Cosmological Production of Black Holes

Lawrence J. Hall and Stephen D. H. Hsu

Department of Physics, University of California, Berkeley, California 94720 and Theoretical Physics Group, Physics Division, Lawrence Berkeley Laboratory, University of California, 1 Cyclotron Road, Berkeley, California 94720

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It is argued that, at a first-order phase transition, false-vacuum bubbles may occasionally collapse to become black holes. If the critical temperature T_c is less than a TeV, these black holes, which have a mass proportional to $M_{\rm Pl}^2/T_c$, could survive until today to be the dark matter. Alternatively, evaporation of black holes could give rise to relic particle abundances.

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The framework of the standard Friedmann-Robertson-Walker big bang, together with a theory of particle physics, allows us to calculate, at least in principle, the present contents of the Universe. However, there are several snags: We do not know the particle physics of baryon-number violation, nor the particle physics corresponding to dark matter. Furthermore, while accurate calculations of relic particle abundances can be made, it is much harder to compute abundances of extended relics produced at phase transitions.

It is critical to consider any mechanism which at high temperatures transforms energy from radiation to a form which redshifts more slowly. For example, suppose that at temperature T_c, during the radiation-dominated era, a fraction f_M of the radiation energy is converted to a form which redshifts like matter. Today this matter will contribute to $\Omega = \rho/\rho_c$ an amount

$$\Omega_M \simeq 10^8 [T_c/(1\ \text{GeV})] f_M \,, \tag{1}$$

where ρ is energy density and ρ_c is the value which makes the Universe critical. At high T_c only a very weak conversion mechanism is required to produce significant Ω_M. Alternatively, if the conversion mechanism at high T_c is too strong, the Universe will be overclosed. Examples include the cosmological monopole problem, which constrains certain phase transitions, and a limit of about 1000 TeV on the mass of point-particle candidates for dark matter. In this Letter we give a new mechanism for primordial black-hole (PBH) production at a first-order phase transition. It produces PBH's only rarely, but this may suffice to give the dark matter.

It is an unproven, but widely believed, result in general relativity that if a mass M is located inside a region of radius $R_s \leq 2GM$, it forms a black hole. Such a black hole emits radiation[1] with a spectrum similar to a thermal one of temperature $T = 1/8\pi GM$.[2] The black-hole lifetime is therefore $\tau \sim G^2 M^3 \simeq (10^{10}\ \text{yr})[M/(10^{39}\ \text{GeV})]^3$. PBH's produced with masses less than 10^{39} GeV will reach a temperature of the Planck mass before today. The final state of such an exploding PBH is unclear; we will not consider the case of PBH remnants as dark matter.[3]

Could the dark matter be dominantly in PBH's of mass $\sim 10^{39}$ GeV? No, they have a temperature today of about 10 MeV, and the diffuse background radiation in this region implies $\Omega < 10^{-9}$ for such masses.[4] Furthermore, if the temperature of the PBH's today is larger than m_e, they will emit significant numbers of e^- and e^+. If our galactic halo were made of these PBH's, the positrons would slow down in the interstellar medium and annihilate to produce 512-keV line radiation. We find that this implies that the halo should be dominated by PBH's of mass greater than 10^{41} GeV. If $\Omega = 1$ in PBH's of mass $\sim 10^{41}$ GeV, they will lead to a diffuse background radiation peaking at energies of 0.25 MeV with a flux of 0.1 $\text{cm}^{-2}\text{s}^{-1}$. This is just below the observed background, and apparently is just unable to explain the observed feature in the MeV region.

Many dark-matter candidates, monopoles and stable point particles, for example, are more dangerous in overclosing the Universe the earlier they are produced. This is not the case for PBH's. At temperatures larger than 10^8 GeV the horizon mass is less than the 10^{41} GeV. Hence, prior PBH production is unimportant for dark matter, unless they can be made to collide and "cannibalize" before evaporating. We find that significant cannibalism does not occur. Consequently, we find that if PBH's are the dark matter today, the scale of the particle physics responsible for the phase transition is less (for our mechanism, much less) than 10^8 GeV.

