QCD Plasma Parameters and the Gauge-Dependent Gluon Propagator

R. Kobes,[(1)] G. Kunstatter,[(1)] and A. Rebhan[(2),(a)]

[(1)]*Physics Department, University of Winnipeg, 515 Portage Avenue, Winnipeg, Manitoba, Canada R3B 2E9*

[(2)]*Institut für Theoretische Physik, Technische Universität Wien, Wiedner Hauptstrasse 8-10, A-1040 Vienna, Austria*

(Received 13 October 1989)

We derive the Ward identities that determine the gauge dependence of the QCD dispersion relations obtained from the ordinary gluon propagator in a certain class of gauges. These identities hold for complex structure functions at both zero and finite temperature. A direct consequence of our analysis is that the gauge dependence of the gluon-plasma damping constant obtained in recent one-loop calculations is due to an inconsistent approximation scheme.

PACS numbers: 12.38.Mh

At high temperatures and pressures, hadronic matter is thought to undergo a phase transition to a deconfined plasma phase consisting of weakly interacting, deconfined quarks and gluons.[1] There has been a great deal of controversy in recent years, however, concerning the stability of this plasma against external perturbations.[2,3] This controversy has been triggered in part by the gauge-fixing dependence of the gluon-plasma dispersion relations at the one-loop level. Although all one-loop calculations yield the same lowest-order plasma frequency for both the transverse gluon mode and the spatially longitudinal plasmon mode, the rate at which the plasma oscillations are damped seems to be gauge-fixing dependent, both with respect to magnitude and sign. Despite the fact that there exist arguments concerning the need to incorporate higher-loop corrections,[4] much of the controversy has focused on the correct definition of dispersion relations in hot QCD. In particular, several groups[5] have proposed dispersion relations derived from modified, manifestly gauge-fixing-independent gluon propagators. In these formulations gauge dependence of the propagator, and hence of the related dispersion relations, cancels algebraically order by order in the loop expansion. As well, calculations based on color-electric and -magnetic correlation functions have been motivated in part by the apparent gauge-fixing dependence of the damping constant.[6] It is worth noting, however, that these modified approaches still do not agree on the value of the one-loop damping constant. Pisarski,[4] on the other hand, has stated that the physical poles in the ordinary gluon propagator would be gauge independent if calculated accurately.

The purpose of this Letter is to clarify the issues raised in the above controversy by deriving a set of generalized Ward identities that govern the gauge dependence, in a wide class of gauges, of the QCD propagator and hence of the QCD plasma dispersion relations. The derivatio. of the identities is nonperturbative and does not distinguish between zero and finite temperature nor between real and imaginary parts of the structure functions. Given certain additional assumptions that will be made explicit in the following, the identities imply the gauge-fixing independence of the *physical* poles in the gluon propagator in the class of gauges considered here. At this level, our discussion simply generalizes earlier claims concerning gauge-fixing independence of the poles based on the S matrix[7,8] or on renormalization-group arguments,[9] neither of which are immediately applicable at finite temperature.

In the context of the gluon plasma, the identities explain the gauge-fixing independence of the lowest-order plasma frequency and imply that the gauge-fixing dependence found in one-loop calculations of the damping constant will be absent in a self-consistent perturbative calculation. Thus it follows that the previously mentioned perturbative modifications to the gluon propagator are unnecessarily complicated from the point of view of gauge-fixing dependence, although it may be convenient for other reasons to consider them. It should be noted that the proof does not establish the stability of the perturbative vacuum nor the validity of perturbation theory in general. It does, however, show that claims of the breakdown of standard perturbation theory at finite temperature made purely on the basis of one-loop calculations are premature.

