Laser Cooling of Atoms in Squeezed Vacuum

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The modification of the statistical properties of the vacuum fluctuations, via quadrature squeezing, can dramatically change the mechanical manifestations of light on atoms moving in a standing laser wave. This phenomenon can be attributed to the altered decay rates of the atomic dipole in squeezed vacuum.

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Laser cooling of atoms in a quasiresonant standing laser wave has been attracting considerable attention during the past few years.[1] Another exciting subject has been the modification of the statistical properties of the vacuum fluctuations of the electromagnetic field. Reduction of these fluctuations in one quadrature phase (90° out of phase) of the field by almost an order of magnitude has been already realized in the laboratory. The role played by the vacuum fluctuations in the atomic dynamics has also been of great interest. Recently, Gardiner[2] has shown that the two quadratures of the atomic dipole decay at different rates when the atom is embedded in "squeezed vacuum," i.e., broadband light with reduced quantum fluctuations in one quadrature component. This state can be produced, in principle, by an optical parametric amplifier with vacuum fluctuations as an input. It is shown here, therefore, that the mechanical manifestations of light on a slowly moving atom in a standing coherent laser wave are dramatically modified when the atom is embedded in such a state. In the following, the physical origin of the optical forces in a standing laser wave is described and an intuitive model of the effects in a squeezed vacuum is offered, the method for calculating the force is presented and the modified force is compared to the force in a normal vacuum. Finally, the possibility of the experimental verification of these results is discussed.

A slowly moving atom ($\mathbf{k}\cdot\mathbf{v}<\Gamma$) in a low-intensity standing-laser-light wave experiences a velocity-dependent force. This "radiation pressure" force is well understood in terms of absorption and spontaneous emission. As first envisioned by Hänsch and Schawlow,[3] the atom experiences an *increased* absorption of photons from the laser beam which is shifted closer to resonance due to the Doppler effect. This velocity-dependent differential absorption can provide a cooling force for laser detunings to the red side of the atomic transition or a heating force for blue detunings. At high intensity, however, stimulated emission can change the sign of the force to a heating force at red detunings and to a cooling force at the blue side of resonance.[4,5] This stimulated force (or "dipole force") has been explained within the framework of the dressed-atom model[5] and equivalently as resulting from two-wave mixing[6] (TWM). The TWM resonance appears in pump-probe spectra as a dispersive line shape (as a function of the probes detuning from the pump). This feature has a width of the excited-state decay rate Γ and shows *decreased* absorption at probe detunings from the pump *closer to the atomic transition* (see Fig. 1, trace *B*). In this process the atom absorbs one photon from one wave and emits a photon into the counterpropagating wave, thus acquiring a momentum kick of $2\hbar\mathbf{k}$. This process usually requires high laser intensity; however, it has been shown to occur at lower intensity when the normal relation between the dipole decay rate Γ_2 and the excited-state decay Γ ($\Gamma_2=0.5\Gamma$) is modified by the inclusion of phase interrupting events ($\Gamma_2>0.5\Gamma$).[6] This effect is due to the appearance of the TWM process at lower order in laser intensity, which is closely related to the dephasing induced extra resonances in four-wave mixing. These resonances, which originally have been studied by Bloembergen and co-workers, are induced whenever the normal decay rates of the atom are modified. Their relevance to the stimu-

