Kinematically Complete Measurement of the Reaction $\pi^- p \rightarrow \pi^+ \pi^- n$ in the Region of Δ Dominance as a Test of Chiral Lagrangians

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The first kinematically complete, good-statistics data for the $(\pi, 2\pi)$ reaction on the proton are presented. They involve double- and triple-differential cross sections as well as $\pi^+\pi^-$ angular correlations. At energies around the Δ resonance, they exhibit a marked difference from phase space. The data are qualitatively described within the framework of an extension of Weinberg's effective chiral Lagrangian.

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The πN interaction at medium energies is beyond the scope of perturbative QCD, since it is dominated by confinement. However, on a less fundamental level, the symmetries of QCD guide the formulation of effective Lagrangians. Among other symmetries, the importance of chiral symmetry has been increasingly realized in recent years.¹ The resulting chiral Lagrangians involve highly nonlinear contributions in the pion field, as already realized in the nonlinear σ model² and extended later on by Weinberg to low-energy πN scattering.³

A natural testing ground of the nonlinear πN dynamics, and thus of the underlying chiral symmetry, should be the excitation of multipion states. The most elementary process is the pion-induced (2π) production on the nucleon. Attempts to extract information on chiral symmetry from these reactions reach back to Olsson and Turner.⁴

Recently, with the formulation of different theoretical approaches to the $(\pi, 2\pi)$ reaction, both on the nucleon⁵ and on nuclei,⁶ the need for exclusive data has been emphasized repeatedly.

The reaction $\pi^- p \rightarrow \pi^+ \pi^- n$ has been measured in previous experiments in the low-energy region, but no kinematically complete data, with good statistics, covering a reasonable part of the phase space have yet been published. Bubble-chamber experiments⁷ delivered less than 100 kinematically complete events below $E_{c.m.}$ = 1400 MeV. Other experiments^{8,9} achieved good statistics, but no kinematical completeness, and therefore only provide total cross sections as well as single- and double-differential cross sections. The OMICRON experiment¹⁰ provides exclusive data, but suffers from medium statistics.

In this Letter, we present the first experiment, which provides full-kinematics, high-statistics data for the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$, slightly above the (3,3) resonance, spread over a reasonable part of the phase space (preliminary data were already presented in Ref. 11).

The experiment was performed at the $\pi M l$ channel of

the Paul Scherrer Institut, at kinetic energies of 247, 284, and 330 MeV. The pions in the beam could be clearly identified, using standard methods (e.g., time of flight). We used a liquid-hydrogen target with a thickness of 630 mg/cm². Background, resulting from the target vessel and cell, was checked by an empty-target measurement and was found to be less than 1%.

The two-part detector arrangement is sketched in Fig. 1. The part of the apparatus for measuring the fourmomentum of the π^+ consists of a magnet spectrometer, equipped with a stack of scintillators and three wire chambers for track reconstruction. The performance of this device is extensively described in Ref. 12.

Especially for this type of three-body reaction, the geometrical and momentum acceptance had to be kept



FIG. 1. Horizontal cut through the apparatus together with particle trajectories; the symbols C and S denote wire chambers and scintillators, respectively. Out of the six panels for detecting the π^- , only those intersecting the x-z plane are sketched.

extremely large, i.e., to 35 msr at 200 ± 80 MeV/c. The typical path length for the π^+ of 2.0 m agrees well with the low momentum of the produced pions. In the scintillator stack, the π^+ is stopped after having passed the dipole field and its characteristic decay cascade is observed. Together with time-of-flight and dE/dx measurements, a well-known identification efficiency of 75% for positive pions is achieved (for details, see Ref. 12). This is of crucial significance, because the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$ is the only way—in this energy region - to produce a positive pion from an incident beam of negative pions and a hydrogen target. Therefore the identification of the π^+ is equivalent with the recognition of a $(\pi, 2\pi)$ event. The ratio of misidentified events (mostly Dalitz pairs from the neutral pion decay) to the real $(\pi, 2\pi)$ events was estimated to be only 10^{-4} .

