Vector and Axial-Vector Meson Dominance in Neutrino Scattering and the Measurement of $\sin^2\theta_W$

Jon Pumplin
Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824
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Contributions to neutrino scattering which are proportional to $1/Q^2$ are estimated using vector and axial-vector meson dominance. They are found to be potentially larger than previous estimates of non-leading twist. They lead to an additional theoretical uncertainty, on the order of $^{+0.01}_{-0.00}$, in the value of $\sin^2\theta_W$ which is extracted from the neutrino cross-section ratio σ^{NC}/σ^{CC}. This suggests that the mass of the top quark is likely to lie near the lower end of its currently allowed range in the minimal standard model.

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The ratio $R=\sigma^{NC}/\sigma^{CC}$ of neutral-current to charged-current neutrino cross sections provides one of the most accurate measurements[1-4] of the fundamental weak mixing parameter $\sin^2\theta_W$. The result is so precise that it can be compared with the measurement based on m_Z to test standard-model radiative corrections at one-loop order, and thereby to set limits on the mass of the top quark.[5] The importance of doing this motivates a careful study of all possible sources of systematic error in the neutrino result.

Except for small corrections due to strange quarks, nonisoscalar targets, and radiative corrections, the standard model predicts[6]

$$R_{\rm DIS}=\frac{\sigma_{\rm DIS}^{\rm NC}}{\sigma_{\rm DIS}^{\rm CC}}=\tfrac{1}{2}-\sin^2\theta_W+\tfrac{5}{9}(1+r)\sin^4\theta_W \qquad (1)$$

in the deep-inelastic (DIS) limit $Q^2\rightarrow\infty$, $\nu\rightarrow\infty$, Q^2/ν =const, where $r=\sigma^{CC,\bar{\nu}}/\sigma^{CC,\nu}$. Cross sections elsewhere in the paper are for neutrinos, since the antineutrino data have larger errors and less sensitivity to $\sin^2\theta_W$.

Neutrino experiments actually measure

$$R_{\rm expt}=\frac{\sigma_{\rm DIS}^{\rm NC}+\sigma_V^{\rm NC}+\sigma_A^{\rm NC}}{\sigma_{\rm DIS}^{\rm CC}+\sigma_V^{\rm CC}+\sigma_A^{\rm CC}}\,, \qquad (2)$$

where σ_V^{NC}, etc., correspond to "higher-twist"[7] (twist 4) effects which we assume here to be given by the vector and axial-vector meson-dominance processes shown in Fig. 1. These processes fall as $1/Q^2$ relative to the leading hard-scattering mechanism, which varies only logarithmically with Q^2 in the DIS limit. Their effect in Eq. (2) is further suppressed because their NC/CC ratio is similar to that for the DIS process, as emphasized by Llwellyn Smith.[6] The meson dominance processes have therefore been neglected, along with all other power-law corrections, in the experimental analyses of R.[1-4] The point of this paper is to make an estimate of them. We find that they can have a significant effect on the determination of $\sin^2\theta_W$.

There are additional contributions to the neutrino cross section from pion exchange and PCAC (partial conservation of axial-vector current).[8] Their observation[9] lends credibility to the analogous processes considered here, but we need not include them because they are important only at very small Q^2, so they do not affect the determination of $\sin^2\theta_W$.

A number of authors have calculated the coherent production of vector and axial-vector mesons in neutrino scattering.[10] The mechanism is similar to Fig. 1, with the meson total cross section replaced by its elastic cross section. The apparent observation[10,11] of such processes in neutrino bubble-chamber experiments supports the existence of the mechanism of Fig. 1. Cross sections appropriate to Fig. 1 can be obtained simply by substituting ρ and a_1 total cross sections for the integrated elastic ones which appear for coherent production. The result for the charged vector current is

$$\frac{d\sigma_\rho^{\rm CC}}{dQ^2d\nu}=\frac{G_F^2}{2\pi^2}\frac{V_{ud}^2}{\gamma_\rho^2}\frac{Q^2}{E^2}\Phi S\left(\frac{m_\rho^2}{Q^2+m_\rho^2}\right)^2\frac{\sigma_T+\epsilon\sigma_L}{1-\epsilon}\,, \qquad (3)$$