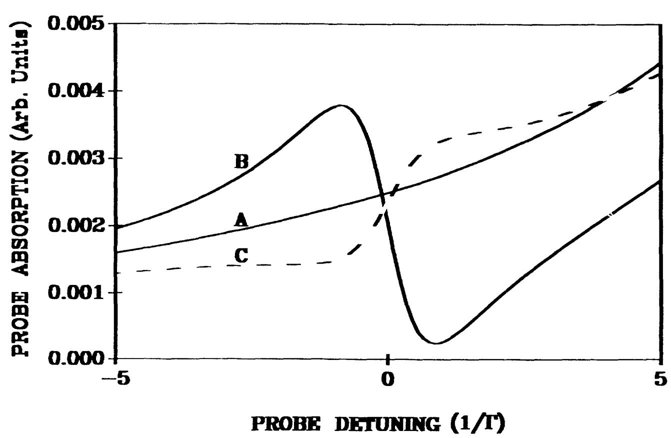

FIG. 1. Probe absorption as a function of its detuning from a pump tuned 20Γ to the red of an atomic transition. Trace *A*, at low pump intensity the probe sees higher absorption at positive detuning closer to the atomic transition. Trace *B*, in normal vacuum at high pump intensity the TWM process is induced leading to less absorption for frequency shifts closer to the atomic transition. Trace *C*, at the same high intensity as in trace *B* but in squeezed vacuum the TWM process can change its line shape leading to an additional cooling force [Eq. (12), Ref. 8 with $\bar{N}=0.1$, $M=0.33$, $\phi=\pi$, and $\Omega=8\Gamma$].

lated force is discussed in more detail in Ref. 6.

Armed with this insight into the connection between TWM and the stimulated force, it is instructive to investigate the effect of squeezing on the TWM process. Gardiner[2] has shown that, in general, squeezing the vacuum fluctuations results in two different decay rates for the two quadratures of the atomic dipole, one of which is larger and the other *smaller* than the normal $\Gamma/2$ value in ordinary vacuum. The implications of this result to a number of nonlinear optical processes[7-9] have been investigated. In particular, Ritsch and Zoller[8] and An and Sargent[9] have studied the affect of squeezing on TWM. Their calculations imply that the line shape of this process becomes phase dependent and can even change to a "dispersive" line shape with opposite sign (larger absorption closer to the atomic transition) as demonstrated in Fig. 1, trace *C*. This indicates that the stimulated force can change sign to provide an additional cooling force instead of heating for red laser detunings from resonance.

In order to calculate the forces acting on a slowly moving atom in a standing wave ($\mathbf{k}\cdot\mathbf{v}\ll\Gamma$), the semiclassical treatment of Gordon and Ashkin[4] is adopted. This procedure yields the average value of the force correct to all orders of the laser intensity and to first order in the atomic velocity. It is assumed that the atom is embedded in a broadband squeezed vacuum which is centered on the atomic transition, so that the squeezed vacuum appears to the atom as a δ-correlated squeezed white noise.[2] The optical Bloch equations (OBE) for a two-level atom with a ground state $|a\rangle$ and an excited state $|b\rangle$ which is embedded in squeezed vacuum are thus given by[2,4,8]

$$\begin{aligned}\langle\dot\rho_{ab}\rangle &= -\gamma\langle\rho_{ab}\rangle-\Gamma M\langle\rho_{ab}\rangle^*+\Omega\langle D\rangle\,,\\ \langle\dot D\rangle &= -\Gamma(2\bar N+1)\langle D\rangle+\Gamma-2[\Omega^*\langle\rho_{ab}\rangle+\Omega\langle\rho_{ab}\rangle^*]\,,\end{aligned}\tag{1}$$

where $\langle D\rangle=\langle\rho_{aa}\rangle-\langle\rho_{bb}\rangle$, $\gamma=\Gamma(2\bar N+1)/2-i\Delta$, Δ being the laser detuning from resonance, $\bar N$ is proportional to the number of photons in the squeezed vacuum, and M is the degree of squeezing (i.e., the amount of correlation between the sidebands) given by $|M|^2\le\bar N(\bar N+1)$, where the equality holds for maximum squeezing[2] (minimum uncertainty state). Finally, $\Omega=\mu_{21}E/\hbar$ is the Rabi frequency, where μ_{21} is the dipole moment and $E=2E_0\cos(\mathbf{k}\cdot\mathbf{x})e^{i\phi}$ is the laser field in a standing wave. The force is given by[4] $F=\alpha i\hbar[\Omega^*\langle\rho_{ab}\rangle-\Omega\langle\rho_{ab}\rangle^*]$, where $\alpha=-\mathbf{k}\tan(\mathbf{k}\cdot\mathbf{x})$ and is calculated by solving the OBE in the steady state and then by introducing the first-order corrections due to the atomic motion.[4] This procedure gives after some algebra the following expression of the force in squeezed vacuum:

