Theory of $1/f$ Magnetic Flux Noise in High-T_c Superconductors

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A theoretical model is proposed to explain the magnitude of excess $1/f$ magnetic flux noise in high-T_c superconducting SQUID's. Based on a model for $1/f$ noise generation due to the multiple elastic scattering of quasiparticles with disordered mobile vortices, we show that the $1/f$ flux noise magnitude has the following features: It increases with T below T_c, sharply falls off with T above T_c, and decreases with an increasing energy gap. Its magnetic-field dependence is also shown to be inversely proportional to the square of the voltage-to-flux transfer function. Our results are shown to be in good agreement with recent experimental measurements.

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Recently there have been a number of experiments on high-T_c SQUID's that show unusual noise behaviors.[1-5] Koch *et al.*[1] found significant evidence that an additional noise mechanism is present in high-T_c SQUID's, which cannot be simply attributed to as the flux noise generated by the "flux-creep" motions, which is a dominant noise mechanism in low-T_c superconductors.[6] In particular, they were able to determine that the $1/f$ flux noise in the Y-Ba-Cu-O high-T_c SQUID's results from the *voltage* fluctuations, which have a significantly larger magnitude than those in classical SQUID's at 4.2 K, and that the $1/f$ voltage noise does not arise from the critical current fluctuations, but from some unknown source. In a related experiment, Ferrari *et al.*[4] reported a systematic experimental study of the flux noise in thin-film loops of Y-Ba-Cu-O superconductors, coupled magnetically to low-T_c SQUID's, over a temperature range 1.3–125 K. The experimental observations of the $1/f$ noise magnitudes can be summarized as follows. (1) Below T_c, the spectral density of the noise increases with temperature and varies from 10^{-9} to 10^{-3} in units of Φ_0^2 Hz^{-1}.[4-6] (2) Just above T_c, however, the noise power drops very rapidly, which can be fitted by a power law with a large exponent, i.e., $S_\phi \sim (T-T_c)^{-g}$ with $g \approx 12$.[4] (3) The $1/f$ voltage noise power S_v is almost independent of the applied flux,[5] but the spectral density of the flux noise S_ϕ shows a minimum where the transfer function has a maximum, which is a rather anomalous feature. (4) The magnitude of the noise depends strongly on samples, e.g., the microstructures of the film, and is the lowest for samples which are predominantly oriented with its c axis perpendicular to the substrate.[4] A more recent experiment[5] indicated that in a better quality film of $Tl_2Ba_2Ca_2Cu_3O_\delta$, which has higher T_c than those of Y-Ba-Cu-O materials, the noise magnitude can be dramatically reduced.[5] There does not exist at present a satisfactory explanation for these unusual noise behaviors in the high-T_c superconductors.

In this Letter, we propose a theoretical picture for the voltage-flux noise phenomenon in high-T_c superconductors, which accounts well for all of the above-mentioned interesting experimental observations. Our theory not only qualitatively accounts for the dependence of the noise magnitude on both the applied magnetic field and sample quality, but also quantitatively gives the temperature dependence of the noise power, including its surprisingly large peak at T_c.

As is well known, an Abrikosov (vortex) state can be formed when an external magnetic field exceeding H_{c1} is applied perpendicular to a thin film. From the nonlinear Ginzburg-Landau (GL) equations, one can easily obtain an isolated vortex wave function of the form $\Delta(r) = \Delta(T) \times \tanh(r/\xi)$, i.e., the pair potential $\Delta(r)$ is zero at the center of the vortex, and approaches its zero-field value $\Delta(T)$ at a distance of several coherence lengths ξ away from the center of the vortex.[7] This spatial variation of the Cooper-pair gap order parameter can be thought of as a potential well for the quasiparticles (QP), of depth $\Delta(T)$ and radius ξ. Quasiparticles could form bound states with energies less than $\Delta(T)$ within such a well, with an energy spacing of order $\hbar^2/2m\xi^2$. Such an intuitive physical picture has been confirmed by more rigorous theoretical calculations and experiments for classical type-II superconductors, and for the high-T_c superconductors.[8,9] In the high-T_c superconductors, however, the coherence length ξ is 2 orders of magnitude smaller than that in a conventional superconductor, so that there is almost no bound-state quasiparticles within a given vortex. This implies that the scattering states of the QP's dominate the transport properties of the system here. Also, the pinning energy for vortices in high-T_c superconductors has been shown experimentally to be typically small, such that a large number of vortices can be thermally activated. As a current exceeding the critical current of the SQUID is applied across the sample, the activated vortices are forced to move around by the Lorentz force, resulting in a macroscopic electrical field. The unbound QP's can then respond to this driving field. If the vortex lattice is *disordered*, and some of the vortices move around due to thermal activations, the transporting QP's scattering with the disordered mobile vortices will give rise to an additional conductance, as well as a conductance noise. This picture for the $1/f$ noise generation is closely related to the "universal conductance fluctuations" (UCF) phenomenon in disordered normal-metal systems, which have recently been studied extensively.[10]