where

$$\epsilon=\frac{4E(E-\nu)-Q^2}{4E(E-\nu)+Q^2+2\nu^2}\,, \qquad (4)$$

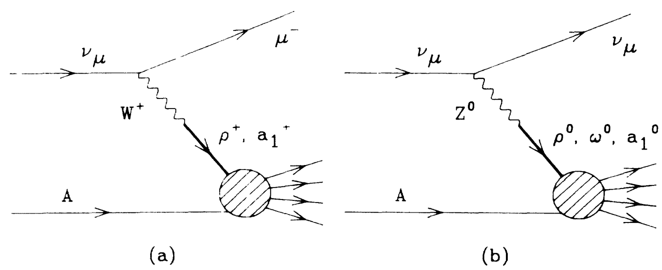

FIG. 1. Vector and axial-vector meson-dominance contributions to (a) charged-current and (b) neutral-current neutrino cross sections. The blob represents the total cross section for the virtual meson on the target nucleus.

with E the incident-neutrino energy. The following paragraphs discuss various factors in Eq. (3) and their estimated uncertainties.

The $W\rho$ coupling at $Q^2 = -m_\rho^2$ is defined as $V_{ud}(g_W/\sqrt{8})\sqrt{2}m_\rho^2/\gamma_\rho$. Its strength $\gamma_\rho^2/4\pi = 2.5 \pm 0.1$ can be obtained from τ decay[12] using the narrow-resonance approximation:

$$\Gamma_{\tau\to\rho\nu} = \frac{G_F^2}{8\pi}\frac{V_{ud}^2}{\gamma_\rho^2}\frac{m_\rho^2}{m_\tau^3}(m_\tau^2 + 2m_\rho^2)(m_\tau^2 - m_\rho^2)^2 . \quad (5)$$

A similar, though slightly inconsistent, value 2.0 ± 0.1 can be obtained from ρ^0 electromagnetic decay:

$$\Gamma_{\rho\to e^+e^-} = (4\pi/3)\alpha^2 m_\rho/\gamma_\rho^2 .$$

σ_T is the total cross section for off-shell transverse ρ scattering. For "large" Q^2, I assume it to be given by $\sigma_T = 28.5$ mb, corresponding to on-shell $\sigma_{\pi^\pm N}$. For "small" Q^2, I assume it to display shadowing,[13] and therefore to be given by $\sigma_T = (28.5 \text{ mb})A^{0.75}/A$, which is a fit[14] to $\pi^\pm$-nucleus inelastic cross-section data. (The dependence $A^{0.75}$ is similar to the naive $A^{2/3}$ of the black-disk limit, but differs from it because surface effects are non-negligible at moderate A. The extra $1/A$ appears because of the convention that cross sections here are *per nucleon.*) The coherent elastic part of the cross section on the nucleus can be neglected here, since it involves an integral over momentum transfer t which is strongly suppressed by the elastic form factor of the nucleus. This suppression occurs because $t_{\min}$ is significant over most of the range of Q^2 and ν.[10] We assume a simple parametrization for the transition between "large"-$t_{\min}$ (A^1 behavior) and "small"-$t_{\min}$ ($A^{0.75}$ behavior) regions. Our results are insensitive to the details of this assumption.

The longitudinal cross section σ_L must be 0 at $Q^2 = 0$. Reasonable guesses[15] for σ_L/σ_T range 0 to $Q^2/(Q^2 + 1 \text{ GeV}^2)$, where I have built in a prejudice that it remains ≤ 1 at large Q^2. The flux factor Φ is slightly ambiguous because the meson scattering is off mass shell. Reasonable choices are ν, $|\mathbf{q}| = (Q^2 + \nu^2)^{1/2}$, or $\nu - Q^2/2M$. The distinction between these choices is of little importance, because most of the cross section comes from a region where $Q^2 \ll 2M\nu \ll \nu^2$.

I take account of all off-shell effects by including an additional suppression factor

$$S = [1 + (Q^2 + m_\rho^2)/S_0]^{-2} \quad (6)$$

in Eq. (3), where $1 \text{ GeV}^2 \leq S_0 \leq \infty$. It is here that a broad range of model dependence enters, from no suppression at all to substantial suppression already at $Q^2 = 0$.