For the second part of the apparatus for observing in coincidence the direction of the outgoing π^- , six detector panels were placed around the target, covering a solid angle of 4 sr. Such a large solid angle is necessary to



FIG. 2. The angular correlation function (a) $W(\phi_{\pi})$ at ϕ_{π} -=115° and (b) $W(\phi_{\pi})$ at ϕ_{π} -=175° for a total energy of 1301 MeV, with a π^+ fixed at 112 MeV/c and (ϕ_{π}, ϕ_{π}) =(78°,0°). The horizontal error bars denote the angular binning. All quantities are given in the center-of-mass system. The comparison with the data includes the full calculation for the bag radii R=0.0 fm (solid line) and R=1.0 fm (long-dashed line), together with the phase-space dependence (dash-dotted line). The contribution from the nonlinear diagrams [Figs. 3(c) and 3(d); dash-multiple-dotted line] and the linear diagrams [Figs. 3(a) and 3(b); dotted line] are shown separately (R=0.6 fm).

cover major parts of the three-body phase space. Each panel consists of a doublet of plastic scintillators with photomultipliers on both sides, together with a multiwire proportional chamber in front of them. The active area is 64×64 cm² at a distance of about 80 cm from the target.

To check the level of spurious π^- , several tests were performed:

(a) The rate of accidental coincidences was measured by mismatching the timing of the two pions. It was less than 1% of the total events.

(b) In order to trace the π^- tracks, a second wire chamber was mounted in front of one panel. It turned out that the number of particles, which did not come from the target, was less than 1%.

(c) The time-of-flight measurements, the pulse heights in the scintillators, and the range of the particles (scanned with absorbers) were consistent with the $(\pi, 2\pi)$ kinematics.

The influence of the π^- decay on the result has been corrected (in first order) by Monte Carlo simulations.

With the measurement of the angles of the outgoing $\pi^+\pi^-$ pair, together with the momentum of the positive pion, the kinematics of the production process is fixed. A sample of about 3.5×10^4 kinematically complete $(\pi, 2\pi)$ events at an incoming pion momentum of 400 ± 7 MeV/c has been taken.¹³ At fixed angles of the π^+ , the angular distribution of the π^- was measured as a function of the π^+ momentum. Thus, the first triple-differential cross sections for this reaction could be extracted with low statistical error. This cross section, depending on five observables $(\vartheta_{\pi^+}, \varphi_{\pi^+}, \vartheta_{\pi^-}, \varphi_{\pi^-})$, is the most exclusive quantity for this reaction (for an unpolarized target).

Instead of working with this quantity, we prefer the angular correlation function W, defined by

$$W(\vartheta_{\pi^{-}}, \phi_{\pi^{-}}, \vartheta_{\pi^{+}}, \phi_{\pi^{+}}, p_{\pi^{+}}) = 4\pi \frac{d^{3}\sigma}{d\Omega_{\pi^{+}} d\Omega_{\pi^{-}} dp_{\pi^{+}}} \Big/ \frac{d^{2}\sigma}{d\Omega_{\pi^{+}} dp_{\pi^{+}}}$$

The advantage in using W is that this quantity is free of uncertainties in the acceptance and efficiency of the spectrometer, the target thickness, and the beam intensity. Thus, W is independent of beam normalization and monitoring. Therefore, the extraction of angular correlation data is very precise.

The double-differential cross section was extracted simultaneously by observation of the total π^+ rate and is in good agreement with Ref. 8.

The angular-correlation data were evaluated as a function of the polar and azimuthal angle of the π^- . These angles correspond to a z axis given by the beam direction, and a x-z plane defined by the outgoing π^+ and the beam.

In this Letter, we present only a very small part of our data, i.e., for a kinematical situation as given in Fig. 2. Here the π^- angular distributions, both in and out of the

x-z plane are shown, together with the phase space of the reaction. The presentation and discussion of the full data set of some hundred experimental points, covering interesting parts of the phase space, and the comparison with other experimental results will be part of a forth-coming paper.

One characteristic of the data is a pronounced deviation from phase space, which signals different production mechanisms. For a more quantitative insight, we follow the chiral πN Lagrangian of Weinberg, allowing in addition the coupling to the Δ isobar and the Roper resonance. Consequently, with $B, B'=N,N^*$, Δ , we start from the Lagrangian

$$\mathcal{L} = \sum_{B} \mathcal{L}_{B} + \mathcal{L}_{\pi} + \sum_{BB'} \mathcal{L}_{\pi BB'}.$$

Here, the \mathcal{L}_B denote the standard Lagrangians for the free baryons, while \mathcal{L}_{π} for the π field is given as³

$$\mathcal{L}_{\pi} = \frac{1}{2} \lambda(\boldsymbol{\phi}_{\pi}^2) [\lambda(\boldsymbol{\phi}_{\pi}^2) (\partial \boldsymbol{\phi}_{\pi})^2 - m_{\pi}^2 \boldsymbol{\phi}_{\pi}^2].$$



FIG. 3. Diagramatic expansion of the $(\pi, 2\pi)$ amplitude. (a),(b) linear *p*- and *s*-wave diagrams; (c),(d) nonlinear thirdand fourth-order graphs; and (e) possible final-state interaction.