The conductance due to the QP's can be evaluated using the Kubo formula. The conductance fluctuations can then be computed, following the UCF-type calculations,[10] from the characteristic diagram of the inset in Fig. 1. Each loop here represents the conductance of the quasiparticles flowing in the z direction of a given sample of length L_z. An effective vertex correction w denoted by a dashed line is inserted to account approximately for the effect of the Josephson junctions in a given SQUID on the transport of the quasiparticles. It is known from experiments that the junction region (link, pad, etc.) serves as an important noise source.[5] In fact, any kind of junction will more or less destroy the local regular lattice of the vortices and function as an effective elastic scatterer for the quasiparticles flowing through it. w depends on the actual structure of the junctions, and is proportional to the vortex density, and an effective scattering potential whose size may be somewhat larger than that from a normal vortex. We thus term w the *"junction correction factor,"* and note that w should be insensitive to temperature. We emphasize that our calculation does not refer to any specific mechanism for the formation of the Cooper pairs, and thus the fundamental mechanisms of high-T_c superconductors. We only assume *phenomenologically* the existence of the Cooper pairs, the corresponding GL order parameter, and the various Green's functions in the superconducting state. Since the high-T_c superconductors are extreme type-II superconductors, and the effect of an external magnetic field has been involved in the formation of the vortices, it is a conventional approximation to ignore the vector potential in deriving the various Green's functions.[8] The retarded and advanced zero-temperature Green's functions of quasiparticles are then given by[11]

$$g^{\pm}(\mathbf{p},\omega)=\frac{\omega\pm i(1/2\Lambda)+\xi_{\mathbf{p}}}{[\omega\pm i(1/2\Lambda)]^2-(\xi_{\mathbf{p}}^2+\Delta^2)}, \tag{1}$$

where $\Lambda^{-1}=\tau^{-1}+\tau_{\text{in}}^{-1}$, τ and τ_{in} are the elastic- and inelastic-scattering times for the QP, $\xi_{\mathbf{p}}=\epsilon_{\mathbf{p}}-\epsilon_F$, and $\Delta(T)$ is the energy gap assumed to be a real constant which is independent of wave vector $\mathbf{k}$ and frequency ω. For simplicity we use the natural units ($\hbar=c=1$). It may be shown that an effective interaction between a quasiparticle and a vortex can be approximately obtained by using the well-known coupled Green's-function expressions.[11] We then obtain for the effective scattering potential $u\approx\Delta^2(\zeta)^{d+1}/v_F$, where ζ is an effective range of a vortex, which is a few coherent lengths long. The elastic-scattering time τ is defined in the usual manner, $1/\tau=2\pi N_0 f_i u^2$, where f_i is the density of vortices and N_0 is the density of states of the quasiparticles. Similarly, treating Δ as a perturbation, we can calculate the diffusion propagator for the interfering quasiparticles, and it takes the form

$$P(q)=\frac{f_i u^2}{\tau(Dq^2+\tau\Delta^2+1/\tau_{\text{in}})}. \tag{2}$$

With the help of the usual Feynman rules,[10] we can now calculate explicitly the conductance fluctuations of the quasiparticles scattering off of moving vortices in a SQUID, at temperatures below T_c. We assume that the number of moving vortices which scatter the QP's is sufficiently large so that the "saturated" limit of the UCF noise generation is reached; namely, we take the fraction of moving vortices $p>\tau/\tau_{\text{in}}$.[10] We shall present our results for both a quasi-two-dimensional film with thickness smaller than the mean free path for the QP's and a three-dimensional sample.

