Wave-Number-in-Cell Simulation of Weak Langmuir Turbulence

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A wave-number-in-cell code was developed for simulation of a model proposed by Vedenov, Gordeev, and Rudakov for studying weak Langmuir turbulence. The model uses a WKB approach for describing the Langmuir wave field. Theoretical results for damping of ion acoustic waves caused by resonant interaction of Langmuir waves with group velocity around the sound speed were confirmed. Simulations corresponding to an initial condition with an intense wave burst demonstrate a formation of almost stationary localized cavities of intense wideband self-trapped Langmuir wave fields. These cavities have features in common with phase-space vortices.

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A simple model for describing Langmuir turbulence was proposed by Vedenov, Gordeev, and Rudakov[1] on semi-intuitive grounds. In normalized form the equations are

$$\partial_t F(x,\kappa,t)+\kappa\,\partial_x F(x,\kappa,t) - \tfrac{1}{2}\,\partial_x n(x,t)\partial_\kappa F(x,\kappa,t)=0\,, \qquad (1)$$

$$\partial_t^2 n(x,t)-\partial_x^2 n(x,t)=\partial_x^2\int F(x,\kappa,t)\,d\kappa\,, \qquad (2)$$

where the present study is restricted to one spatial dimension. Physically $\int F\,d\kappa$ can be interpreted as a wave action density, while n denotes the bulk plasma density. In this physical model $F(x,\kappa,t)$ denotes the density of wave packets at position x at time t with central wave number κ. These wave packets (or quasiparticles) are in the WKB limit propagating in a plasma with varying index of refraction caused by the varying density $n(x,t)$, and their orbits are consequently perturbed. Gradients in density give rise to an effective "force" $-\frac{1}{2}\partial_x n(x,t)$ acting on the wave packets, see Eq. (1). The spatially modulated wave field will in turn perturb the plasma if the wave intensity is sufficiently large. A gradient in wave intensity will thus act on the light component of the plasma, i.e., the electrons, by ponderomotive forces. The resulting electron displacement gives rise to an ambipolar electric field, which subsequently also sets the ions in motion, and a perturbation of the bulk plasma density results. This process in the quasineutral limit is accounted for by the term on the right-hand side of Eq. (2).

The normalizing quantities in Eqs. (1) and (2) are L for length, T for time, $\frac{1}{3}n_0 m/M$ for density, $(\omega_p/v_T)\times(m/3M)^{1/2}$ for wave numbers, and $\frac{2}{3}\sqrt{3}(n_0 v_T^2 m/\omega_p)\times(v_T/\omega_p)(m/M)^{1/2}$ for the distribution function of wave action density, where m/M is the electron-to-ion–mass ratio, ω_p is the electron plasma frequency, n_0 is the unperturbed plasma density, and $v_T=(3T_e/m)^{1/2}$ is the electron thermal velocity. The ion component is assumed to be cold. The system of Eqs. (1) and (2) does not possess any characteristic length or time scale, so the only constraint on L and T comes through $L/T=(m/M)^{1/2}\times v_T/\sqrt{3}$. A characteristic length scale is imposed by the initial condition. With the present normalizations a wave packet will have a group velocity $v_g=Kv_T^2/\omega_p$ equal to the sound speed $C_s=(T_e/M)^{1/2}$ for a normalized wave number $\kappa=K(m/M)^{1/2}\omega_p/v_T\sqrt{3}=1$, where K is the unnormalized wave number. The WKB assumption in (1) will be satisfied in the limit of $L\to\infty$ for all plasmon wave numbers κ, except for a set of zero measure where $\kappa=0$.

Equations (1) and (2) can also be derived[2] as a limiting case of a general formalism based on the so-called Zakharov equations,[3] where, in particular, the dispersive term in the equation for the high-frequency Langmuir waves corresponds to the mixing of waves propagating with different velocities, i.e., different wave numbers κ, in Eq. (1).

A laboratory experiment by Michelsen, Pécseli, and Rasmussen[4] on the interaction between electron plasma waves and ion acoustic waves was interpreted by analytical results based on Eqs. (1) and (2).

