Statistical Distribution of Trajectories in the Time-Intensity Plane during Semiconductor-Laser Gain Switching

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We show that a semiconductor laser switched from below to above threshold by a gradual increase of the optical gain, which follows a stepwise change of the injected current (gain switching), exhibits, besides a random distribution of the turn-on times, peculiar features in the time-intensity evolution. In particular, we find that the switching trajectories in the time-intensity plane cluster under a line at a nonzero angle off the time axis. This new effect can be used to discriminate transient fluctuations due to spontaneous emission against those arising from other effects.

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Soon after its realization, the laser was investigated as a prototype of a system undergoing a second-order phase transition. When a laser is switched from below to above threshold, spontaneous emission fluctuations play an essential role in driving the laser towards the final steady state in which coherent radiation is established inside the cavity.[1] Two different procedures can be followed to switch on a laser: Q switching and gain switching. Q switching is mainly employed for gas and solid-state lasers, while gain switching is the usual way to switch on the semiconductor lasers used as sources in optical communication systems. The study of the transient behavior of semiconductor lasers is then an important task from the point of view of the applications, and a lot of work has been recently spent in this direction.[2]

The stochastic dynamics of laser switching has been well studied both experimentally[3,4] and theoretically[5,6] when the change of laser gain is faster than all the characteristic times involved in the laser dynamics. Two different time intervals characterize this phenomenon. The initial one is dominated by spontaneous emission fluctuations, and gives rise to a random delay time between the switching of the cavity and the buildup of the optical pulse. In this stage of the evolution, the intensity is very small and the equations governing the phenomenon are linear. During the second period, the system follows a trajectory, in the time-intensity plane, which is practically deterministic but its dynamics is strongly nonlinear, due to the large stimulated emission inside the cavity. Being the same as the value of population inversion at the moment of the photon buildup, any individual realization appearing in this period is identical to the other ones, but shifted in time by a random amount determined by the initial evolution.

Only recently, the case has been faced in which the laser gain is swept slowly, with respect to the rise time of the optical pulse, from below to above threshold (gain switching).[7,8] The influence of fluctuations in determining the macroscopic evolution while the laser is switched on has also been pointed out.[9] A detailed stastistical theory, however, has been performed for gain switching of both gas[10] and semiconductor[11] lasers only by considering the initial portion of the optical pulse, for which a linear evolution can be assumed.

In this paper, the statistics of the intensity fluctuations during the transient which follows a sudden switch of the injected current from below to above threshold in a semiconductor laser is considered also in the nonlinear region of the optical pulse emission. Because of the slow dynamics of the number of minority carriers, the optical gain does not immediately follow the abrupt change of the current, but increases slowly in a first stage of the evolution. A peculiar structure of the switching trajectories is found experimentally; as in the case of Q switching, where the optical net gain is changed abruptly, the trajectories spread out at the beginning of the leading edge of the optical pulse, but here they accumulate under a straight line which is not parallel to the time axis. The initial spontaneous emission does not produce a random rigid translation of any switching trajectory, but each of them attains a maximum of the intensity strictly dependent on the random delay between the current step and the emission of the optical pulse. This is a consequence of the fact that in gain switching the population inversion is still growing with time when the photon buildup begins so that the later the switching occurs, the larger the values the population inversion and, consequently, the photon density will attain during the first overshoot peak.

The experiment was performed by applying a fixed current step to a laser diode biased at various current levels below threshold. The rise time of the electrical step is about 100 ps so that, after this initial delay, the density of excited carriers, and hence the gain, increases almost linearly from the bias value until it is affected by the growth of the photon density through the stimulated emission.[12] The experimental results are shown in Fig. 1, where some pictures of a sampling oscilloscope screen are displayed. The need for a sampling oscilloscope arises from the short times involved in semiconductor-laser dynamics. The use of such an oscilloscope, which gives a single dot for each realization of the process, al-