(1) Quasi-two-dimensional case (2D):

$$\langle\delta G^2\rangle\Big/\left(\frac{e^2}{h}\right)^2=\frac{1}{L_z^2}\frac{2\pi N_0^2 v_F^2\tau^3}{\tau\Delta^2+1/\tau_{\text{in}}}w^2. \tag{3}$$

(2) Three-dimensional case (3D):

$$\langle\delta G^2\rangle\Big/\left(\frac{e^2}{h}\right)^2=\frac{A}{L_z^3}\frac{2}{\sqrt{3}}\frac{\pi N_0^2 v_F\tau^{5/2}}{(\tau\Delta^2+1/\tau_{\text{in}})^{1/2}}w^2, \tag{4}$$

where A is the cross section of the wire forming the ring in a SQUID.

For T just above T_c, the above calculation needs to be modified. First, the QP Green's functions become those for the normal state, i.e., $g^{\pm}=1/(E-\xi_{\mathbf{p}}\pm i/2\tau)$. Second, the energy gap now exists only because of the fluctuation effect, i.e., the superconducting order parameter has the property $\langle\Delta\rangle=0$ but $\langle\Delta^2\rangle\neq 0$. Following the similar lines of reasoning which led to Eqs. (3) and (4), we obtain the following for the conductance fluctuations in the two different geometries of interest:

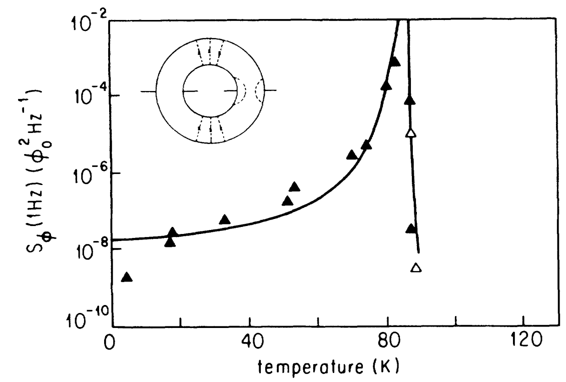

FIG. 1. The solid curve is the $1/f$ flux noise power calculated from our theory, using parameters in correspondence with those in the experiments of Ref. 4, and the solid triangles are the actual data. The open triangles at temperatures above T_c represent the white thermal noise, which are plotted here as reference. Inset: The Feynman diagram used to calculate the conductance fluctuations due to quasiparticles.

(1) Quasi-two-dimensional case (2D):

$$\langle \delta G^2 \rangle \bigg/ \left[\frac{e^2}{h} \right]^2 = \frac{1}{L_z^2} 2\pi N_0^2 v_F^2 \tau^3 \tau_{\text{in}} w^2 \qquad (5a)$$

$$\sim w^2 \tau^3 \tau_{\text{in}} ; \qquad (5b)$$

(2) Three-dimensional case (3D):

$$\langle \delta G^2 \rangle \bigg/ \left[\frac{e^2}{h} \right]^2 = \frac{A}{L_z^3} \frac{2}{\sqrt{3}} \pi N_0^2 v_F \tau^{5/2} \sqrt{\tau_{\text{in}}} w^2 \qquad (6a)$$

$$\sim w^2 \tau^{5/2} \sqrt{\tau_{\text{in}}} . \qquad (6b)$$

The spectral density of the flux noise $S_\phi(f)$, which is generated through the above quasiparticle conductance noise (and therefore the voltage noise), can then be written as

$$S_\phi(f) = \frac{S_v(f)}{(\partial V/\partial \phi)^2} = \frac{V^2}{(\partial V/\partial \phi)^2} \frac{\langle \delta G^2 \rangle R^2}{f \ln(\omega_{\max}/\omega_{\min})} , \qquad (7)$$

where R is the resistance of the SQUID, $\ln(\omega_{\max}/\omega_{\min})$ is usually assumed to be of the order 10–10^2,[10] and $\partial V/\partial \phi$ is the so-called voltage-to-flux transfer function. Equations (3)–(7) form the essential results of our theory. We now discuss the significance of the above results in interpreting the $1/f$ flux noise measurements in high-T_c SQUID's and loops.