Equation (1) is formally identical to the Vlasov equation where the self-consistent force is derived from a dynamic equation (2) for the driven sound waves instead of the usual Poisson equation. Important features of (2) are the homogeneous solutions, "free sound waves," which give rise to correlations between widely separated regions of space.

The set of Eqs. (1) and (2) were solved numerically by a procedure similar to the one used in collisionless plasma simulations. Thus a "wave-number-in-cell" simulation was developed for the self-consistent movement of quasiparticles as described by their position x and respective wave number κ, according to the characteris-

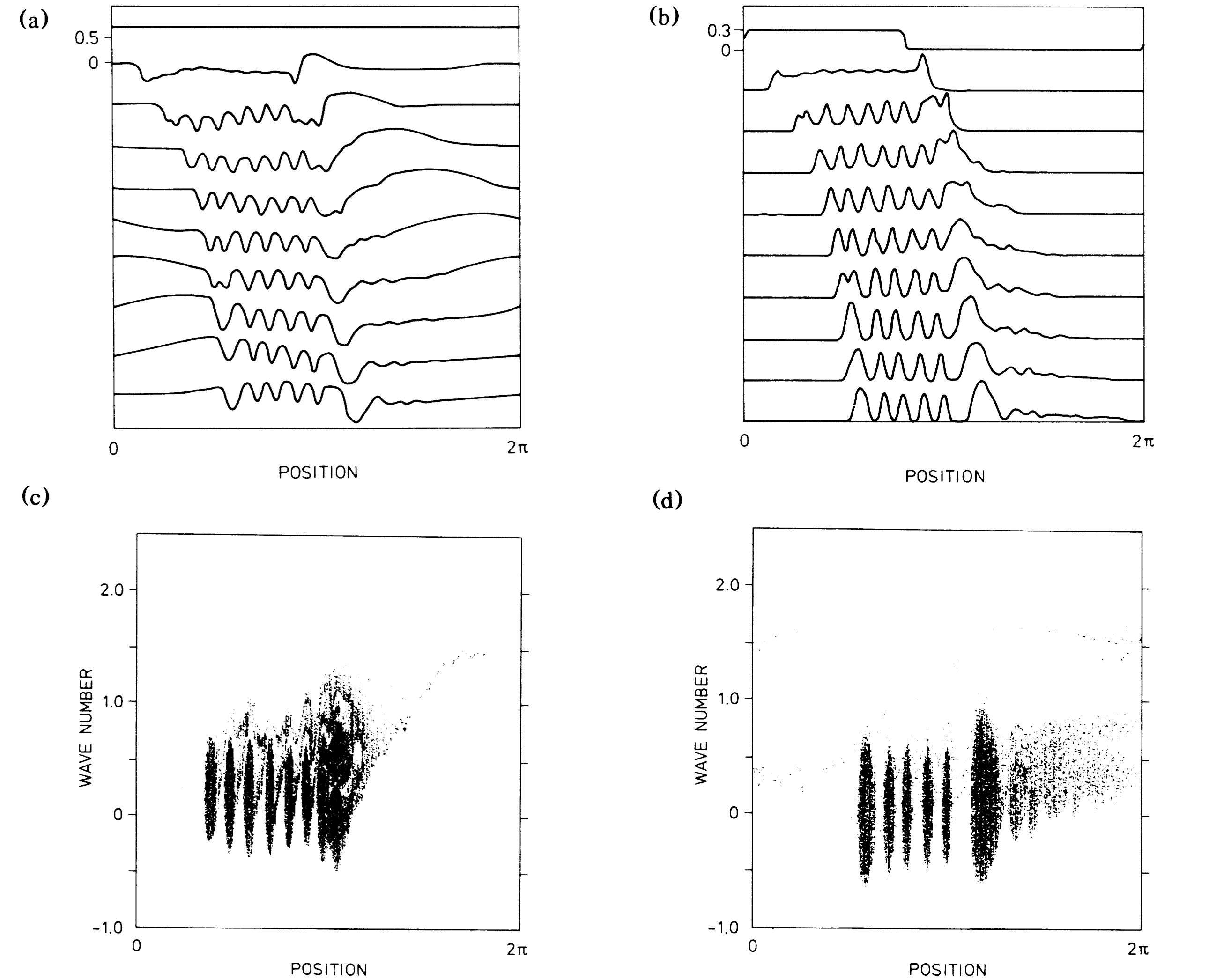

FIG. 1. (a) Time evolution of plasma density and (b) $\rho = \int F\,d\kappa$ for the case where the initial condition consists of an intense wave burst. The time difference between curves is $\Delta t = 0.7854$. A wave-number–position space of the quasiparticles at (c) $t = 2.3562$ and (d) $t = 7.0684$.

