Hyperextended Inflation

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(Received 16 February 1990)

We present a dynamical mechanism for completing inflationary phase transitions via bubble nucleation which can satisfy all known constraints for a wide spectrum of models and parameters. The approach is a generalization of the recent "extended inflation" model which corrects a serious flaw. We find an essentially model-independent bubble-size distribution which may be important for large-scale structure.

PACS numbers: 98.80.Cq, 04.50.+h

The Achilles' heel of inflationary cosmology has been the mechanism for completing the first-order phase transition that drives the superluminal expansion. Inflation relies on the extraordinary expansion that results when the Universe is trapped in a metastable, false-vacuum phase caused by a strongly first-order phase transition. The expansion of the scale factor $R(t)$ by e^{60} or more is needed to resolve the cosmological horizon, flatness, and monopole problems.[1] While sufficient expansion is not difficult to achieve in realistic models, exiting the false-vacuum phase and restoring a hot, Friedmann-Robertson-Walker universe has proved to be embarrassingly difficult. The original "old inflation" model fails to exit at all.[1] "New inflation"[2] and variants thereof (e.g., chaotic inflation,[3] quantum cosmology,[4] etc.) allow completion of the phase transition, but at the cost of fine tuning of parameters in order to obtain acceptably small density perturbations.

"Extended inflation"[5] is a recent proposal intended to produce a successful inflationary phase transition for a much wider and more natural range of models. Unlike new inflation, the phase transition is completed by ordinary bubble nucleation—there is no need to fine tune the transition so that the barrier between the false- and true-vacuum phases disappears as the Universe supercools. Unfortunately, it was soon found that the bubbles can lead to unacceptable distortions of the microwave background.[6,7]

The goals of this paper are twofold. First, we consider a natural but important extension of extended inflation. As in extended inflation, a modification of conventional Einstein gravity is considered in which a scalar field ϕ is nonminimally coupled to the scalar curvature $\mathcal{R}$. Such corrections to Einstein gravity arise in virtually every known unified theory that couples particles to gravity, including supersymmetry and superstring models. If the nonminimal couplings are not present at tree level, they are typically generated by quantum corrections. Usually these corrections to Einstein gravity are ignored, but now it appears that they can play a critical role in the early history of the Universe. Only quadratic couplings of the form $\xi\phi^2\mathcal{R}$ were introduced in extended inflation, whereas higher-order couplings are generally expected as well. In this paper, we consider a completely general interaction, $f(\phi)\mathcal{R}$, where $f(\phi)=M^2+\xi\phi^2+\xi'(\phi^4/M^2)+\cdots$ for small ϕ. Our initial concern was that quartic and higher-order couplings may interfere with extended inflation. In fact, we find that the quartic and higher-order couplings greatly enhance the scenario and permit inflation to be completed for a much wider range of models and initial parameters.

Our second goal is to address a serious flaw of extended inflation. Bubble sizes at the end of the phase transition are distributed in a nearly scale-invariant spectrum,[6,7] with the deviation in the exponent scaling as 32ξ. The bubbles represent large-density perturbations $\delta\rho/\rho\approx 1$ on wavelengths equal to their radius. To suppress bubbles of astrophysical size that would unacceptably distort the microwave background, $\xi>0.005$ is required. Otherwise, only $\xi<0.1$ is needed to satisfy all other constraints. Not only does the bubbles' constraint narrow the spectrum of acceptable models, but the allowed range of ξ conflicts with astrophysical tests[8] of Einstein versus Brans-Dicke[9] gravity which require $\xi<0.00025$.

We will present a simple scenario utilizing the expanded nonminimal couplings which will result in an acceptable distribution of bubble sizes for a much wider range of initial parameters. The bubble spectrum is essentially model independent, assuming no special tuning of parameters. We will also introduce a mechanism—a somewhat novel approach to induced gravity—which can result in negligibly small deviations from Einstein gravity after inflation.