From the above equations, we see that the conductance noise power $\langle \delta G^2 \rangle$, and hence the voltage noise power, is *independent* of the applied flux density below T_c. Further, the flux noise power S_ϕ obtained here is inversely proportional to the square of the transfer function $(\partial V/\partial \phi)$, in contrast to "conventional" flux noise from critical current fluctuations from an arbitrary source, which is only weakly dependent on the transfer function. Therefore, S_ϕ has a minimum where the transfer function is a maximum as indicated by experiment.[5] These features obtained here are in correspondence with the findings in Ref. 5.

We can also see from Eq. (3) [or Eq. (4)] and Eq. (7) that $S_\phi \sim 1/\Delta^{10}\zeta^8$ [or $\sim 1/\Delta^9\zeta^8$], i.e., the noise power *decreases* with *increasing* energy gap sharply when all other parameters remain fixed. This prediction is supported qualitatively by experimental observations in Ref. 4 where the noise magnitude decreases markedly as the degree of crystallographic orientation of the film is improved. This we can argue results from the fact that for better oriented samples, the energy gap is enhanced due to the evidence for the large in-plane energy gap ($\sim 8kT_c$) and a relatively small out-of-plane energy gap ($\sim 3kT_c$). This trend is also well supported by the measurements in Ref. 5, where by making a SQUID from $Tl_2Ba_2Ca_2Cu_3O_\delta$ instead of Y-Ba-Cu-O the flux noise becomes appreciably reduced. As the $Tl_2Ba_2Ca_2Cu_3O_\delta$ sample has a transition temperature near 125 K, whereas the Y-Ba-Cu-O compounds are superconducting at about 90 K, and assuming reasonably that the energy gap of the former is larger than of the latter, we again are able to explain such an observed trend from our theory.

Even more remarkable is our theoretical prediction of a divergencelike peak of the $1/f$ flux noise near T_c. Just above T_c, using the 2D example, we get from Eq. (5),

$$\langle \delta G^2 \rangle \sim w^2 \tau^3 \tau_{\text{in}} \sim w^2 \tau_{\text{in}} \frac{1}{\langle \Delta^2 \rangle^6}$$

$$\sim w^2 \tau_{\text{in}} \tau_0^{12} \sim \frac{w^2 \tau_{\text{in}}}{(T - T_c)^{12}} ,$$

where we have used the relation between τ and Δ discussed before, τ_0 is the lifetime of the fluctuating order parameter Δ which has approximately the temperature dependence[12] $\tau_0 \sim 1/(T - T_c)$ just above T_c, whereas the inelastic time τ_{in} is roughly constant in the vicinity of T_c.[12] The sharp decrease of the noise power predicted here is in good agreement with the experimental observations by Ferrari *et al.*,[4] namely, $S_\phi \sim (T - T_c)^{-12}$.

It is also important to estimate the numerical magnitudes of $S_\phi(f)$, based on realistic assumptions about the various experimental parameters. As we are only concerned with the leading contribution to the noise magnitude, the factor $1/\tau_{\text{in}}$ is assumed to be small in comparison with $\tau\Delta^2$ and can thus be dropped. Taking the parameters to correspond to those in the Koch *et al.*[1] experiment: $w \sim 100 f_i u^2$, $v_F \sim 10^8$ cm/s, $L_z \sim 40$ μm, $\tau \sim 10^{-3}$ cm, $\Delta \sim k_B T_c \sim 10^2$ K, $\ln(\omega_{\max}/\omega_{\min}) \sim 10$, $\partial V/\partial \phi \sim 10^{-8}$ V/Φ_0^2, $V \sim 10^{-3}$ V, and $R \sim 0.1$ Ω, we obtain an estimate for the noise power $S_\phi(1\text{ kHz}) \approx 10^{-7} \times \Phi_0^2$ Hz^{-1} in the quasi-2D case. The measured noise power is of the order $(10^{-5}$–$10^{-6})\Phi_0^2$ Hz^{-1}.[5] Thus we see that our model can account for the experimentally observed noise magnitudes rather well. When comparing our results with the experiment by Ferrari *et al.*,[4] we need to exercise some care in our interpretations, as their high-T_c thin-film ring is not connected directly to a current source. However, as the high-T_c thin-film ring is strongly coupled *magnetically* to the low-T_c SQUID, and as the latter *is* subject to an external current, the activated vortex motions in the low-T_c SQUID induce similar motions of vortices in the high-T_c ring on the top, which in turn produce a macroscopic electrical field. The QP's in the high-T_c ring thus respond to this electrical field to produce conductance fluctuations. This QP conductance noise again converts into flux noise which couples magnetically back to the low-T_c SQUID and gets measured directly. Insofar as the magnetic coupling between the high-T_c ring and the low-T_c SQUID has been accounted for, we are justified in applying our model to their geometry. We also notice that their high-T_c ring size is larger, roughly 1 mm, and their junction correction factor can be argued to be about an order of magnitude smaller, because of the absence of Josephson junctions. If we keep the other parameters the same as above, then we obtain $S_\phi(1\text{ Hz}) \sim 10^{-8}\Phi_0^2$ Hz^{-1}. This