Cosmological production mechanisms of PBH's are not new. Carr showed that the Harrison-Zeldovich scale-invariant density-perturbation spectrum, which is commonly taken for the origin of large-scale structure, leads to PBH formation as the perturbations enter the horizon.[5] The resulting spectrum of PBH's is a steeply falling power law, implying an $\Omega \ll 1$ in PBH's today. More interesting from the viewpoint of dark matter is the mechanism of Hawking, Moss, and Stewart.[6] They produce horizon-size PBH's at a first-order phase transition by the unlikely event that many neighboring bubbles grow to horizon size before percolating. Providing the phase transition occurs at $T_c < 10^8$ GeV, the PBH masses will be large enough to survive until today. Critically closing the Universe with these PBH's requires a choice for the bubble nucleation rate. In this Letter we provide an alternative production mechanism, in which the PBH masses are much less than the horizon mass at formation, and where the density of PBH's is largely independent of the bubble nucleation rate.

We illustrate our mechanism in a simple field theory with a real scalar field ϕ coupled to a Dirac fermion ψ:

$$\mathcal{L} = \tfrac{1}{2}\,\partial^\mu\phi\,\partial_\mu\phi + i\bar{\psi}(\not{\partial} - m')\psi + \mu^2\phi^2 + \gamma\phi^3 - \lambda\phi^4 + \alpha\bar{\psi}\psi\phi\,,$$

where all the parameters are chosen to be real and positive. At high T the vacuum is at $\langle\phi\rangle = 0$, while at lower temperatures this becomes a false vacuum with a true vacuum appearing for positive $\langle\phi\rangle = v(T)$. At the critical temperature T_c, the two phases coexist and are separated by a transition region in which ϕ interpolates between the two vacua. The energy per unit area of the boundary region is the surface tension $\sigma \equiv \Delta^3$. The scale Δ is a function of the parameters of the scalar potential, and is typically $\simeq T_c$.

At T_c, there will be a nonzero difference in vacuum energy: $B(T_c) = V(\phi=0) - V(\phi = v(T_c)) \neq 0$. This is because the vacuum, or bag, pressure tending to collapse the regions with $\phi = 0$ is countered by the pressure due to the fermions being transmitted and reflected from the boundary. For convenience we take $m' \ll T_c$ and ignore m' from now on. The fermions have a mass difference $m = \alpha v(T_c)$ across the phase boundary and the fermion pressure arises from the change in momentum of fermions which are reflected or transmitted at the boundary.

Suppose that bubble nucleation is appreciable at some temperature T_N which is not very much less than T_c. As the bubbles grow and convert false to true vacuum, we assume that the released energy reheats the bulk of the Universe and does not accumulate as domain-wall kinetic energy. Throughout this Letter we assume a mechanism which always maintains the Universe at a single homogeneous temperature. Finally, we assume that the temperature reaches T_c while the fraction of the Universe in the true vacuum f_T is much less than $\frac{1}{2}$. This will prevent further bubble nucleation, establishing an era of quasistatic equilibrium[7] (QSE). Such a phase transition is of great interest: The transition proceeds very slowly at a rate governed by the rate of expansion of the Universe:

$$\dot{f}_T = \frac{3}{B(T_c)} H(p+\rho)\,, \tag{2}$$

where H is the Hubble parameter during this era, p is the pressure, and ρ is the energy density. The rate at which vacuum energy is released is equal to the rate of doing work of expansion together with the rate at which energy density must be created to fill the increased physical volume. For $p \simeq \rho \simeq T_c^4$, $H = t_H^{-1}$, the era of quasistatic equilibrium lasts for a time $\tau_{\rm QSE}/t_H \simeq B(T_c)/T_c^4$. We consider the case that $\tau_{\rm QSE}/t_H \lesssim 1$, since in this case no significant inflation takes place, and for many simple estimates the expansion of the Universe can be ignored. In the case that the mass acquired by the fermion crossing the boundary, $m = \alpha v(T_c)$, is less than T_c, we find $B(T_c)/T_c^4 = m^2/T_c^2$, so that this condition is straightforward to satisfy. The controlled nature of the slow-burn phase transition allows us to make simple calculations.

