Hierarchical Radiative Quark and Lepton Mass Matrices

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Assuming that only the top quark gets a tree-level mass due to electroweak gauge-symmetry breaking, a model is presented where all other known quarks and leptons acquire radiative masses in a hierarchical manner. This is the first such model where the 3×3 charged-current quark mixing matrix is realistically generated without the fine tuning of parameters.

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There is now experimental evidence that the top quark is heavy, $m_t > 77$ GeV,[1] and that a fourth light neutrino is ruled out.[2] This means that there are probably only three generations of quarks and leptons, and of these, only the top quark is not light relative to the electroweak mass scale,

$$v = (\sqrt{2}G_F)^{-1/2} = 246\ \text{GeV}. \tag{1}$$

However, the proper question to ask is perhaps not why t is so heavy, but rather why the other fermions are so light. A possible answer is that their masses are of radiative origin and in such a way that a natural hierarchy exists from the heaviest to the lightest. In the same spirit, small mixing angles should be generated naturally as well. Many models now exist where some quarks and/or leptons acquire their masses radiatively.[3] Most require the presence of heavy fermions beyond those of the three-generation standard model and few explain why the 3×3 charged-current quark mixing matrix, i.e., the Kobayashi-Maskawa (KM) matrix,[4] is nearly diagonal as observed. To give an example of how both problems may be overcome, a model is presented in this paper where there are only three generations of quarks and leptons transforming in the usual way under the standard SU(3)×SU(2)×U(1) gauge group, but the scalar sector is greatly enriched and a discrete $Z_8 \times Z_5$ symmetry is imposed to forbid tree-level masses for all fermions except the top quark. As Z_8 is broken softly and spontaneously, other fermions acquire masses hierarchically as shown schematically in Fig. 1. The Z_5 is not broken, and because of the gauge symmetry and the representation content of this model, it results in the separate conservation of the conventional baryon number and a somewhat unconventional lepton number.

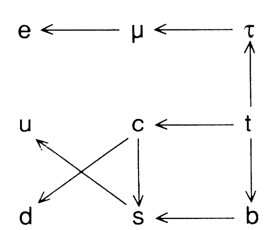

FIG. 1. Schematic representation of how all other fermions acquire radiative masses from a single tree-level mass for the top quark.

Consider first the quarks. They are assumed to transform under Z_8 as shown in Table I. The scalar bosons $\phi_{1,2}$ are doublets of the form (ϕ^0, ϕ^-) transforming as $(1,2,-\frac{1}{2})$ under SU(3)×SU(2)×U(1), whereas the ζ's are color triplets with $\zeta_{1,2} \sim (3,1,-\frac{1}{3})$, $\zeta_3 \sim (3,1,-\frac{4}{3})$, $\zeta_{4,5} \sim (3,1,\frac{2}{3})$, and $\psi = (\psi^{2/3}, \psi^{-1/3}, \psi^{-4/3})$ is a color triplet as well as an SU(2) triplet. All Yukawa interactions are assumed to be invariant under Z_8. Hence the only tree-level mass comes from the term $(\bar{t},\bar{b})_L t_R \phi_1$ (which also serves to define t_R) as ϕ_1^0 acquires a nonzero vacuum expectation value, thus also breaking Z_8. Indeed, Z_8 is assumed to be also broken softly; i.e., all terms of dimensions 2 and 3 in the Lagrangian are allowed, subject only to the requirement of gauge invariance. In particular, the term $\zeta_1^* \zeta_2$ enables b to pick up a radiative mass as shown in Fig. 2. (This mechanism was first proposed in Ref. 5, but in that model, no distinction is made among the three left-handed doublets; hence it is unnatural for t_L to couple only to b_L and not to an approximately equal mixture of b_L, s_L, and d_L. In other words, fine tuning would be needed there to make the KM matrix nearly diagonal as observed.) Note that the term $t_R b_R \zeta_2$ serves to define b_R. We now have[6]

$$\frac{m_b}{m_t} = \frac{f_L f_R \sin 2\theta}{16\pi^2}[F(m_1,m_t) - F(m_2,m_t)], \tag{2}$$

TABLE I. Assignments of quarks and scalars under Z_8 for generating quark masses.