In extended inflation and our improved approach, the nonminimally coupled field ϕ is completely independent of the field σ, the order parameter for the inflationary phase transition. Since ϕ is coupled to the curvature, its expectation value contributes a correction to the effective gravitational constant, $G \equiv M_P^{-2} = [f(\phi)/16\pi]^{-1}$, where M_P is the Planck mass. We consider a Lagrangian density

$$\mathcal{L} = -f(\phi)\mathcal{R} + \tfrac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + 16\pi\mathcal{L}_{\rm matter}\,, \tag{1}$$

where $\mathcal{L}_{\rm matter}$ contains contributions of all other matter fields including the field σ, which drives the inflationary phase transition. During inflation, we assume that σ is trapped in the false-vacuum phase of its effective potential $V(\sigma)$. We define the vacuum energy to be $V(\sigma) \equiv \rho_F \equiv M_F^4$. Unlike new inflation models,[2] a non-negligible energy-density barrier, $\sim M_F^4$, is assumed to separate the false- and true-vacuum phases. Hence, the bubble nucleation rate per unit volume λ is exponentially small. We assume that the potential for ϕ is negligible or zero.[10] The semiclassical equations are simple to analyze if we recast the model in a form reminiscent of Brans-Dicke theory.[9] Let $\Phi \equiv f(\phi)$ replace the scalar field. [Φ has the dimensions of (mass)^2.] The Lagrangian density is

$$\mathcal{L} = -\Phi\mathcal{R} + \frac{\omega(\Phi)}{\Phi}\partial_\mu\Phi\,\partial^\mu\Phi + 16\pi\mathcal{L}_{\rm matter}\,; \tag{2}$$

$\omega(\Phi)$ can be written as $f/2(f')^2$, where $f' \equiv df/d\phi$.

For simplicity, we will consider $f(\phi) = \xi(\phi^2 + \phi^{n+2}/M^n)$, $n > 2$. A leading constant only adds to the net amount of inflation (which is unnecessary) and a greater sum of higher-order couplings does not substantially change the scenario. For $\phi \ll M$, the quadratic coupling dominates and $\omega(\Phi) = 1/8\xi = $const, the standard Brans-Dicke result.[5,9] For $\phi \gg M$, though,

$$\omega(\Phi) = [2(n+2)^2\xi^{2/(n+2)}]^{-1}(\Phi/M^2)^{-n/(n+2)}$$

becomes Φ dependent. The equations of motion are

$$\ddot\Phi + 3H\dot\Phi = \frac{1}{3+2\omega}[8\pi(\rho - 3p) - \omega'\dot\Phi^2]\,, \tag{3}$$

$$H^2 + \frac{k}{R^2} = \frac{8\pi\rho}{3\Phi} - \frac{\dot\Phi}{\Phi}H + \frac{\omega}{6}\left(\frac{\dot\Phi}{\Phi}\right)^2, \tag{4}$$

where $H = \dot R/R$ is the Hubble parameter, $\omega' \equiv d\omega/d\Phi$, and ρ and p are the energy density and pressure due to the matter fields. During the false-vacuum phase, $V(\sigma)$ dominates the energy density and $\rho - 3p \approx 4\rho_F$. For $\phi \ll M$ and $\omega = 1/8\xi$, the solutions are the same as those exploited in extended inflation:

$$\begin{aligned} \Phi(t) &= \Phi_b(1 + H_b t/\alpha)^2\,,\\ R(t) &= R_b(1 + H_b t/\alpha)^{\omega+1/2}\,, \end{aligned} \tag{5}$$

where $\alpha^2 = (3+2\omega)(5+6\omega)/12 \approx \omega^2$ and the subscript b will be used to denote values when inflation begins. $R(t)$ grows as a power law in time, rather than the exponential obtained for Einstein gravity.