$$\mathbf{F}=-\frac{\alpha\hbar[\Delta+\Gamma M\sin(2\phi)]P}{\Phi(2\bar N+1+P)}\times\left[1-\frac{4(|\gamma|^2-\Gamma^2M^2)M\cos(2\phi)P+\Gamma^2(2\bar N+1)^2\Phi(2\bar N+1-P)-2(|\gamma|^2-\Gamma^2M^2)P^2}{\Gamma(2\bar N+1+P)^2(|\gamma|^2-\Gamma^2M^2)\Phi}\alpha\mathbf{v}\right],\tag{2}$$

where $\Phi=2\bar N+1-2M\cos(2\phi)$ and $p=2\Phi|\Omega|^2/(|\gamma|^2-\Gamma^2M^2)$ being the modified saturation parameter. It is instructive to examine the new expression of the force by comparing it to the force in ordinary vacuum. In this limit ($\bar N=M=0$) the force is reduced to the well-known expression of the force [Ref. 4, Eq. (18)] given by

$$\mathbf{F}=-\alpha\hbar\Delta\frac{P}{1+P}\left[1-\frac{\Gamma^2(1-P)-2|\gamma|^2P^2}{\Gamma(1+P)^2|\gamma|^2}\alpha\mathbf{v}\right].\tag{3}$$

Note that in the normal vacuum limit the first term in the numerator of the velocity-dependent part of Eq. (2) is zero while the other two terms are reduced to those of Eq. (3). The striking appearance of the additional term in squeezed vacuum is analogous to the result of Ref. 6. In this case, the introduction of classical phase noise results in the appearance of an extra term $-4|\gamma|^2(\Gamma_2/\Gamma-0.5)P$ ($\Gamma_2=\Gamma/2+\Gamma_o$, where Γ_o is the rate of the phase interrupting events). This term can give the stimulated force usually given by $-2|\gamma|^2P^2$ at lower intensity when $\Gamma_2/\Gamma>0.5$ as phase noise is added. Notice that when $\Gamma_2/\Gamma<0.5$ this term can also be induced but with opposite sign. This is indeed the case with quadrature squeezing, which can result in either larger or smaller phase noise than the vacuum level. This in turn introduces two different decay rates for the two quadratures of the atomic dipole. One of these, $\Gamma_{2x}=\Gamma(\bar N+M+0.5)$, is larger and the other, $\Gamma_{2y}=\Gamma(\bar N-M+0.5)$, *is smaller* than the normal $\Gamma_2=\Gamma/2$ value. Therefore, the sign of the extra term in Eq. (2), $4(|\gamma|^2-\Gamma^2M^2)M\cos(2\phi)P$, can be controlled by the relative phase ϕ of the driving field with respect to the squeezed vacuum. This modification of the force can be further correlated with the TWM line shape which becomes strongly dependent on the laser phase ϕ and can even change sign as shown in Fig. 1, trace *C*. The physical implications of these results indicate that the stimulated force cannot only occur at lower laser intensity, it can change sign to provide an additional cooling force at red laser detuning from resonance.

Other important modifications of the force in squeezed vacuum are described by the term $\Delta+\Gamma M\sin(2\phi)$. This term gives rise to a force at *zero detuning* as well as strong variations of the force at small detunings [$\Delta<\Gamma M\times\sin(2\phi)$]. These effects can be understood by noting that the dephasing-induced line shape of TWM at resonance is absorptive in normal vacuum, but it can be