In order to derive the desired identities, and to illustrate their general nature, we temporarily use the condensed notation of DeWitt[10] in which ϕ^i denotes the complete set of fields in a theory, and the index i includes all continuous and discrete labels of the fields. The summation convention then also implies integration over the space-time variables with appropriate measure. We henceforth denote functional differentiation with respect to the field ϕ^i by a comma with appropriate indices. Consider an arbitrary gauge theory with action $S[\phi]$ and gauge invariance under $\delta\phi^i = D^i_\alpha[\phi]\delta\xi^\alpha$. A compact equation for the generating functional of connected functions for this theory, with complete gauge-breaking terms, is given by

$$\exp(iW_F[J]) = \int \mathcal{D}\mu[\phi]\,\mathrm{Det}(F^\alpha{}_{,i}[\phi]D^i_\beta[\phi])\exp(i\{S[\phi] + \tfrac{1}{2}\eta_{\alpha\beta}F^\alpha[\phi]F^\beta[\phi] + J_i\phi^i\})\,. \quad (1)$$

The gauge-fixing conditions $F^\alpha[\phi]$ are arbitrary functionals of the quantum fields such that the Faddeev-Popov operator matrix $F^\alpha{}_{,i}[\phi]D^i_\beta[\phi]$ is invertible. Note that background field and pure axial gauges are not directly implementable by gauge conditions as in Eq. (1); their inclusion requires special attention, and will be discussed separately.[11]

An arbitrary, infinitesimal change ΔF in the gauge-fixing condition can be absorbed by a gauge transformation of the path-integral variables of the form[10] $\Delta\phi^i = D^i_\alpha[\phi]\mathcal{G}^\alpha{}_\beta[\phi]\Delta F^\beta[\phi]$, where $-\mathcal{G}^\alpha{}_\beta$ is the inverse of the Faddeev-Popov operator. It then follows that under these variations the change in the generating functional comes purely from the source term, so that[12]

$$\Delta W[J] \equiv W_{F+\Delta F}[J] - W_F[J] = -J_i \Delta X^i[J] , \qquad (2)$$

where

$$\Delta X^i[J] = \langle D^i_\alpha[\phi]\mathcal{G}^\alpha{}_\beta[\phi]\Delta F^\beta[\phi]\rangle . \qquad (3)$$

The change in the propagator $D^{ij} = \delta^2 W/\delta J_i \delta J_j$ under an arbitrary change in F at $J=0$ is obtained by differentiating both sides of Eq. (2) with respect to the external source:

$$\Delta D^{ij} = -\Delta X^i_{,m} D^{mj} - \Delta X^j_{,m} D^{im} , \qquad (4)$$

where

$$\Delta X^i_{,m} \equiv \delta \Delta X^i / \delta \bar\phi^m , \qquad (5)$$

$\bar\phi \equiv \langle \phi^i \rangle$, and we have used the fact that $\delta\bar\phi^i/\delta J_k = D^{ik}$. Note that it is not necessary here to distinguish between zero and finite temperature: At zero temperature, $\langle\ \rangle$ denotes the vacuum expectation value of time-ordered products, while at finite temperature, it denotes the thermal average.

We now consider the case of the Yang-Mills theory by replacing the generic field ϕ^i by the Yang-Mills vector potential $A^a_\mu(x)$, and the generator of gauge transformations is simply the covariant derivative operator $D^i_\alpha \equiv D^\mu_{ab} = \delta_{ab}\partial_\mu - g f_{abc} A^c_\mu$. Here $\{a,b=1,\ldots,N\}$ represent color indices, while $\{\mu,\nu=0,1,2,3\}$ represent space-time indices. Assuming global color symmetry and translational invariance, we can neglect the color indices and go to momentum space, so that Eq. (4) implies

$$\Delta D^{\mu\nu}(k) = -D^{\mu\lambda}(k)\Delta X^\nu_{,\lambda}(k) - \Delta X^\mu_{,\lambda}(k) D^{\lambda\nu}(k) , \quad (6)$$

where $\delta^{ab}\Delta X^\mu_{,\nu} = \delta\langle D^\mu_{ac}\mathcal{G}^c_d \Delta F^d\rangle/\delta\bar A^\nu_b$. These are the desired Ward identities governing the gauge-fixing dependence of the gluon propagator in gauges that can be unambiguously implemented by a generating functional of the form given in Eq. (1).