When f_T approaches $\frac{1}{2}$, bubble collisions become frequent. We assume that at T_N the nucleation rate was fast enough so that when the bubbles collide they are sufficiently small that a period of bubble coalescence takes place. The original scale of the bubbles is erased and the final scale of the bubbles depends on the dominant dynamics of bubble coalescing. During coalescence of two bubbles of size r, bubble walls must move a distance r in time $\simeq \tau_{\rm QSE}$. Bubble-wall speeds are limited by frictional drag: The transmitted fermions impart a momentum of order $(m/T_c)^2 T_c$ to the wall. This is a severe limitation, so that bubbles coalesce by settling up bulk fluid flows rather than having walls move rapidly with respect to the fluid.

We find that in time $\tau_{\rm QSE}$ bubbles coalesce by fluid flow up to an average scale $r_c = \bar{\epsilon} t_H$, where

$$\bar{\epsilon} \simeq \left[\left(\frac{\Delta}{T_c}\right)\left(\frac{m}{T_c}\right)^2\left(\frac{T_c}{M_{\rm Pl}}\right)\right]^{1/3}. \tag{3}$$

After the era of coalescence, further expansion of the bubbles causes f_T to become greater than $\frac{1}{2}$. The true vacuum percolates to produce a connected region of true vacuum containing shrinking bubbles of false vacuum. We will argue that eventually a few of these shrinking bubbles will form black holes.

Consider first the idealized situation of a density $n_B = r_c^{-3}$ of identical spherical, collapsing bubbles of radius $r(t)$. Since $1 - f_T = n_B r^3$,

$$\dot{f}_T \simeq r^2 \dot{r}/\bar{\epsilon}^3 t_H^3\,. \tag{4}$$

As the bubbles get smaller, $\dot{r}$ must increase in order to satisfy the condition of quasistatic equilibrium, Eq. (2). When the bubble walls reach the speed of light, QSE will be lost and the bubble radius will be $\hat{r} \simeq \bar{\epsilon}^{3/2}(T_c/m)t_H$.

Next we consider the case of a distribution of initial bubble radii about the average $\bar{r} = \bar{\epsilon} t_H$. If the temperature in the true vacuum during QSE is uniform throughout the Universe, the rate of collapse of bubbles will be approximately independent of their size. This of course neglects surface tension which causes smaller bubbles to contract slightly faster than larger bubbles. Therefore, at the end of QSE, while $\bar{r}$ has decreased to $\bar{\epsilon}^{1/2}(T_c/m)\bar{r}$ (and smaller bubbles have already disappeared), a bubble with initial radius $r_i > \bar{r}$ has only decreased to $r_i - \bar{r}$. As QSE ends, we are left with the bubbles on the large end of the size distribution, most of which have barely decreased in radius.

After QSE ends, work is done on the bubbles as they undergo relativistic collapse,

$$W = P_{\rm net}\Delta V = [B(T) - P_{\rm matter}]\Delta V . \quad (5)$$

There are two separate effects which contribute to $P_{\rm net}$. One is the increase in $B(T)$ as the temperature falls beneath T_c: $B(T) \simeq B(T_c) + B'(T_c)(T - T_c)$. The time δt necessary for relativistic collapse of a bubble is at least $\delta t \gtrsim \bar{\epsilon} t_H$. The corresponding temperature drop will be

$$\delta T \sim (M_{\rm Pl})^{1/2}(t_H^{-3/2})\delta t \sim T_c\,\delta t/t_H \sim \epsilon T_c$$

so $\delta B(T) \sim |B'(T_c)|\,\epsilon T_c$, which we will approximate as $\sim(\Delta^3 T_c)\bar{\epsilon}$. The second contribution comes from the fact that the bubble walls become increasingly porous to fermions as they become highly relativistic. Consider the scattering of fermions from a bubble wall, which we have idealized as a step-function potential. In this case we have the following results from elementary quantum mechanics:

$$E_\psi < m \rightarrow \text{reflection},\ \ \Delta p \simeq 2E_\psi ,$$

$$E_\psi > m \rightarrow \begin{cases} \text{probability } (m/E_\psi)^4 \text{ of reflection} , \\ \text{probability } [1-(m/E_\psi)^4] \text{ of transmission} \quad (\text{with } \Delta p \simeq m^2/2E_\psi) . \end{cases}$$