In Ref. 5, it was shown that the slowing from exponential to power-law inflation is sufficient to allow a phase transition from false- to true-vacuum phase to be completed by ordinary bubble nucleation. Briefly, the fractional volume remaining in the false-vacuum phase after time t is

$$p_f(t) = \exp\left[-\int_{t_b}^{t}\lambda R^3(t')\frac{4\pi}{3}\left[\int_{t'}^{t}\frac{dt}{R}\right]^3 dt'\right]. \tag{6}$$

The physical volume trapped in the false phase is, therefore, $V_F = p_f R^3$. If $R(t)$ grows exponentially and λ is indeed small, V_F increases exponentially forever and the transition is never completed. If $R(t)$ grows as a power for a sufficiently long time, though, V_F begins to decrease exponentially (and the transition is completed) once the exponential decrease in $p_f(t)$ outruns the power-law growth in $R(t)$. Typically, $R(t) \propto t^{\omega+1/2}$ has grown much more than the necessary 60 e-foldings before completion. The problem, though, is the distribution of bubbles that results:[7]

$$F(x > x_0) \sim \frac{1}{(1 + H_e x_0)^{4/\omega}}\,, \tag{7}$$

where $F(x > x_0)$ is the fractional volume occupied by bubbles of proper radius greater than x_0 and H_e is the value at the end of inflation. To avoid distortion of the microwave background, we require $F(x > x_0)$ to be less than 10^{-4} for bubbles of supercluster scale, $H_e x_0 \gtrsim 10^{25}$. This implies $\omega < 25$ (or $\xi > 0.005$), in conflict with observational bounds.[8]

We find that quartic and higher-order nonminimal couplings have the desired effect of slowing the expansion even more dramatically. The couplings can be incorporated in a rather natural scenario that results in an acceptable bubble-size distribution for a much wider choice of initial conditions and parameters: First, ϕ is much less than M so the quadratic coupling dominates $f(\phi)$. The expansion proceeds as in extended inflation in this initial stage. Second, ϕ increases to near M, and the higher-order couplings become important. The scenario requires that the nucleation rate λ is so small that $p_f(t)$ is still very close to unity by the "crossover" time, $t = t_c = \omega_c/H_c$ when $\phi \sim M$. That is, $p_f(t_c)$ is still very close to unity so that the final fractional volume occupied by bubbles produced before t_c is negligibly small. Since λ varies exponentially[11] for relatively small adjustments of the energy barrier of $V(\sigma)$, this condition can be satisfied for natural, untuned choices of $V(\sigma)$ and $\omega_b > 1$. For $t > t_c$, ω becomes Φ dependent, and the solutions are for $n = 2$,

$$\begin{aligned} \Phi(t) &\approx \Phi_c \exp(2\Delta t/t_c)\,,\\ R(t) &\approx R_c \exp[\omega_c(1 - e^{-\Delta t/t_c})]\,, \end{aligned} \tag{8a}$$

for $n > 2$,

$$\Phi(t) \approx \Phi_c (1-\gamma \Delta t/t_c)^{(2n+4)/(2-n)} ,$$
$$R(t) \approx R_c \exp\{\gamma' \omega_c [1-(1-\gamma\Delta t/t_c)^{2n/(n-2)}]\} , \tag{8b}$$

where $\Delta t \equiv t - t_c$, $\gamma = (n-2)/(n+2)$, and $\gamma' = (n+2)/2n$. The striking feature is the rapid deceleration of $R(t)$ and the rapid acceleration of $\Phi(t)$. Third, as $\Phi(t)$ diverges, $\omega(\Phi) \propto \Phi^{-\gamma}$ approaches zero and the character of the solutions changes once again. In the first equation of motion, Eq. (3), the prefactor approaches a constant for $\omega \ll 1$. Consequently, the solutions reassume the form they had in extended inflation, $\Phi(t) \propto t^2$ and $R(t) \propto t^{\omega + 1/2}$. Inflation is completed when ω drops below $\frac{1}{2}$ and $R(t)$ no longer scales superluminally. The key feature is that, for essentially arbitrary initial values of ω, natural choices of $V(\sigma)$ (and, thereby, λ) can be made so that $\omega < 25$ and typically $O(1)$ during the final throes of inflation when the majority of bubbles are nucleated. From previous calculations, we anticipate that an acceptable bubble distribution results for ω in this range, and, hence, a successful inflationary transition results. Of course, the effects of a small ω in the later history of the Universe are worrisome; we will introduce a mechanism to address this issue after considering other constraints.