In the Abelian case, $\Delta X^\mu_{,\nu}(k) \propto k^\mu$ because $D^\mu \to \partial^\mu$ is field independent. In any linear gauge the photon self-energy is purely transverse, so that Eq. (4) can be used[11] to derive the well-known result that the photon self-energy is gauge independent to all orders. No such simple conclusions can be drawn in the non-Abelian case, since the corresponding gauge generators are field dependent. Indeed, even the Abelian fermion self-energy is, in general, gauge-fixing dependent due to the field dependence of the fermion gauge generator.

Returning to the case of non-Abelian Yang-Mills theories, we now restrict the discussion to the class of gauge theories in which the momentum-space gluon propagator can be decomposed as follows:

$$D_{\mu\nu}(k) = \alpha A_{\mu\nu} + \epsilon \frac{\tilde n_\mu \tilde n_\nu}{\tilde n^2} + \beta(\tilde n_\mu k_\nu + k_\mu \tilde n_\nu) + \delta \frac{k_\mu k_\nu}{k^2} , \qquad (7)$$

with $\tilde n_\mu = P_{\mu\nu} n^\nu$, $P_{\mu\nu} = g_{\mu\nu} - k_\mu k_\nu / k^2$ being the projection operator with respect to four-momentum, and

$$A_{\mu\nu} = P_{\mu\nu} - \tilde n_\mu \tilde n_\nu / \tilde n^2 = \begin{pmatrix} 0 & 0 \\ 0 & -\delta_{ij} + k_j k_j / \mathbf{k}^2 \end{pmatrix} \qquad (8)$$

projects out the spatially transverse physical gluon mode. The matrix $\tilde n_\mu \tilde n_\nu$ is also transverse with respect to the four-momentum k_μ and orthogonal to $A_{\mu\nu}$. At zero temperature, the above decomposition restricts the (possibly nonlinear) gauge condition to depend at most on the momentum four-vector and on one arbitrary, fixed vector n_μ. At finite temperature O(3,1) is broken to O(3) by the presence of the heat bath whose rest-frame velocity is n_μ, so that the gauge conditions can depend at most on the velocity of the heat bath and derivatives.

One can use Dyson's equation, $D_{\mu\nu}(k) = (D_{(0)}^{-1\mu\nu} + \Pi^{\mu\nu})^{-1}$, to express the structure functions of $D^{\mu\nu}$ in terms of the bare inverse propagator:

$$(D_{(0)}^{-1})^{\mu\nu} = k^2 P_{\mu\nu} + C_{(0)} \frac{\tilde n^\mu \tilde n^\nu}{\tilde n^2} + B_{(0)}(\tilde n^\mu k^\nu + \tilde n^\nu k^\mu) + D_{(0)} \frac{k^\mu k^\nu}{k^2} , \qquad (9)$$

and the proper self-energy,

$$\Pi^{\mu\nu} = \Pi P^{\mu\nu} + \chi \frac{\tilde n^\mu \tilde n^\nu}{\tilde n^2} + \Phi(\tilde n^\mu k^\nu + \tilde n^\nu k^\mu) + \Lambda \frac{k^\mu k^\nu}{k^2} . \qquad (10)$$

The result is

$$\alpha^{-1} = k^2 + \Pi \equiv \mathcal{T}(k) , \qquad (11)$$

$$\epsilon^{-1} = \mathcal{L}(k) , \qquad (12)$$

$$\beta^{-1} = -\frac{D_{(0)} + \Lambda}{B_{(0)} + \Phi} \mathcal{L}(k) , \qquad (13)$$

$$\delta^{-1} = \frac{D_{(0)} + \Lambda}{k^2 + \Pi + C_{(0)} + \chi} \mathcal{L}(k) , \qquad (14)$$

with

$$\mathcal{L}(k) \equiv k^2 + \Pi + \chi + \tilde n^2 k^2 \left[\frac{B_{(0)}^2}{D_{(0)}} - \frac{(B_{(0)} + \Phi)^2}{D_{(0)} + \Lambda} \right] . \qquad (15)$$