The parameters of Eq. (3) for the ρ-dominated charge-vector-current cross section σ_ρ^{CC} have now been specified. To obtain $\sigma_{a_1}^{CC}$, it is only necessary to substitute $a_1(1260)$ for $\rho(770)$ in Eqs. (3)-(6). I assume the weak current couples equally to vector and axial-vector states, so that $\Gamma_{\tau\to\rho\nu}$ and $\Gamma_{\tau\to a_1\nu}$ would be equal in the limit of large m_τ. This implies $\gamma_\rho/m_\rho = \gamma_{a_1}/m_{a_1}$, and hence $\gamma_{a_1}^2/4\pi = 6.7$, which is consistent with the value 7.6 ± 2.4 which can be obtained from $\Gamma_{\tau\to a_1\nu}$ using the analog of Eq. (5).

Figure 2 shows the charged-current cross sections calculated using Eq. (3), assuming no off-shell suppressions ($S_0 \to \infty$), $\sigma_L/\sigma_T = 0$, and $A = 21.6$ which is the $A^{0.75}$-weighted average number of nucleons for the marble target used in the CHARM experiment. Also shown is the DIS cross section, calculated using the Duke-Owens[16] fit 1 to structure functions. For simplicity, the contributions of s quarks to the DIS cross section is ignored.

The dotted curves show predicted cross sections for an enviably high monochromatic neutrino energy $E = 400$ GeV. Contributions from ρ and a_1 are seen to fall off at large Q^2—as expected of higher-twist processes—while the leading-twist process persists to very large Q^2. Hence ρ and a_1 would have very little effect on the integrated cross section.

The solid curve shows the predicted cross section using the neutrino energy spectrum of the CHARM experiment.[1] (That energy spectrum contains two peaks: 83% of the probability is in a peak with energy-weighted average energy 46 GeV, while the remaining 17% is in a peak with energy-weighted average energy 127 GeV.) *With this actual experimental energy spectrum, the DIS process is seen to fall at large Q^2 almost as rapidly as the higher-twist ones.* Integrating the cross sections of Fig.

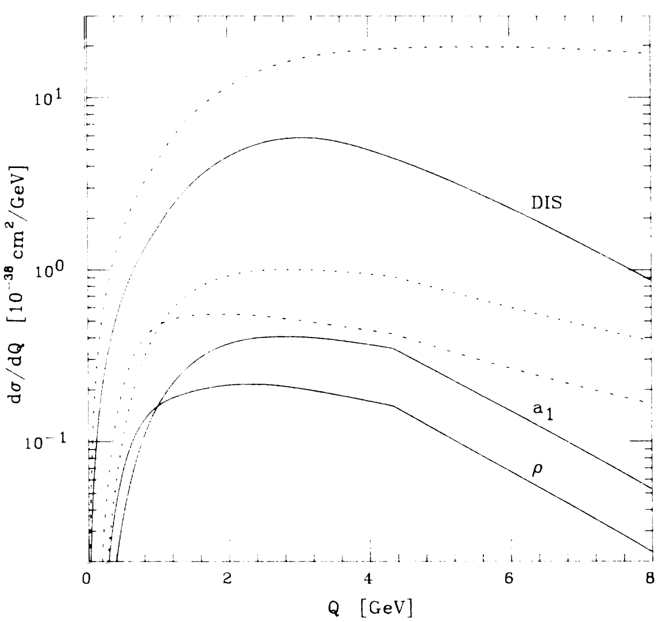

FIG. 2. Contributions to the charged-current neutrino cross section as a function of $Q \equiv (Q^2)^{1/2}$. The solid curves are for the neutrino energy spectrum of the CHARM experiment (Ref. 1). The corresponding dotted curves are for a fixed incident neutrino energy $E = 400$ GeV.

2 over all Q leads to ρ and a_1 contributions amounting to sizable fractions of DIS: 3.90% and 7.29%.

The cross sections due to ρ and a_1 are sensitive to any extra off-shell suppression at large Q^2, since they otherwise contain significant contributions from fairly large Q^2. Using the suppression factor of Eq. (6) with $S_0 = 20$, 10, and 4 GeV^2 leads to $\sigma_\rho^{CC}/\sigma_{DIS}^{CC} = 1.75\%$, 1.19%, and 0.60%; and $\sigma_{a_1}^{CC}/\sigma_{DIS}^{CC} = 2.68\%$, 1.62%, and 0.66%, respectively. A suppression factor of $\exp(-B\Delta^2)$, where $\Delta = (Q^2+m^2)/2\nu$ and $B = 5\ \text{GeV}^{-2}$, which operates at small ν, and which might arise according to a formation-time argument, reduces $\sigma_\rho^{CC}/\sigma_{DIS}^{CC}$ $(\sigma_{a_1}^{CC}/\sigma_{DIS}^{CC})$ to 2.5% (4.0%).