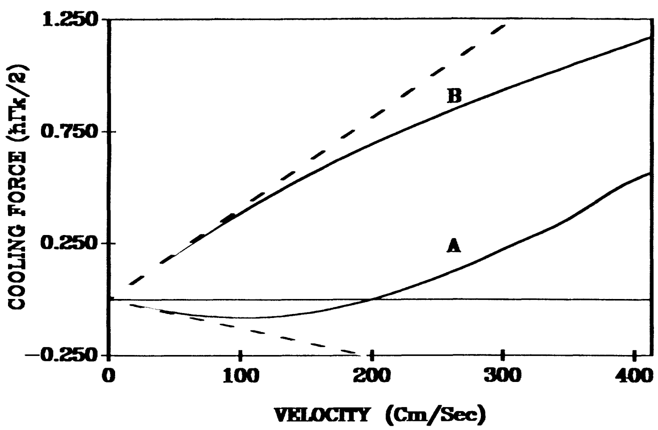

FIG. 2. The velocity dependence of the spatially averaged force, in normal vacuum trace A and squeezed vacuum trace B, obtained by numerical solution of the OBE. The dashed lines are the result of the analytic solution. The parameters used for this figure are $\Delta=-3\Gamma$, $\Omega=1.36\Gamma$, $\Gamma=10^7$ Hz, and $\lambda=5890$ Å for both traces and $\bar{N}=1$, $M=\sqrt{2}$, and $\phi=0$ for trace B.

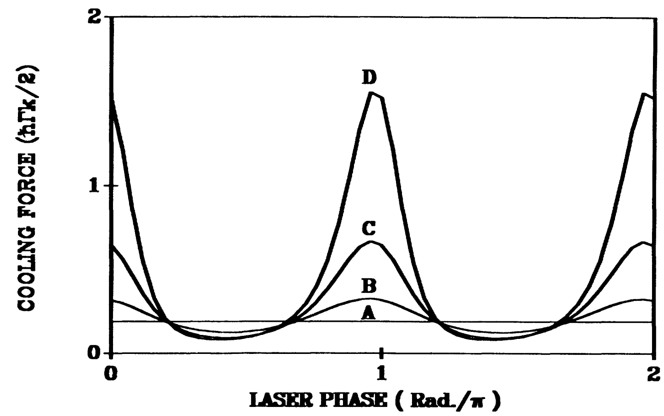

FIG. 3. The spatially averaged force as a function of the laser phase ϕ for increasing amount of correlation between the sidebands: Trace A, $M=0$ (thermal light no correlation); trace B, $M=0.5$; trace C, $M=1$; and trace D, $M=\sqrt{2}$ (maximum squeezing). Other common parameters used are $\Delta=-3\Gamma$, $\Omega=1.5\Gamma$, and $\bar{N}=1$.

transformed to a dispersive line shape in squeezed vacuum, giving rise to a force at resonance. In addition, it has been shown that the TWM can have subnatural linewidth at small detuning.[2,7,8] This indicates that one can obtain arbitrarily large cooling forces at small detuning as the number of photons in the squeezed vacuum $\bar{N}$, and therefore the amount of squeezing, is increased. This can be understood by noting that $\Gamma_{2\Gamma}=\Gamma(\bar{N}-M+0.5)$ in the limit of $\bar{N}\gg 1$ and maximum squeezing becomes arbitrarily small, $\Gamma_{2\Gamma}=\Gamma/8\bar{N}$.

In the analytic solution shown above the force is calculated only to first order in velocity (i.e., a linear velocity dependence is assumed), this is correct only for small velocities $\mathbf{k}\cdot\mathbf{v}\ll\Gamma$. The numerical solution of the OBE, however, can provide the full velocity dependence of the force. This solution[10] is shown in Fig. 2 for ordinary (trace A) and squeezed vacuum (trace B). This figure demonstrates that the stimulated force which gives a heating force in normal vacuum for velocities on the order of $\mathbf{k}\cdot\mathbf{v}<\Gamma/2$ (as expected from the TWM line shape, Fig. 1, trace B), can be transformed to a cooling force in squeezed vacuum. The dashed lines in the figure are the results of the spatially averaged analytic solution which show good agreement with the numerical solution at small velocities.

Figure 3 demonstrates the interesting dependence of the force on the driving laser phase ϕ [using the analytic solution Eq. (2) with $\mathbf{k}\cdot\mathbf{v}=\Gamma/2$]. This is shown for a constant number of photons in the squeezed vacuum, $\bar{N}=1$, but for various values of the squeezing parameter M. Trace A plots the force for thermal light $M=0$ (i.e., no correlation between the sidebands) with no variation on the phase, as expected. Traces B-D, however, show large variations of the force with the laser phase for increasing degree of squeezing up to the maximum value of M [$M^2=\bar{N}(\bar{N}+1)$]. This dependence is due to the different amount of phase noise that the induced dipole sees at a different quadrature phase. Figure 3 also shows that even a modest amount of squeezing can induce large effects on the force.