In Eq. (15) we have used the fact that the parameters $B_{(0)}$, $C_{(0)}$, and $D_{(0)}$ satisfy $B_{(0)}^2 = C_{(0)} D_{(0)} / \tilde n^2 k^2$. It is clear that apart from kinematical factors the physical poles in the gluon propagator are determined by the structure functions $\mathcal{T}$ and $\mathcal{L}$, whose zeros determine the dispersion relations of the spatially transverse (plane-wave) and spatially longitudinal (plasmon) collective modes, respectively, of the quark-gluon plasma.[13,14] Note that at zero temperature, in covariant gauges, Lorentz symmetry requires $\alpha=\epsilon$, so that there is only one independent propagating transverse mode. In noncovariant gauges α is not necessarily equal to ϵ, but the Ward identities guarantee that, in general, they will have the same physical poles, so again there is only one physical mode. At finite temperature, the structure function ϵ represents an additional collective "plasmon" mode, which propagates independently from the spatially transverse mode α.

Projecting Eq. (6) onto the orthogonal matrices $A_{\mu\nu}$ and $\tilde{n}_\mu \tilde{n}_\nu$, respectively, yields the Ward identities that determine the gauge dependence of the QCD dispersion relations:

$$\Delta\mathcal{T}(k) = -\mathcal{T}(k) A^{\lambda}_{\nu}(k)\Delta X^{\nu}_{,\lambda}(k) \equiv \mathcal{T}(k)\Delta Y(k) \quad (16)$$

and

$$\Delta\mathcal{L}(k) = -\mathcal{L}(k) 2\left[\frac{\tilde{n}_\nu \tilde{n}^\lambda \Delta X^{\nu}_{,\lambda}}{\tilde{n}^2} - \frac{B_{(0)}+\Phi}{D_{(0)}+\Lambda}\tilde{n}_\nu k^\lambda \Delta X^{\nu}_{,\lambda}\right] \equiv \mathcal{L}(k)\Delta Z(k) \,. \quad (17)$$

Thus, as long as the coefficients ΔY and ΔZ of Eqs. (16) and (17) are well behaved in the neighborhood of the solutions, the dispersion relations are gauge-fixing independent: A solution of $\mathcal{T}(k)=0$ is also a solution of $(\mathcal{T}+\Delta\mathcal{T})(k)=0$, and similarly for solutions of $\mathcal{L}(k)=0$.

It should be noted that this proof of gauge-fixing independence does not cover all zeros in $\mathcal{T}$ and $\mathcal{L}$. One must exclude from the proof such zeros which coincide with poles in ΔY or ΔZ, since in this case a cancellation in Eqs. (16) or (17) may occur. This could result in gauge-dependent poles such as those found by DeTar, King, and McLerran[15] in the electron propagator in superaxial gauges. However, such gauge-dependent kinematical poles will generally be exceptional, as the pole structure of ΔY and ΔZ is essentially that of the ghost propagator [cf. Eq. (3)]. As well, a potential simple zero of $\mathcal{L}$ at $k^2=0$ must also be excluded from this proof, as it may be canceled on the right-hand side of Eq. (17). It can, in fact, be shown that such a pole cancels in the equations of motion of the field strengths.[13] Similarly, a zero in $\mathcal{L}$ coinciding with that in $D_{(0)}+\Lambda$ must also be excluded due to a cancellation in Eq. (17), and indeed one can use Eq. (6) to prove that

$$\Delta(D_{(0)}+\Lambda) = 2[D_{(0)}+\Lambda+(B_{(0)}+\phi)\tilde{n}^2]k_\alpha \Delta X^{\alpha}_{,\beta}k^\beta \,, \quad (18)$$

so that this zero is, in general, gauge-fixing dependent. Equation (18) furthermore shows that only the structure functions α and ϵ of Eqs. (11) and (12) associated with the transverse part of the gluon propagator can have gauge-fixing-independent poles.