tics of (1). The integral in (2) is obtained in each spatial grid point, and the equation is solved by fast Fourier transform in the spatial variable using periodic boundary conditions. A smoothing of small scale variations by a filter as, e.g., $\frac{1}{4}(n_{i-1}+2n_i+n_{i+1})_{\text{old}} \rightarrow n\,|_{i\,\text{new}}$ allows the simulations to be performed by a modest number of simulation particles, which would otherwise give rise to an enhanced noise level due to particle discreteness. There is no phase change in the signal associated with this filtering. Standard methods[5] are used for the propagation of quasiparticles.

With the code we verified theoretical results[1,6] for damping and growth of ion sound waves caused by resonant interaction of quasiparticles, a process very similar to Landau damping and growth. An equilibrium solution to (1) and (2) is $n=0$ and $F=F_0(\kappa)$. Solutions of the dispersion relation

$$u^2 - 1 = \frac{1}{2}\oint_{-\infty}^{\infty} \frac{F_0'(\kappa)}{\kappa - u}\,d\kappa\,, \tag{3}$$

with $u \equiv \Omega/K$, are compared with results from the simulations for the case where $F_0(\kappa) = A(2\pi\Delta)^{-1/2} \times \exp[-\frac{1}{2}(\kappa-\kappa_0)^2/\Delta]$. The symbol $\oint$ denotes the usual Landau contour of integration, while $F_0'(\kappa) \equiv dF_0(\kappa)/d\kappa$. Only those solutions close to the sound-wave branches $u^2=1$ were considered. Deviations from the theoretical curves are due to difficulties in creating consistent initial conditions which require $F = \frac{1}{2}n \times F_0'(\kappa)/(\kappa - u)$. The difficulty is due to the random-number generators which are used to set up the initial distributions of quasiparticles. The code is found to be particularly suitable for describing large-amplitude phenomena. Figure 1 shows the evolution of density n and the corresponding variation of $\rho \equiv \int F\, d\kappa$ as functions of x for different times. Initial conditions were $n(t=0)=0$ and $\rho=0.1$ in an interval $[0,0.4L]$, where L is the length of the system. Initially F is chosen to be a Gaussian with a standard deviation of 0.075 around $\kappa=0.75$, i.e., a characteristic group velocity below C_s. The figures contain several of the important features of general results. The intense wave burst is digging a cavity which traps parts of the wave field, while the rest is slowly dispersing. The plasma removed from the cavity escapes in the form of free sound pulses. Inside the cavity a modulational instability develops. The instability saturates in an array of density cavities [Fig. 1(a)], filled with a "hot" plasmon gas [Fig. 1(c)] which maintains an equilibrium very much similar to the Bernstein-Grene-Kruskal phase-space equilibria.[7] The figures show clearly a coalescence of some of these cavities, which is a process well known for phase-space vortices.[8] Figures like Fig. 1(c) show that the interaction which leads to coalescence between cavities is mediated by an exchange of quasiparticles. Extending the time duration of the calculations in Fig. 1, it is observed that the velocity of the cavities go to zero even if initial burst velocity was close to C_s. The excess momentum is carried away by the sound pulse and a weak population of free plasmons. Because of the periodicity of the calculations, the free sound pulses will eventually catch up with the plasmon-filled cavities. This interaction left the cavities unaffected in all simulations carried out so far. It should be noted though that with the initial conditions used, the free sound pulse was always spatially wider than the cavities and the interaction had the form of a "tidal" effect caused by the sound pulse. The presence of quasiparticles with $\kappa=0$, as seen in Figs. 1(c) and 1(d), is, strictly speaking, not consistent with the WKB approximation inherent in (1) and (2). However, these particles contribute only a little to the integral in (2) and this formal inconsistency does not affect the overall plasmon dynamics.