Note that these results already indicate that very energetic particles ($E_\psi \gg m$) barely interact with the interface between true and false vacuum. For the extremely relativistic case, in which the domain wall has a velocity $\beta_{\rm DW} \sim 1$, we can still apply the results above if we calculate the scattering process in the frame of the domain wall and then Lorentz transform back to the cosmological frame. In the case where $\gamma_{\rm DW} \gg 1$, we find that virtually all fermions penetrate the wall, and that the momentum change is $\Delta p \simeq m^2/2E_\psi(1+\beta_{\rm DW})$. The above considerations imply that $P_{\rm matter}$ in Eq. (5) decreases as the collapse becomes relativistic. This means that we can approximate $P_{\rm net}$ in Eq. (5) by

$$P_{\rm net} \simeq (\Delta^3 T_c)\bar{\epsilon} + T_c^4 (m/T_c)^2 . \quad (6)$$

The dominant term for small $\bar{\epsilon}$ is the latter, which yields the following mass formula:

$$M_{\rm PBH} \simeq P_{\rm net}\Delta V \simeq T_c^4 (\epsilon t_H)^3 (m/T_c)^2$$
$$\simeq (\Delta/T_c)^3 (m/T_c)^8 M_{\rm Pl}^2/T_c . \quad (7)$$

This, of course, applies only for bubbles with $\epsilon > \bar{\epsilon}$. Bubbles smaller than or only slightly larger than $\bar{r} = \bar{\epsilon} t_H$ will collapse without forming black holes. Bubbles with $\epsilon \gg \bar{\epsilon}$ produce heavier PBH. In general, the mass distribution of black holes formed can be given in terms of the initial distribution of bubble sizes after percolation.

Given the mass estimate Eq. (7), we can now estimate how far a bubble must collapse before falling into its Schwarzschild radius r_S. Using Eqs. (3) and (7),

$$\frac{r_S}{r_i} = \frac{r_S}{\epsilon r_H} \simeq \left(\frac{\Delta}{T_c}\right)^2 \left(\frac{T_c}{M_{\rm Pl}}\right)^{2/3} , \quad (8)$$

where we drop factors of m/T_c from now on. We see that for high T_c (i.e., grand unification scales or higher) this factor is not extremely small. However, to produce PBH's which are interesting as dark matter, Eq. (7) requires a much lower T_c. This leads to $r_S/r_i \ll 1$, which can only be achieved for bubbles which are extremely spherical at collapse.

Bubbles which have fractional asphericities greater than r_S/r_i at the point of collapse will probably self-collide, rather than form PBH's. In considering coalescence, we calculated the maximum size of bubbles that could become roughly spherical in a Hubble time. To understand exactly how spherical such a bubble can become, it is necessary to include the effects of damping on the motion of bubble walls. During the QSE era, we can treat an individual bubble as a stationary, nearly spherical membrane with surface tension $\sigma \sim \Delta^3$ and radius $r = \epsilon r_H$. We can now study the damping of an arbitrary perturbation on our membrane by examining the excited normal modes. We can characterize a given mode by its amplitude δr and wavelength λ. A bubble minimizes its surface energy by becoming spherical. For fixed δr, it is clearly the longest-wavelength modes ($\lambda \sim r$) that are hardest to eliminate. The equation of motion for such a mode is

$$(\sigma + T_c^4 \delta r)\ddot{\delta r} + \gamma \dot{\delta r} + \frac{\sigma}{r}\left(\frac{\delta r}{r}\right) = 0 , \quad (9)$$

where γ is the damping factor and $\sigma + T_c^4 \delta r$ is the effective mass per unit area (surface tension plus mass per unit area bulk fluid) to be moved.