Thus, in using the identities (16) and (17) to examine gauge independence of poles in the gluon propagator, one must know something of the structure of ΔY and ΔZ. With this in mind, one can use these identities to make definite statements concerning the gauge-fixing independence of the physical poles in the gluon propagator. It is instructive to examine first how gauge dependence can be distributed across different loop orders of the gauge-independent dispersion relations $\mathcal{T}(k)=0$ or $\mathcal{L}(k)=0$. The identities (16) and (17) necessarily hold order by order in the loop expansion, even when higher-loop effects can contribute to the same order in the coupling constant, as is the case in a high-temperature expansion. Expanding, for example, Eq. (16),

$$\begin{aligned} &\Delta\mathcal{T}(k) = \mathcal{T}(k)\Delta Y(k)\,, \\ &\mathcal{T} = \mathcal{T}^{(0)}+\mathcal{T}^{(1)}+\mathcal{T}^{(2)}+\cdots\,, \\ &\Delta Y = \Delta Y^{(0)}+\Delta Y^{(1)}+\Delta Y^{(2)}+\cdots\,, \end{aligned} \quad (19)$$

where the superscripts correspond to any formal expansion parameter, one finds that at lowest order $\Delta\mathcal{T}^{(0)} = \mathcal{T}^{(0)}\Delta Y^{(0)}$. In the loop expansion $\mathcal{T}^{(0)}(k)$ is gauge independent, and accordingly $\Delta Y^{(0)}\equiv 0$. Thus, to one loop

$$\Delta\mathcal{T}^{(1)} = \mathcal{T}^{(0)}\Delta Y^{(1)}. \quad (20)$$

This equation guarantees gauge independence of the pole if the loop expansion is a consistent expansion: To the order of Eq. (20) the pole will be determined from $\mathcal{T}^{(0)}=0$, $\mathcal{T}^{(1)}$ being of higher order, and so the right-hand side of Eq. (20) vanishes.

However, gauge dependence can appear in the loop expansion when $\mathcal{T}^{(0)}$ and $\mathcal{T}^{(1)}$ are of the same order, so that in Eq. (20) the pole is determined from $\mathcal{T}^{(0)}+\mathcal{T}^{(1)}=0$. This is the situation of QCD at high temperature, where a mass of order gT is generated at one loop which cannot be considered small but instead sets the scale for the dispersion relation. An explicit calculation shows[11] that $\Delta Y^{(1)}, \Delta Z^{(1)} \sim O(g^2 T)$, which in the context of these identities explains the observed gauge-fixing independence of the plasma dispersion relations to order gT, as well as the gauge-fixing *dependence* to order g^2T in a loop expansion.

An illustration of the identities is provided by a recent calculation of the leading-order gluon damping constant, where a partial resummation of higher-loop effects has been performed in order to ensure that $\mathcal{T}^{(0)}=0$ at the plasmon mass.[16] The resulting "on-shell" dispersion relations were indeed gauge-fixing independent, as required by Eq. (20).

In a completely self-consistent expansion scheme one needs $\mathcal{T}^{(n+1)} \ll \mathcal{T}^{(n)}$. In the case of high-temperature QCD this means such an expansion is not in powers g, but rather in those powers of g in excess of the powers in T. In such an expansion the pole structure of $\Delta Y^{(1)}$ and

