1.05\pm0.07$, and Ho, $\nu=0.57\pm0.04$, $\gamma=1.14\pm0.1$, agree, within experimental errors, with those of TMF exponents. The value of β found for Tb by Dietrich and Als-Nielsen,[18] $\beta=0.25\pm0.01$, is also TMF.

Altogether, these results support the existence in three dimensions of a tricritical surface separating, in the whole parameter space, a first-order region from a basin of attraction of the O(4) fixed point. Concerning specific-heat measurements, it has been found that for Dy, $\alpha=-0.2$,[19] $\alpha=0.18$,[20] and $\alpha=0.24$;[21] for Ho, $\alpha=0.27\pm0.02$;[22] and for Tb, $\alpha=0.20\pm0.03$.[23] Apart from the value $\alpha=-0.2$, found for Dy, which is consistent with $N=4$, the other values of α disagree with both the O(4) and TMF ones, $\alpha=-0.22$ and $\alpha=0.5$, respectively. This dispersion may be due to the proximity of the tricritical surface or a sign that the transition is weakly first order.

Of course, in a real material there are anisotropies which make the spins neither completely XY nor Heisenberg. So far, our discussion has ignored this fact. We shall argue that a scheme similar to the Heisenberg case may also apply in the XY case. Frustrated XY-spin systems have been shown to possess, in general, a discrete Z_2 Ising-like degeneracy in addition to the O(2) one. It has been argued that, near two dimensions, two transitions associated to Z_2 and O(2) breaking should occur.[26] The values of the critical temperatures T_{Z_2} and T_{XY} are, of course, nonuniversal. Since the RG analysis predicts a single first-order phase transition near $D=4$,[26,27] we suggest that for a definite model, increasing the dimension D, the two lines of continuous transitions and the first-order line meet in some manner at a tricritical point (tetracritical). The position of this point should depend on the system under consideration. Results from MC simulations for both STA and BCT lattices with XY spins proceed in this sense. A single first-order transition was found in BCT,[2] whereas a continuous transition with exponents $\gamma=1.1\pm0.1$, $\nu=0.53\pm0.03$ occurs in STA.[1] These values are close to TMF. Moreover, experiments on the layered antiferromagnet $CsMnBr_3$,[28,29] where the spins are believed to be XY-like, show a second-order phase transition with exponents close to the TMF ones: $\gamma=1.01\pm0.08$, $\nu=0.54\pm0.03$, $\beta=0.21\pm0.02$ (Ref. 28) and $\beta=0.25\pm0.01$.[29] Although the situation is more complicated for XY spins since more than one transition is involved, we think that our scheme is qualitatively correct. For a given model, we predict that the phase transitions are either second order with standard Ising or O(2) universality classes, or first order or even second order with TMF exponents.

To summarize, our simple hypothesis allows us to shed light on the critical behavior of helimagnets and related canted-spin systems. The lack of universality of experimental results on Ho, Dy, and Tb is interpreted as the consequence of the existence of a tricritical point in the phase diagram of these systems. The transition can be either first order or second order with TMF or O(4) exponents provided the spins are Heisenberg-like. It remains to investigate the relevant parameters on which the onset of the first-order regime, as well as the occurrence of both O(4) and tricritical transitions, depend. This can be done by means of MC simulations. In systems where in-plane anisotropy is believed to be strong the situation might be more complicated. Even so, we have proposed a natural generalization of our hypothesis. Note that in real systems the situation can be complicated by multicriticality as is the case in $CsMnBr_3$.[30,31] It is quite surprising to find so many experimental results with TMF exponents since we expect that tricritical behavior is obtained by tuning at least one external parameter. This seemingly bizarre fact allowed in our scenario certainly deserves more investigation. In principle, there is a test which allows one to discriminate between our hypothesis and the existence of a new universality class: If one sees TMF-like exponents, since $D=3$ is the upper critical dimension for tricritical phenomenon, one should also see logarithmic corrections to leading scaling behavior.

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