There is less sensitivity to the other ambiguities in the model. For example, the choices $\sigma_L/\sigma_T = 0$, $Q^2/(Q^2+2\ \text{GeV}^2)$ lead to $\sigma_{a_1}^{CC}/\sigma_{DIS}^{CC} = 3.9\%$, 5.1%, respectively, for $S_0 = \infty$; or 0.98%, 1.5% for $S_0 = 10\ \text{GeV}^2$. The choices $\Phi = \nu$, $(Q^2+\nu^2)^{1/2}$, and $\nu - Q^2/2M$ lead to $\sigma_{a_1}^{CC}/\sigma_{DIS}^{CC} = 3.9\%$, 4.0%, and 2.6%, respectively, for $S_0 = \infty$; or 0.98%, 0.99%, and 0.82% for $S_0 = 10\ \text{GeV}^2$.

A cut requiring $\nu > 10$ GeV is included in Fig. 2, and in the above cross-section ratios. This approximates the hadronic-energy cut $E_h > 10$ GeV made by the CERN-Dortmund-Heidelberg-Saclay (CDHS) Collaboration,[2] and the cut $E_h > 9$ GeV made in part of the CHARM data.[1] The intersection of this cut with the kinematic boundary $Q^2 < 2M\nu$ is responsible for the kinks which appear in the ρ and a_1 curves.

Thus far, we have only considered higher-twist contributions to the charged-current cross section [Fig. 2(a)]. Contributions to the neutral current [Fig. 2(b)] are simply related to these by

$$\frac{\sigma_V^{NC}}{\sigma_V^{CC}} = \tfrac{1}{2}\left[(1-2\sin^2\theta_W)^2 + (\tfrac{2}{3}\sin^2\theta_W)^2\right], \tag{7a}$$

$$\frac{\sigma_A^{NC}}{\sigma_A^{CC}} = \frac{1}{2}. \tag{7b}$$

The approximate agreement noted above between the determinations of γ_ρ from τ decay or from the electromagnetic decay of ρ^0 supports the validity of these quark-model relations. A Kobayashi-Maskawa matrix-element factor V_{ud}^{-2}, where $V_{ud} \approx 0.975$, has been omitted, to be consistent with the neglect of s quarks in the calculation. The second term in brackets comes from the contribution of $\omega(783)$, which is taken to be proportional to the ρ, since their masses are so similar. Note that there is no quantum-mechanical interference between the contributions to the cross section from ρ, ω, and a_1, since the Pomeron which is responsible for these total cross sections is assumed to have isospin 0 and natural parity.

We now have expressions for each of the higher-twist cross sections σ_V^{CC}, σ_A^{CC}, σ_V^{NC}, and σ_A^{NC} which appear in Eq. (2). We are therefore in a position to study their effect on measuring $\sin^2\theta_W$ via Eq. (1). A simple way to do this is to first "correct" the experimental data for higher twist, and then proceed with the traditional analysis. To this end, we rewrite Eq. (2) in the form

$$R_{DIS} = R_{expt} + (a\sigma_A^{CC} + b\sigma_V^{CC})/\sigma_{DIS}^{CC}, \tag{8}$$

where $a = R_{expt} - \sigma_A^{NC}/\sigma_A^{CC}$ and $b = R_{expt} - \sigma_V^{NC}/\sigma_V^{CC}$. The CHARM experiment[1] (with $E_h > 9$-GeV cut) finds $R_{expt} = 0.3052 \pm 0.0033$, while the CDHS Collaboration[2] finds a very similar $R_{expt} = 0.3072 \pm 0.0033$. Using Eqs. (7) even with a generous range of 0.235 ± 0.015 for $\sin^2\theta_W$ leads to rather well-defined values for the coefficients in Eq. (8): $a = -0.194 \pm 0.004$ and $b = -0.153 \pm 0.020$.