Figure 4 demonstrates the dramatic difference of the intensity dependence of the spatially averaged force, for normal (trace A) and squeezed vacuum (traces B and C), showing that the force continues to increase with laser intensity in squeezed vacuum while it saturates and changes sign to a heating force in ordinary vacuum. Note that, even for small $\bar{N}$, the force in squeezed vacuum can reach a value more than *2 orders of magnitude* larger than the maximum value of the cooling force due to radiation pressure force in ordinary vacuum $\hbar k\Gamma/2$. Moreover, the force increases monotonically as the

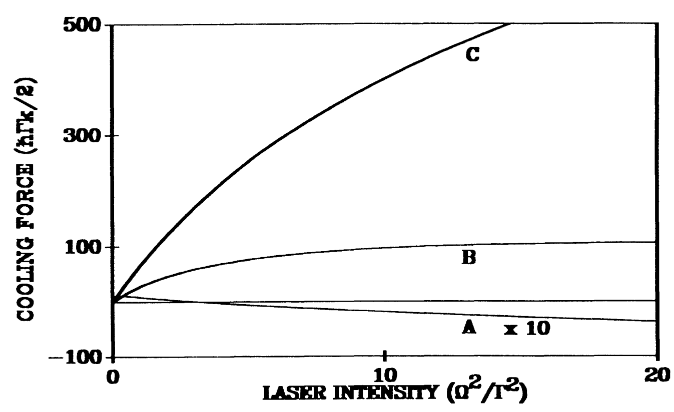

FIG. 4. The spatially averaged force as a function of the laser intensity at $\Delta=-0.5\Gamma$: Trace A, in normal vacuum; trace B, squeezed vacuum $\bar{N}=2$, $M=\sqrt{6}$, and $\phi=0.985$; trace C, larger squeezing $\bar{N}=10$, $M=\sqrt{110}$, and $\phi=0.985$.

amount of squeezing increases (trace C). Note, however, that only the average force is calculated here; the fluctuations of the force give rise to diffusion of the atomic momentum[4] which increases the equilibrium temperature of the laser-cooled atoms. Since in normal vacuum the fluctuations of the force at high intensity are dominated by the fluctuations of the stimulated force, a modification of the diffusion of momentum due to the squeezed-vacuum fluctuations is to be expected. If these fluctuations can be kept small, laser cooling of *two-level* atoms below the Doppler limit, $k_b T = \hbar\Gamma/2$, can be achieved. This will be an important addition to the newly discovered cooling mechanisms of *multilevel* atoms.[1]

As to the experimental verification of these interesting phenomena: Although 90% squeezing has already been achieved in the laboratory, it is important to note that the calculation presented here is carried out with the assumption that the atom is embedded in squeezed vacuum. In practice, the output of present sources of squeezed light (degenerate parametric oscillators) can couple only to part of the 4π sr enveloping the atom. A possible solution to this problem has been proposed by Parkins and Gardiner[11] who suggested coupling the squeezed modes to the atom in an optical cavity of small dimension. The other important assumption here is that the spectrum of the squeezing is much broader than that of the atomic transition. Theories which include a finite bandwidth of squeezing[11] have shown that the essential features due to squeezing are preserved, even for a bandwidth of squeezing only a few times larger than the width of the atomic transition. In addition, the results here indicate that even a modest amount of squeezing can produce pronounced effects on the force.

In conclusion, this Letter demonstrates that the reduction of the quantum fluctuation of the vacuum by quadrature squeezing can profoundly modify the mechanical manifestations of light. In particular, the stimulated force can change sign to provide a cooling force much larger than the maximum radiation pressure force in normal vacuum. This interesting phenomenon can, in principle, be observed experimentally, however, its potential for laser cooling depends on the behavior of the force fluctuations in squeezed vacuum.

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