Z_8	Quarks	Scalars
-3	b_R, s_R, d_R	
-2	$(c,s)_L$	$\zeta_4^{2/3}$
-1	t_R, c_R, u_R	
0	$(t,b)_L$	$\phi_2, \zeta_1^{-1/3}$
$+1$		ϕ_1
$+2$	$(u,d)_L$	$\psi, \zeta_3^{-4/3}, \zeta_5^{2/3}$
$+3$		
$+4$		$\zeta_2^{-1/3}$

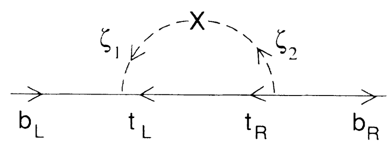

FIG. 2. Diagram for generating a radiative mass for the b quark.

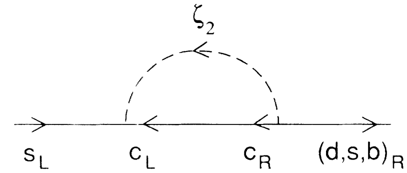

FIG. 3. Diagram for radiative mass terms linking s_L to d_R, s_R, and b_R through m_c.

where $f_{L,R}$ are the Yukawa couplings, $m_{1,2}$ are the mass eigenvalues of $\zeta_{1,2}$, and θ their mixing angle. The function F is given by

$$F(a,b)=\frac{a^2\ln(a^2/b^2)}{a^2-b^2}\,. \tag{3}$$

The next step is to find radiative mass terms involving the c and s quarks. Since m_c is not much smaller than m_b, it will be assumed that it comes also from m_t. This is achieved through the interactions

$$c_Lt_L\psi^{-4/3}+\frac{1}{\sqrt{2}}(c_Lb_L+s_Lt_L)\psi^{-1/3}+s_Lb_L\psi^{2/3} \tag{4}$$

and $t_Rc_R\zeta_3$ which also serves to define c_R. Note that $t_Rt_R\zeta_3$ is forbidden because the color indices must be antisymmetric to have the decomposition $3\times3\times3\to1$ under SU(3). The mixing of $\psi^{-4/3}$ with ζ_3 occurs through the term $\psi\cdot(\phi_2\phi_2)\zeta_3^*$ with $\langle\phi_2^0\rangle\neq0$. Similarly, if we replace b by s, t by b, ζ_1 by $\psi^{2/3}$, and ζ_2 by ζ_4 in Fig. 2, we obtain a radiative mass term for s through the mixings $\psi\cdot(\tilde\phi_2\tilde\phi_2)\zeta_5^*$ and $\zeta_4^*\zeta_5$, where $\tilde\phi_2=(-\phi_2^+,\bar\phi_2^0)$. Note that the $b_Rs_R\zeta_4$ term serves to define s_R. Note also that $\psi^{-1/3}$ does not mix with $\zeta_{1,2}$ at tree level; hence c_L (s_L) is not connected to t_R (b_R) through m_b (m_t), and no mixing occurs between the second and third generations at this stage. However, as shown in Fig. 3, s_L is connected to d_R, s_R, and b_R through m_c. (Recall that b_R and s_R were defined previously by other interactions, and hence c_R couples in general to all three through ζ_2.) This diagram is finite despite the absence of mixing in the scalar exchange because m_c itself is a loop effect. It provides a mass term linking s_L to b_R of the same order of magnitude as m_s, hence the KM-matrix entry for $b\to c$ will be of the order m_s/m_b which is a few percent, in agreement with observation.

If we replace the $s_Lc_L\zeta_2$ vertex in Fig. 3 by the $d_Lc_L\zeta_1$ vertex, we obtain a connection through $\zeta_{1,2}$ mixing between d_L and d_R, s_R, and b_R. Similarly, u_L is connected to u_R and c_R through the effective $\bar s_Ld_R$, $\bar s_Ls_R$, and $\bar s_Lb_R$ masses of Fig. 3. As a result, the 3×3 mass matrix in the (u,c,t) sector is essentially diagonal, whereas the one linking $(\bar d,\bar s,\bar b)_L$ to $(d,s,b)_R$ is given by

$$\mathbf{M}_d=\begin{bmatrix} zm_1 & zm_2 & zm_3\\ m_1 & m_2+m_4 & m_3\\ 0 & 0 & m_5\end{bmatrix}, \tag{5}$$