The coefficients a and b have opposite signs, so the effects of ρ and a_1 tend to cancel each other. This happens because the average of the NC/CC ratios in Eqs. (7) is quite close to that of Eq. (1). Indeed, that average is given exactly by setting $r = 1$ in Eq. (1), which makes sense because the higher-twist terms are the same for neutrinos and antineutrinos. Hence $a+b$ is proportional to $\sin^4\theta_W$, and is therefore small. Since σ_V^{CC} and σ_A^{CC} might be expected to be equal, the a and b terms in Eq. (8) might be expected to effectively cancel each other. This cancellation is the basis for the usual expectation[6] that higher-twist contributions to R can be neglected. The possibility we find here of a significant effect on R arises because the cancellation is incomplete, since (i) $a+b$ is actually not zero because of the $\sin^4\theta_W$ term, and (ii) σ_V^{CC} and σ_A^{CC} are actually different because of the mass difference between $\rho(770)$ and $a_1(1260)$. The dimensional scale of the $1/Q^2$ terms is thus set here by $m_{a_1}^2 - m_\rho^2$, rather than the much smaller m_{quark}^2.

Using Eq. (8) and the charged-current cross sections calculated above, we obtain $R_{DIS} - R_{expt} = -0.0050$, -0.0016, -0.0008, and -0.0002 for $S_0 = \infty$, 20, 10, and 4 GeV^2, respectively. This quantity is more sensitive to the off-shell parameter S_0 than are the cross sections themselves, because the difference between ρ and a_1 contributions is small at small Q^2, and even changes sign there. Using Eq. (1), these results translate to changes in the value of $\sin^2\theta_W$ which should be extracted from the CHARM data of $+0.0078$, $+0.0025$, $+0.0013$, and $+0.0003$. *Since the actual extent of off-shell suppression is unknown, we must interpret this as a systematic theoretical error in the measurement of* $\sin^2\theta_W$. Because of additional uncertainties in $\sigma_L/\sigma_T = 0$ and Φ, the actual error range is somewhat larger: approximately 0 to $+0.010$. Note that $\sin^2\theta_W$ can only *increase* as a result of the meson-dominance correction.

I have used the CHARM experiment as an example. Nearly identical results apply to the CDHS experiment,[2] which used a very similar neutrino spectrum. The fact that they used a target of iron rather than marble matters little, because the nuclear t_{min} effect implies rather little shadowing of the meson-dominance terms. (Even a shadowing behavior of $A^{0.75}$ for the meson-dominance terms would suppress them by only a factor of 0.8 in CDHS relative to CHARM.) The remaining two high-statistics experiments[3,4] are also similar, but have somewhat larger statistical errors, so that the effect considered here is of marginal importance for them.

To conclude, measurements of $\sin^2\theta_W$ based on neutrino cross sections contain a systematic theoretical uncertainty due to the nonperturbative higher-twist processes. These processes could, according to a meson-dominance calculation, allow the actual value of $\sin^2\theta_W$ to be larger than the published experimental results by as much as 0.01. This maximum value occurs only if there is no off-shell suppression: $S \rightarrow 1$ in Eqs. (3) and (6). That would perhaps be surprising, but it cannot at present be ruled out. Note, for example, that the pointlike couplings of the weak current make the process of Fig. 1 different from a purely hadronic one such as $\pi^- p \rightarrow \pi^0 X$. The analogous ρ-exchange diagram for that process is suppressed exponentially at large Q^2 in the triple-Regge limit; but that suppression is associated with the "wave-function" effect of requiring the π^0 to come out intact.

The result of this paper has an important implication in the minimal standard model, since electroweak radiative corrections generate a difference between the $\sin^2\theta_W$ discussed here and the $\sin^2\theta_W^*$ which can be obtained from the accurately measured mass of Z^0.[5] The difference is sensitive to the mass of the top quark (and mildly sensitive to the mass of the Higgs boson). Our result suggests that $\sin^2\theta_W$ measured by neutrino scattering may be somewhat larger than previously thought, which in turn hints that the top quark is most likely to be found near the low end of the range of masses which is currently allowed.

Experiment[17] E733 at the Fermilab Tevatron, whose data are now being analyzed, employs significantly higher neutrino energies, for which the power-suppressed contributions will be negligible. Results of that experiment, and its possible successors, are eagerly awaited.

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