Furthermore, the various pion-baryon-baryon vertices, with $B, B' = N, N^*$, are given by ^{3,14}

$$\mathcal{L}_{BB'\pi} = -\frac{g_{BB'\pi}}{2\sqrt{m_B m_{B'}}} \bar{B}' \lambda(\phi_{\pi}^2) \left[-\gamma_5 \tau \partial \phi_{\pi} + x^2 \frac{g_{BB'\pi}}{2\sqrt{m_B m_{B'}}} \tau \cdot \phi_{\pi} \times \partial_{\mu} \phi_{\pi} \right] B,$$

$$\mathcal{L}_{B\Delta\pi} = \frac{g_{B\Delta\pi}}{2\sqrt{m_B m_{\Delta}}} \bar{B} \mathbf{T} \cdot \partial_{\mu} \phi_{\pi} \Delta^{\mu}, \quad \mathcal{L}_{\Delta\Delta\pi} = -g_{\Delta\Delta\pi} \bar{\Delta}_{\mu} \gamma_5 \boldsymbol{T} \cdot \phi_{\pi} \Delta^{\mu},$$

with $\lambda(\phi) = [1 + (g/2m_n)x\phi_{\pi}^2]$ and $x = g_V/g_A \approx 0.8$. Above, Δ_{μ} is the Rarita-Schwinger spinor for a spin- $\frac{3}{2}$ particle; **T** and **T** represent appropriate isospin matrices.

 \mathcal{L} still involves the π field to all orders. For practicality, \mathcal{L}_{π} is expanded in powers of the pion field up to ϕ_{π}^4 . The corresponding set of diagrams is summarized schematically in Fig. 3. Treating the baryons in the static limit and accepting the cloudy bag model form factor¹⁵ for off-shell pions, the further evaluation is straightforward.

For practical evaluation, we use the standard values¹⁶ $f_{\pi} = 93$ MeV and $g_{\pi NN}^2/4\pi = 14$ and $g_{\pi N\Delta}^2/4\pi = 74$; the other coupling constants are extracted from the corresponding partial decay widths, together with SU(6) relations for $g_{\Delta\Delta\pi}$ and $g_{N^*N^*\pi}$. More detailed information on the model is given in Ref. 17. We stress that the results are fairly insensitive against a moderate variation of $g_{\pi NN^*}, g_{\pi\Delta N^*}$ and particularly— $g_{\Delta\Delta\pi}, g_{N^*N^*\pi}$ and the bag radius R as the most uncertain quantities.

Comparing the data with this calculation (Fig. 2), the following features are evident:

(i) The calculation reproduces—in a systematic way—the qualitative features of the data.

(ii) With the accuracy of the data, however, structure both in the azimuthal and in the polar sector is clearly missing.

(iii) Neither the nonlinear contributions nor the "classical" s- and p-wave diagrams account even qualitatively for the data.

(iv) The nonlinear diagrams, particularly the pole term, dominate the cross section; however, the calculations also indicate a significant influence of the linear p-wave diagrams.

Summarizing, our experiment provides the first exclusive and precise data for the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$

TABLE I. Numerical values of the data points shown in Fig. 2. ΔW denotes only the statistical error of W; the systematical error of W is about 5%. The angular resolution in ϕ and ϑ is < 1°. The double-differential cross section for this kinematic situation is $d^2\sigma/d\Omega_x + dT_x + = 800 \pm 38$ nb/MeV sr.

φ	W	ΔW
176.7	2.981	0.1845
168.2	2.795	0.1653
152.9	2.642	0.1672
128.1	2.298	0.1135
111.7	1.380	0.0817
92.20	1.118	0.0826
72.91	0.5716	0.0928
62.20	0.2688	0.0776
49.30	0.3221	0.0609
θ	W	ΔW
158.1	1.586	0.1661
142.5	2.310	0.1686
124.5	2.984	0.2824
110.4	2.978	0.2437
93.12	2.872	0.1633
68.48	2.586	0.1508
47.83	1.571	0.1243
28.81	0.6746	0.0798

in the region of the Δ resonance. Based on a sample of about 10⁵ kinematically complete events, a set of high-statistics data for the angular-correlation function is now at our disposal (compare upper and lower sections of Table I).

Our theoretical approach¹⁷ gives a surprisingly good first-order description of the data, without providing quantitative insight. Various possible refinements of the approach include a nonstatic treatment of the baryonic degrees of freedom or a more rigorous treatment of the nonlinear dynamics; such calculations are currently being performed. It is clear that the data presented here will serve as a stringent guide for further theoretical efforts.

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