(1) Sufficient inflation, reheating, and $\delta\rho/\rho < 10^{-5}$.— The constraints that require fine tuning in conventional inflation models are easily satisfied here. The net inflation is (a) the inflation before t_c, $R(t_c)/R(t_b) = [\Phi(t_c)/\Phi(t_b)]^{1/16\xi}$ [see Eq. (5), plus (b) the inflation after t_c, $R(t_e)/R(t_c) = e^{\omega_c}$ [see Eq. (8)], where $\omega_c \equiv \omega(\phi \approx M) \approx 1/32\xi$. Supercooling begins at $\Phi(t_b) \sim M_F^2$ when the Hawking temperature falls below the critical temperature for the σ-field phase transition. Hence, the necessary 60 e-foldings of inflation[1] are obtained if either (a) $\omega_b \equiv 1/8\xi > 2$ and $[\Phi(t_c)]^{1/2} \approx \sqrt{\xi} M > 10^{25/\omega_b} M_F$ or (b) $\omega_c = \omega_b/4 > 60$, where ω_b is the value at the beginning of inflation. For example, (a) $\omega_b = 10$ and $M > 10^3 M_F$ or (b) $\omega_b > 240$ satisfies the constraints. Reheating occurs by bubble-wall collisions and subsequent rethermalization.[11] The average temperature should be $\sim M_F$, the critical temperature of the σ-field phase transition. The contribution to $\delta\rho/\rho$ due to fluctuations in ϕ is a nearly scale-invariant, adiabatic spectrum of fluctuations with amplitude $\delta\rho/\rho = 2 \times 10^{-2} H^2/\dot{\phi}$ evaluated during the last 60 e-foldings of inflation.[12] At t_c, the constraint is already satisfied if $M > 240(\omega_b)^{3/4} M_F$. However, this bound is much too conservative: Smaller values of M are acceptable because $\delta\rho/\rho \propto H^2/\dot{\phi}$ decreases exponentially after t_c (which typically include the last 60 e-foldings).

(2) Bubble-size distribution.— By the assumptions of the scenario, only bubbles produced after t_c occupy a non-negligible fraction of the Universe, $p_f(t_c) \approx 1$. Superluminal expansion ceases once ω falls below $\frac{1}{2}$. Although the transition may not yet be complete, bubbles produced after this time remain less than H^{-1} in radius, too microscopic to affect large-scale structure. We shall refer to the time when the expansion becomes subluminal as t_s. First, consider the large bubbles produced between $t = t_c$ and $t = t_s$. The bubble-size distribution can be expressed as $dN/d(H_e x) = \epsilon(t_0)/(1+H_e x)^4$, where $x = x(t_e, t_0)$ is the physical radius of a bubble produced at time t_0 measured at the end of inflation. In old inflation, ϵ is time invariant and the distribution is scale invariant;[13] in extended inflation, $\epsilon(t_0) \propto (1+H_e x)^{-4/\omega_b}$ produces a small deviation from scale invariance,[7] resulting in too many large bubbles unless $\omega_b < 25$. Here

$$\epsilon \propto 1/H^4(t_0) \propto e^{4t_0/t_c}$$

for $n = 2$ and

$$\epsilon \propto (1-\gamma\Delta t/t_c)^{(4n+8)/(2-n)}$$

for $n > 2$. Using $x(t,t_0) \equiv R(T)\int_{t_0}^{t} [dt'/R(t')]$, we find that $\epsilon \propto [\ln(1+Hx)\omega_e]^{-4\beta}$ and, hence, the fractional volume occupied by bubbles of radius greater than x_0 is

$$F(x > x_0) \approx \frac{\mathcal{V}_T(t_s)}{[(1/\omega')\ln(1+H_e x_0)+1]^{4\beta - 1}} , \tag{9}$$

where $\omega' = \omega_e \approx \frac{1}{2}$, and $\beta = 1$ for $n = 2$ or $\omega' = \gamma' \omega_e$ and $\beta = (n+2)/2n$ for $n > 2$. $\mathcal{V}_T(t_s)$ is of order unity times the volume fraction of true vacuum at t_s ($H_e x_0 \leq 1$). The fraction of the Universe that still remains in the false phase after t_s is percolated with bubbles with radius $H_e x < 1$. For x_0 of supercluster scale, $M_F \approx 10^{14}$ GeV, and $\Phi \approx M_P^2$, we have that $H_e x_0 \approx 10^{25}$ and $F(x > x_0) < 10^{-5\beta}$ for $\omega_e \sim 1$, which is small enough that the microwave background should not be unacceptably distorted. Yet, if $\mathcal{V}_T(t_s)$ is not too small, the distribution may provide a sufficient number of moderate-sized bubbles to be of interest for large-scale structure.