where m_5 comes from Fig. 2, m_4 from the analog of Fig. 2 as discussed earlier, and $m_{1,2,3}$ from Fig. 3. The parameter z is naturally smaller than 1 because it involves $\zeta_{1,2}$ mixing versus just ζ_2 exchange. It is also proportional to the $d_Lc_L\zeta_1$ coupling divided by the $s_Lc_L\zeta_2$ coupling which may vary within an order of magnitude. Hence z may naturally be of order 10^{-1}, and $m_{1,2,3,4}/m_5$ of order 10^{-2}-10^{-1}. To show that this is indeed a realistic hierarchy, we find from $\mathbf{M}_d$ that the quark masses m_d, m_s, and m_b and the KM-matrix entries V_{us}, V_{cb}, and V_{ub} are related to $m_{1,2,3,4,5}$ and z as follows:

$$m_b\simeq m_5\,, \tag{6}$$

$$m_s\simeq[m_1^2(m_2+m_4)^2]^{1/2}\,, \tag{7}$$

$$\frac{m_d}{m_s}\simeq\frac{zm_1m_4}{m_1^2+(m_2+m_4)^2}\,, \tag{8}$$

$$V_{cb}\simeq m_3/m_5\,, \tag{9}$$

$$V_{ub}/V_{cb}\simeq z\,, \tag{10}$$

$$V_{us}\simeq z\left[\frac{m_1^2+m_2(m_2+m_4)}{m_1^2+(m_2+m_4)^2}\right]. \tag{11}$$

Using the experimental values $V_{cb}\simeq0.05$, $V_{ub}/V_{cb}\simeq0.1$, $V_{us}\simeq0.22$, $m_d/m_s\simeq\frac{1}{20}$, $m_s\simeq150$ MeV, and $m_b\simeq5$ GeV, we then obtain

$$\mathbf{M}_d\simeq\begin{bmatrix} 6 & -33 & 25\\ 58 & -138 & 250\\ 0 & 0 & 5000\end{bmatrix} \tag{12}$$

in MeV, which clearly displays the hierarchy we have assumed.

Since quarks carry baryon number $B=\frac{1}{3}$, the color-triplet scalars we have introduced must have $B=-\frac{2}{3}$. Hence soft terms such as $\zeta_1\zeta_2\zeta_4$ and $\zeta_3\zeta_4\zeta_5$ would not conserve B and would induce neutron-antineutron oscillations. To avoid this possible disagreement with experiment, we impose an exact Z_5 discrete symmetry as shown in Table II. The three generations of leptons are assumed to be distinct under Z_5, and there are two additional color-triplet scalars $\eta_{1,2}^{-1/3}$ as well as four more scalar doublets $\phi_{3,4,5,6}$. Their Z_8 assignments are given in Table III. The τ lepton gets a radiative mass proportional to m_t through $\eta_{1,2}$ mixing as shown in Fig. 4. The muon then gets a radiative mass proportional to m_τ

TABLE II. Assignment of quarks, leptons, and scalars under the assumed exact Z_5 discrete symmetry.

Z_5	Particles
−2	ζ, ψ, ϕ_6
−1	e, ν_e, ϕ_3
0	τ, ν_τ, $\phi_{1,2}$
+1	q, μ, ν_μ, η, ϕ_4
+2	ϕ_5

through $\phi_{3,4}$ mixing as shown in Fig. 5. Similarly, the electron becomes massive from m_μ and $\phi_{5,6}$ mixing. Thus the schematic representation of how each charged fermion acquires mass given in Fig. 1 is now complete.