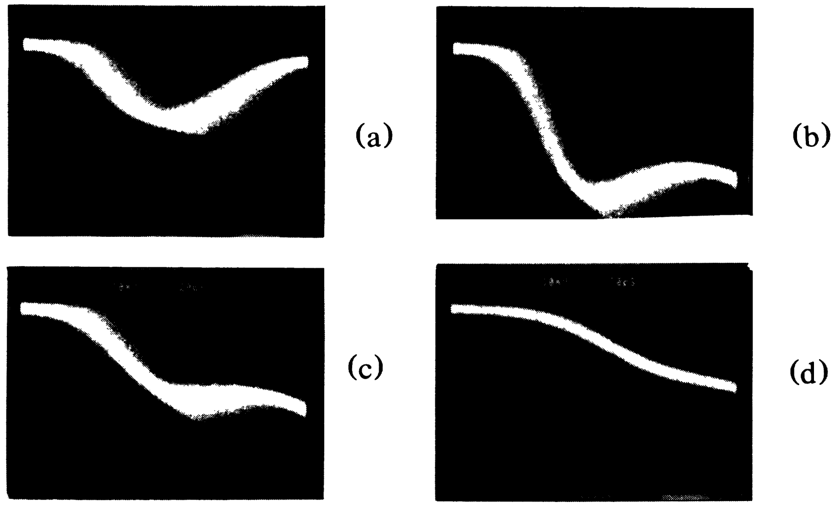

FIG. 1. Experimental traces of the laser switch on displayed by a sampling oscilloscope. The laser is a DFB laser with a threshold current of 28.9 mA. The driving conditions are pulse current 60 mA and bias current (a) 1 mA, (b) 21 mA, (c) 28.1 mA, and (d) 29 mA. The horizontal scale is 20 ps/div.

lows us to perform a statistical analysis of the ensemble of the trajectories even if the single trajectory is not experimentally accessible. The pictures shown in Fig. 1 are obtained using the same equipment described in Ref. 2 and show the leading edge of optical pulses emitted by a distributed-feedback (DFB) laser (Fujitsu FLD 150 F, working at 1.55 μm) under different driving conditions. In all the pictures the pulse current is set at 60 mA, while the bias is (a) 1 mA, (b) 21 mA, (c) 28.1 mA, and (d) 29 mA; the threshold current is 28.9 mA. Figure 1(d), which corresponds to a biasing slightly above threshold, shows that the fluctuations displayed in Figs. 1(a)–1(c) are really the consequence of the passage of the laser through a critical point during the switching. Furthermore, it shows that time jitter due to the electronic apparatus is negligibly small in our working conditions.[13] In the pictures, the number of sampling dots per unit area is proportional to the probability that a single trajectory crosses it. The increased density of dots for times a bit shorter than those corresponding to the attainment of the maximum intensity is then a mark that all the trajectories cluster in that region, however far apart they are at the beginning of the pulse. Moreover, all the trajectories stay under a straight-line envelope

TABLE I. Meaning of the symbols and values of the physical constants used in the simulation.

Symbols	Meaning	Values and units
c	Speed of light in vacuum	3×10^{10} cm s^{-1}
n_g	Group refractive index	4
α	Linewidth enhancement factor	4
τ_p	Photon lifetime	2.6×10^{-12} s
γ	Fraction of spontaneous emission coupled into the lasing mode	2.2×10^{-5}
$C_{\rm th}$	Electronic current density at threshold	4.1×10^{26} cm^{-3}s^{-1}
$C_{\rm on}$	Electronic injected current density in the on state	$2C_{\rm th}$
$C_{\rm bias}$	Electronic injected current density in the bias state	$0.9C_{\rm th}$
$\tau_{\rm sp}$	Spontaneous carrier lifetime	2×10^{-9} s
η	Mode confinement factor	0.5
A	Stimulated emission factor	5.6×10^{-6} cm^{3}s^{-1}
n_0	Carrier density at transparency	6.8×10^{17} cm^{-3}

making an angle with the horizontal axis which is a function of the bias current and is not zero, as it is in the case of fast switching of the optical gain.[14]

To explain this behavior of the stochastic dynamics of gain switching, let us consider the stochastic rate equations for the complex field E inside the cavity and for the number of minority carriers n, which in a single-mode semiconductor laser read[15]

$$\frac{dE}{dt}=\frac{c}{2n_g}(1-i\alpha)g(n,|E|^2)E - \frac{E}{2\tau_p}+\left(\frac{\gamma}{\tau_{sp}}n\right)^{1/2}F(t)\,, \quad (1)$$

$$\frac{dn}{dt}=c-\frac{n}{\tau_{sp}}-\frac{c}{n_g}g(n,|E|^2)|E|^2\,, \quad (2)$$