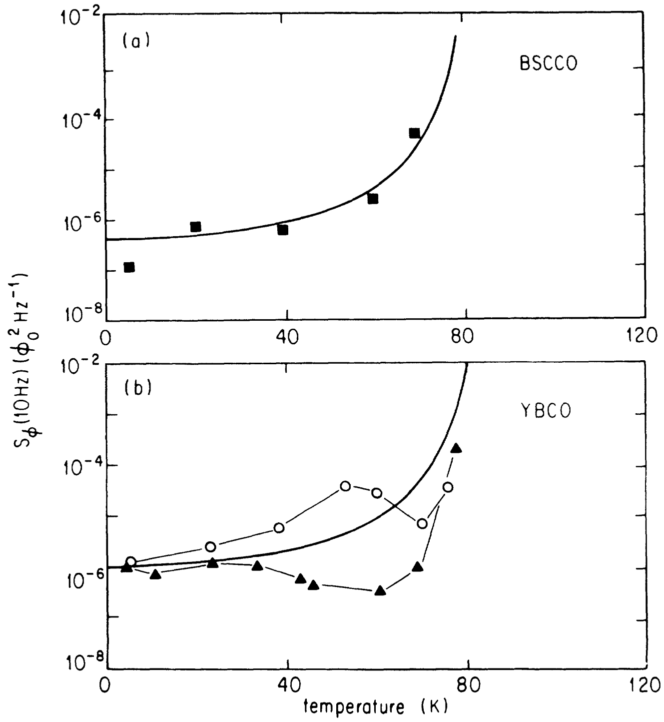

FIG. 2. $1/f$ flux noise powers vs temperature for two samples from Ref. 13. (a) A Bi-Sr-Ca-Cu-O system; (b) a Y-Ba-Cu-O system. The solid curves are the corresponding predictions calculated from our theory.

value corresponds well with the experimental values[5] in the temperature range from 15 to 45 K [$S_\phi(1\ \mathrm{Hz}) \sim 10^{-8}\Phi_0^2\ \mathrm{Hz}^{-1}$].

Finally, we plot the numerical calculations from our theory against the experimental measurements with the different configurations.[4,13] In Fig. 1, the experimental data for the entire temperature range 1.3–89 K represented by the triangles are from the thin-film ring of Y-Ba-Cu-O with $T_c = 85$ K, which are magnetically coupled with classical SQUID's at 4.2 K.[4] The direct noise measurements in high-T_c SQUID's made from Bi-Sr-Ca-Cu-O and Y-Ba-Cu-O and the corresponding theoretical curves from our theory are shown in Figs. 2(a) and 2(b), respectively.[13] We see that our theoretical calculation for the noise power is in good agreement with the measured values. The divergence at T_c should be cut off by the inelastic-scattering time which we have so far neglected. It should be pointed out that there is a discrepancy between the theoretical results and the experimental measurements near $T=0$, which in our picture can arise because the fraction of moving vortices p becomes smaller, and therefore the UCF saturation condition is no longer valid.

In summary, we have presented a theoretical analysis for the spectral density of the $1/f$ flux noise in high-T_c SQUID's. Our basic mechanism is one in which the multiple elastic-scattering events between the quasiparticles and moving vortices give rise to voltage fluctuations which are then related to the flux fluctuations. The main features of our theory are that the $1/f$ flux noise magnitude increases with temperature below T_c, sharply falls off above T_c, and is dependent on sample quality through the parameters such as the energy gap, the transfer function, the junction resistance, and the junction correction factor w. The voltage noise power is found to be *independent* of the applied flux. All available experimental data are shown to be consistent with the predictions of our model.

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