The damping factor represents the rate of energy dissipation into the bulk fluid. Each bounce of the membrane produces a sound wave in the fluid which carries off an energy per unit area of $\sim T_c^4 \delta r \beta^2$, leaving the membrane with kinetic energy $\sim \sigma\beta^2$. Here β is the velocity of the membrane when $\delta r = 0$. Since initially $T_c^4 \delta r \gg \sigma$, we expect each succeeding bounce to have a drastically smaller amplitude than the previous one. This approximation holds until $\sigma \sim T_c^4 \delta r$, which yields $\delta r/r \sim r_S/r_i$.

In general, ignoring dissipation, the time scale for a particular mode to bounce is $\tau_{\rm bounce}/\tau_{\rm QSE} \sim (\delta r/r)^{1/2}$. Thus, in the approximation where we treat the background particles as a bulk fluid, initially roughly spherical bubbles ($\delta r/r \ll 1$) can become extremely spherical. While this approach is naive, only about 1 in $10^8[T_c/(1\text{ GeV})]$ bubbles need be this spherical in order to account for ρ_{critical} today.

Interesting consequences may result from PBH production even if they evaporate while the Universe is still hot. Consider a phase transition at T_c in which f is the probability that a false-vacuum bubble collapses to a PBH of mass M_{Pl}^2/T_c. For $f > (T_c/M_{\text{Pl}})^{1/2}$, the PBH's dominate the energy density of the Universe at a temperature fT_c and subsequently evaporate, reheating the Universe to a temperature $T_R \simeq T_c(T_c/M_{\text{Pl}})^{1/2}$. A particle X, with mass $m_X < T_c$, will be produced in the evaporation with a relative abundance

$$f_X = \frac{n_X}{T_R^3} = \frac{1}{g}\left(\frac{T_c}{M_{\text{Pl}}}\right)^{1/2}, \tag{10}$$

where g is the number of degrees of freedom lighter than T_c.[8] This abundance can be large enough to have important consequences. For example, X particles might decay out of thermal equilibrium to generate the cosmological baryon asymmetry at low temperatures. It is easy to arrange for $T_R \ll m_X$, avoiding washout of the asymmetry. This can be used for a baryogenesis scheme where the X particles are TeV top squarks.[9]

If the X particles are stable, the above abundance may be larger than the Lee-Weinberg freezeout value, and may give $\Omega_X = 1$. For example, for $T_c = 10^5$ GeV, and $f > 10^{-7}$, the PBH evaporation could lead to $\Omega_X = 1$, with X being 50-GeV neutrinos. The reheat temperature of 10 MeV would be sufficient to give a fresh start to nucleosynthesis.

In this Letter we have considered the collapse of false-vacuum bubbles in a first-order phase transition which undergoes an era of quasistatic equilibrium. We find that, during the initial era of nonrelativistic collapse, a few bubbles may sphericalize to a sufficiently high degree that the work done on them during the later relativistic collapse can result in their becoming black holes. The required collapse factor and the resulting PBH mass are given in Eqs. (8) and (7), respectively. For T_c above a TeV, the PBH are likely to be produced, but are unlikely to survive until today. Their evaporation may lead to a production of relic particles. For T_c beneath a TeV, the PBH can be heavy enough to survive until today and, with even a very small probability of a bubble becoming a PBH, can give $\Omega_{\text{PBH}} = 1$. It is possible that this might arise in QCD, although we have not studied that case here. For QCD, if holes survive until today to be the dark matter, it is likely that they would have a mass close to the observational bound of 10^{41} GeV. To detect such holes from the particles they evaporate would require their mass to be very close to the limit. Holes in the range of 10^{49}–10^{56} GeV, which would occur for lower T_c, could be detected by lensing of background stars.[10]

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