Because of the exact Z_5, μ-e transitions such as $\mu \to e\gamma$ and $\mu \to eee$ are strictly forbidden here. [In fact, if μ were not distinguished from e by an exact symmetry, then the diagonalization of the (e,μ) mass matrix would not result in a diagonal magnetic dipole moment matrix, and the $\mu \to e\gamma$ rate would be typically a few orders of magnitude above the experimental upper limit in models of radiative lepton masses.[7]] A careful examination of all allowed interaction terms consistent with exact Z_5 but softly and spontaneously broken Z_8 shows that B is exactly conserved, whereas the three lepton numbers L^e, L^μ, and L^τ are separately conserved *except* for the terms $\eta_1\eta_2\zeta_4$, $\eta_1\eta_2\zeta_5$, $\phi_1^\dagger\phi_3^\dagger\phi_4\phi_6$, $\phi_2^\dagger\phi_3\phi_4^\dagger\phi_5$, and their conjugates. Consequently, there is really only one conserved lepton number L as shown in Table IV, but since the above interactions all involve new particles, separate L^e, L^μ, and L^τ conservation for ordinary processes is still an excellent approximation. The ν_τ has $L=0$ and acquires a radiative Majorana mass in two loops [mechanism (4) of Ref. 3] through the $\eta_1\eta_2\zeta_4$ coupling, and we estimate its mass to be a few eV. The ν_e combines with $\bar{\nu}_\mu$ to acquire a radiative Dirac mass in four loops through the $\phi_2^\dagger\phi_3\phi_4^\dagger\phi_5$ coupling, and we estimate its mass to be of order 10^{-7} eV. (Details will be given elsewhere.) It is interesting to note that this is a dynamical realization of an old idea for assigning lepton numbers.[8]

Finally, it should be pointed out that the quark mass matrices of this model are still, in general, complex; we have only assumed them to be real for simplicity in the earlier discussion. Hence the usual *CP* nonconservation is present in the KM matrix as needed for the K-$\bar{K}$ system, etc. This model is also consistent with all present experimental constraints. In particular, flavor-changing

TABLE III. Assignment of leptons and scalars under Z_8 for generating lepton masses.

Z_8	Leptons	Scalars
−3	μ_R	ϕ_4
−2	e_R	
−1	$(\nu_\mu,\mu)_L$	ϕ_6
0	$(\nu_\tau,\tau)_L$	η_1, ϕ_2
+1		η_2, ϕ_1
+2	τ_R	ϕ_5
+3	$(\nu_e,e)_L$	ϕ_3
+4		

neutral currents are absent at tree level. However, some processes which are very rare in the standard model are not as rare in this model. Details will be presented elsewhere.

In past attempts,[3] the scalar sector is less complicated but then many results are less satisfactory. For example, m_t and m_b have usually the same origin in these models and in some m_c/m_b is a radiative effect. Also, fermions beyond the three generations of quarks and leptons are usually needed and in most cases the resulting KM matrix is arbitrary and not naturally of the form observed. In this model, the emphasis is on obtaining the most realistic quark and lepton mass matrices without going beyond the known three generations. For example, the empirical fact that m_b, m_c, and m_τ are comparable in magnitude leads to the assumption that they are all one-loop radiative masses proportional to m_t as shown in Fig. 1. There is no understanding at this level for the scalar bosons of this model. The idea is only that new physics at the electroweak scale could explain the observed quark and lepton mass matrices.

In summary, it has been shown in this paper that within the context of the standard SU(3)×SU(2)×U(1) gauge model with three generations of quarks and leptons, it is possible to start out with a tree-level top-quark mass and obtain hierarchical radiative masses and mixing angles for all other fermions. The specific model proposed is based on $Z_8 \times Z_5$ but it should only be considered as an illustrative example. It demonstrates the viability that new physics at the electroweak scale may help to explain the long-standing puzzle of the enormous range of fermion masses and mixing angles.

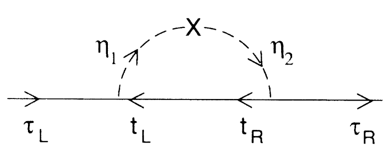

FIG. 4. Diagram for generating a radiative m_τ.

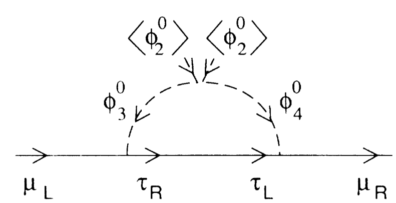

FIG. 5. Diagram for generating a radiative m_μ.

TABLE IV. Assignment of the one exactly conserved lepton number L of this model.

L	Particles
-2	ϕ_5
-1	μ, ν_μ, ϕ_4
0	τ, ν_τ, η
$+1$	e, ν_e, ϕ_3
$+2$	ϕ_6

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