where

$$g(n,|E|^2)=\eta\frac{n_g}{c}\frac{A(n-n_0)}{1+|E|^2/\chi}\,. \quad (3)$$

In Eqs. (1)-(3), which are stochastic differential equations in the Ito sense, the units of E are such that $|E|^2$ is the photon number inside the cavity. The meaning of the above symbols is explained in Table I. χ, the saturation photon density, which arises from the intraband dynamics of semiconductors,[16] has been chosen as 7 times the photon density at the stationary steady state. $F(t)$ is a complex zero-mean Gaussian white-noise term, whose correlation functions are $\langle F(t)F^*(t')\rangle=\delta(t-t')$ and $\langle F(t)F(t')\rangle=0$. The Langevin noise term has been omitted in the equations for the carriers (2) because its effect on both dynamical and stationary statistical properties is negligible.

From a mathematical point of view, the origin of the nonhorizontal envelope of trajectories in the time-intensity plane arises from the fact that the system is described by two coupled equations in which the dependence of the gain on the carriers cannot be adiabatically eliminated.

In order to compare the experimental results with the rate-equation model, we have performed a simulation on 10^5 trajectories of the set of stochastic differential equations (1)-(3) by modeling our experiment as a sudden switch, at $t=0$, of the injected current from a value $C_{\rm bias}<C_{\rm th}$ to a value $C_{\rm on}>C_{\rm th}$, with $C_{\rm th}$ being the threshold current. The numerical algorithm used for the integration has been the Heun method[17] with an integration step of 2 ps, and the values of the physical parameters used in the simulation are listed in Table I. The initial distributions of photons and carriers have been chosen after 1 ns of thermalization of the system at $C_{\rm bias}=0.9C_{\rm th}$. The results are reported in Fig. 2, where a three-dimensional plot of the probability distribution of the intensity versus time is shown. The time origin has been set at the deterministic time at which the threshold is reached. This time exceeds the time at which the current is switched by a value

$$t_{\rm th}=\tau_{sp}\ln[(C_{\rm on}-C_{\rm bias})/(C_{\rm on}-C_{\rm th})]\,.$$

The accumulation of trajectories under the straight line of the envelope is clearly shown as a peak of the probability distribution around the maximum of the intensity overshoot. It has to be noted that for a fixed time slightly exceeding this maximum, the probability distribution of the intensity has two relative maxima, because, at that time, while most of the realizations are decreasing, there are some which are still around their maximum of intensity. This behavior resembles the transient bimodality of Broggi and Lugiato,[18] even if in our case the two maxima of the probability distribution are both centered on two nonstationary states. The agreement between experiment and simulation is good. Even if less clearly, the presence of two maxima of the probability distribution is also shown by the experimental results where the accumulation of points below the already mentioned line is still visible when most of the realizations are decreasing.

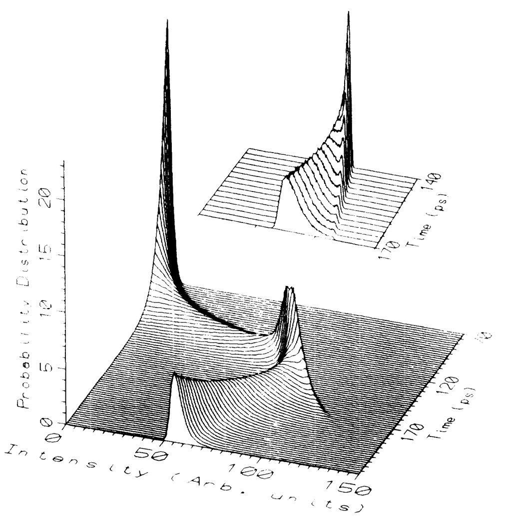

FIG. 2. Probability distribution of the intensity vs time obtained by simulation. Inset: An enlargement showing, from a different viewpoint, the double-peaked region of the pair distribution function.

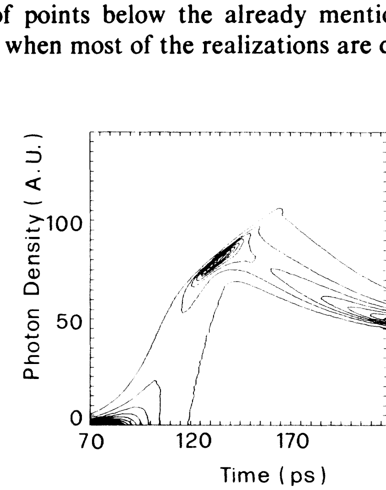

FIG. 3. The same as Fig. 2, but as a two-dimensional plot of level curves. The lower level curve corresponds to a pair distribution function equal to 0.2, and the step is 1.