The results of Fig. 2 demonstrate that Eqs. (1) and (2) in the linearized limit contain kinetic effects, which can be understood as effects equivalent to the Landau damping of sound waves caused by resonant interaction of plasmons considered as quasiparticles. The simulations for a *strongly* nonlinear regime as in Fig. 1 demonstrate that structures evolve which have their analogy in nonlinear kinetic equilibria in collisionless plasmas. We also found that the interaction of these structures are quite similar to that characterizing phase-space vortices.[8] Equations (1) and (2) also formally describe the interaction between broadband *electromagnetic* radiation and a plasma. A number of phenomena[9] can be approached with advantage along the lines of the present study.

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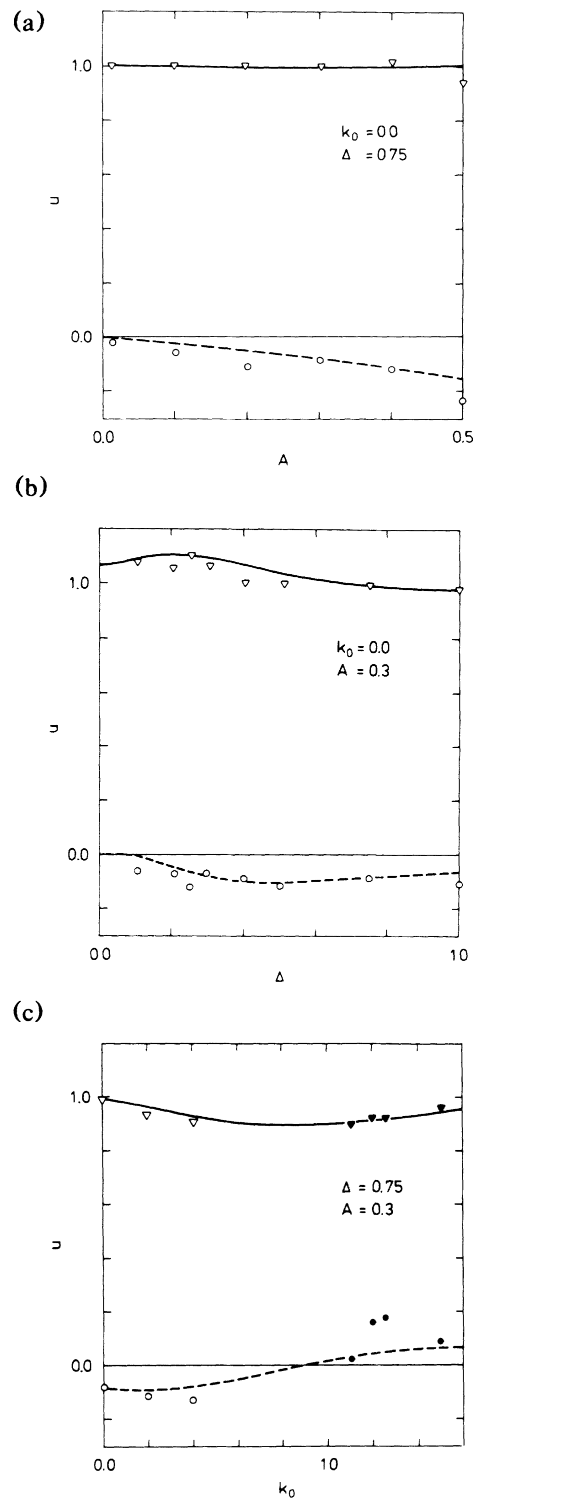

FIG. 2. Comparison between simulated results and the calculated dispersion relation for $F_0(\kappa)=A(2\pi\Delta)^{-1/2}\exp[-\frac{1}{2}(\kappa-\kappa_0)^2/\Delta]$ with varying parameters. Real parts of the phase velocity u are given by —— and ▽, while the imaginary parts are indicated by --- and ○. Solid symbols refer to unstable conditions.

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