$\Delta Z^{(1)}$ is essentially that of the (resummed) ghost propagator,[11] which does not receive corrections of order g^2T^2, and hence is independent of the poles in $\mathcal{T}$ and $\mathcal{L}$ describing the collective modes of the plasma. Therefore, since apart from external ghost propagators ΔY and ΔZ consist of one-particle-irreducible functions whose internal propagators are off the plasmon mass shell even if the external momentum is on it, this renders plausible the assumption that ΔY and ΔZ are well behaved near the plasmon mass shell $k^2 \sim g^2T^2$. By this it follows that in such a consistent resummation the dispersion relations will be gauge-fixing independent. Supporting this argument, Braaten and Pisarski[17] have recently shown in covariant and Coulomb gauges that the complete resummation of loop effects will result to lowest order in gauge-fixing-independent dispersion relations. The analysis presented here shows that the plasma parameters will be gauge independent in a self-consistent perturbative expansion in any one of the class of gauges considered here. Of course, this class does not exhaust all possible gauge choices; the extension of these considerations to other types of gauges and illustrations of the identities in specific examples is in progress.[11]

We would like to thank R. Pisarski for helpful discussions. This research was supported in part by the Natural Sciences and Engineering Research Council of Canada.

---

[a]Present address: Theory Division, CERN, CH-1211 Geneva 23, Switzerland.

[1]Larry McLerran, Rev. Mod. Phys. **58**, 1021 (1986).

[2]For a current overview, see "Status of Perturbative Thermal QCD," edited by K. L. Kowalski, N. P. Landsman, and Ch.G. van Weert [Physica (Amsterdam) **158A** (1989)], and references therein.

[3]M. E. Carrington, T. H. Hansson, H. Yamagishi, and I. Zahed, Ann. Phys. (N.Y.) **190**, 373 (1989).

[4]R. Pisarski, Phys. Rev. Lett. **63**, 1129 (1989); see also Ref. 2.

[5]S. Nadkarni, Phys. Rev. Lett. **61**, 396 (1988); T. H. Hansson and I. Zahed, Phys. Rev. Lett. **58**, 2397 (1987); Nucl. Phys. **B292**, 725 (1987); R. Kobes and G. Kunstatter, Phys. Rev. Lett. **61**, 392 (1988); U. Kraemmer, M. Kreuzer, and A. Rebhan, Ann. Phys. (N.Y.) (to be published); H.-Th. Elze, CERN Report No. CERN-TH. 5223/88, 5423/89 (to be published); S. Catani and E. d'Emilio, Princeton University Report No. PUTP 1134, 1989 (to be published).

[6]K. Kajantie and J. Kapusta, Ann. Phys. (N.Y.) **160**, 477 (1985); U. Heinz, K. Kajantie, and T. Toimela, Phys. Lett. B **183**, 96 (1987); Ann. Phys. (N.Y.) **176**, 218 (1987); H.-Th. Elze, K. Kajantie, and T. Toimela, Z. Phys. C **37**, 601 (1988).

[7]G. 't Hooft and M. Veltman, Nucl. Phys. **B50**, 318 (1972).

[8]B. W. Lee and J. Zinn-Justin, Phys. Rev. D **5**, 3121 (1972).

[9]D. Gross, in *Methods in Field Theory,* edited by R. Balian and Jean Zinn-Justin (North-Holland, New York, 1976), p. 141.

[10]B. S. DeWitt, Phys. Rev. **162**, 1195 (1967).

[11]R. Kobes, G. Kunstatter, and A. Rebhan (to be published).

[12]See, for example, C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980), p. 604.

[13]H. A. Weldon, Phys. Rev. D **26**, 1394 (1982).

[14]R. Kobes, G. Kunstatter, and K. Mak, Phys. Lett. B **223**, 433 (1989); Z. Phys. C **45**, 129 (1989).

[15]C. DeTar, J. King, and L. McLerran, Nucl. Phys. **B249**, 644 (1985).

[16]U. Kraemmer, M. Kreuzer, A. Rebhan, and H. Schulz, Technische Universität Wien report, 1989 (to be published).

[17]E. Braaten and R. Pisarski, Phys. Rev. Lett. **64**, 1338 (1990).