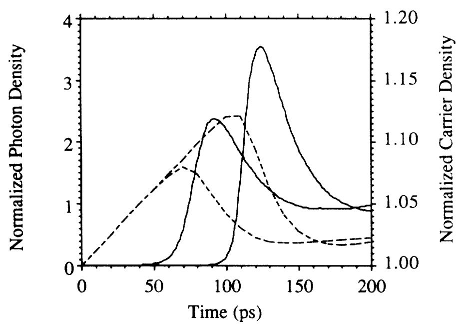

FIG. 4. Photon density, normalized with respect to its stationary value (solid lines), and carrier density, normalized with respect to its threshold value (dashed lines), vs time for two different realizations of the laser switching.

Figure 3 is the same as Fig. 2, but presents the results as a two-dimensional plot of level curves. The presence of an envelope for the trajectories is due to the fact that the evolution of the system is determined by only one random variable, that is, an effective initial condition for the photon density.[5,19] The carrier density grows, in fact, linearly and its values are perfectly deterministic until the photon density becomes large enough to make the term $g|E|^2$ no longer negligible in Eq. (2). A perfect correlation between the time of the emission of the optical pulse and the carrier density at the same instant is then present, so that the maximum value the photon density reaches during the first intensity overshoot is a function of the time at which this maximum is attained. This point is better understood by looking at Fig. 4, where the laser intensity and the number of minority carriers versus time are reported for two different realizations. It is clear how the later the photons build up, the larger the overshoot of the number of minority carriers and, consequently, of the laser intensity is.

The peculiar structure of the time jitter affecting any individual trajectory of gain switching of semiconductor lasers can be usefully employed to discriminate intrinsic laser fluctuations from those due to distributing external sources. The rise time of the optical pulse is, in fact, of the order of some tens of picoseconds, so that both the electronic jitter and the noise of the vertical amplifier of the oscilloscope could be misinterpreted as intrinsic laser fluctuations. The electronic jitter and the amplifier noise, however, give rise only to a spreading of the sampling dots, independent of either the time axis position or the intensity. The probability distribution of the switching trajectories shown in this paper can then be considered as the signature of the fluctuations due to the intrinsic switching dynamics of semiconductor lasers.

In this Letter we have shown how, while the laser is switched on, a laser system described by two coupled equations, exhibits, besides the well-known random distribution of turn-on time, a peculiar structure of the ensemble of the turn-on trajectories. This behavior, which in the case of semiconductor lasers can affect the performances of high-bit-rate optical communication systems, can, in general, account for the dynamics of the decay of different physical systems from an unstable equilibrium point.

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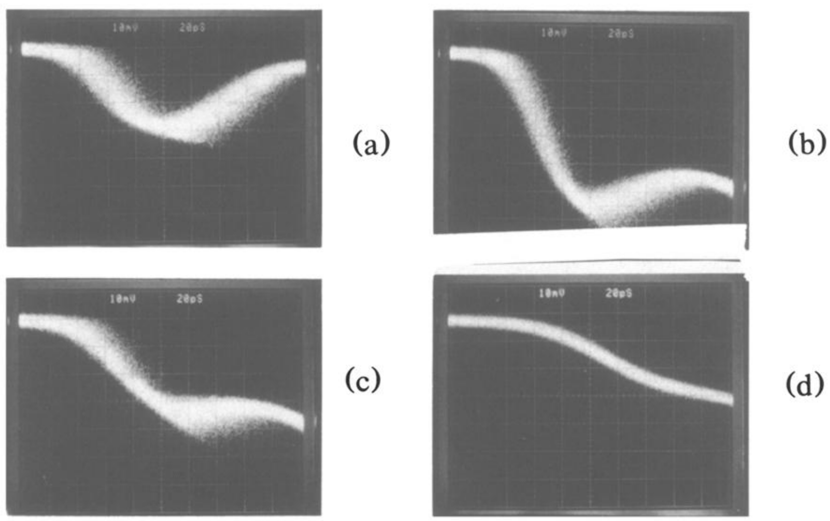

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