We wish to emphasize that the predicted distribution is nearly n independent and only depends on broad, easily obtained conditions on λ. Note that the form for $F(x > x_0)$ assumes ω decreases from some large value to $\omega < 0.5$ at the end of inflation, which applies for most choices of $f(\phi)$. It is also possible to choose $f(\phi)$ so inflation ends with $\omega \approx \text{const} > 1$, which produces a bubble distribution as in Eq. (7), but a broader range of choices lead to the distribution derived above.

(3) $\omega > 500$ today.— The most worrisome aspect of the scenario is that ω_e is much less than the observational bound based on time-delay measurements,[8] $\omega > 500$. Therefore, we introduce a simple mechanism— a somewhat unconventional approach to induced gravity— that could make $\omega > 500$ today even though $\omega_e \sim 1$.

The mechanism takes advantage of higher-order contributions to $f(\phi)$. Recall that $\omega(\phi) = f(\phi)/2[f'(\phi)]^2$. Previously, we only discussed the case where ω is constant (in extended inflation) or decreases with increasing

ϕ. While this may be true as the first ϕ-dependent terms in $f(\phi)$ become important, it is certainly conceivable that $f(\phi)$ reaches a maximum at some value, ϕ_m, where $\phi_e < \phi_m$. A simple example is $f(\phi) = M^2 + \xi\phi^2 - \alpha(\phi^4/M^2)$, with $\alpha < \xi$. By definition, $f'(\phi_m) = 0$, but there is no reason to expect $f(\phi_m) = 0$. With such a nonmonotonic $f(\phi)$, ω diverges as ϕ approaches ϕ_m. As $\omega \rightarrow \infty$, ϕ ceases to vary with time and the model is indistinguishable from Einstein gravity with $M_P^2 = f(\phi_m)/16\pi = \Phi_m$. Rather than fixing M_P via an ordinary potential for ϕ, as in conventional induced gravity, we have fixed M_P by freezing out the ω-dependent kinetic energy for ϕ.

This mechanism can be useful in driving ω to an acceptably large value after superluminal expansion ceases at time t_s. First, Φ increases considerably after t_s in typical cases, at least until the time when bubbles percolate or, equivalently, when the argument of $p_f(t)$ grows larger than unity. Φ then grows by an additional factor of 5 or so as the vacuum-dominated epoch converts into a radiation-dominated epoch (see Appendix of Ref. 6). With this additional growth in Φ after t_s, the higher-order terms become important and, by the mechanism described above, ω can be driven to > 500.

Once the Universe fully enters the radiation-dominated epoch after bubble percolation, the equation of state becomes $\rho - 3p = 0$. From the equations of motion, we find that $\dot{\Phi} \rightarrow 0$ (independent of the value of ω), and the Universe is indistinguishable from conventional Friedmann-Robertson-Walker universes. The change in Φ and, hence, G, after matter domination is negligibly small. After the Universe cools to the matter-dominated epoch, $\rho - 3p = \rho \neq 0$, and $\Phi \propto t^{2/(4+3\omega)}$. As Φ increases further, ω diverges even more, so that there continues to be no discernible distinction from ordinary Einstein gravity.

Our numerical computations have verified that all inflationary and astrophysical constraints can be satisfied for plausible, untuned choices of $f(\phi)$ and $V(\sigma)$. For example, $M = 10^{18}$ GeV, $M_F = 10^{14}$ GeV, $f(\phi) = M^2 + \phi^2 - 0.1(\phi^4/M^2)$, and $V(\sigma) = M_F^4 + M_F^2\sigma^2 - 2.5M_F\sigma^3 + 1.1\sigma^4$ satisfy all known constraints. (Coefficients are chosen so that $v = 0$ at the minimum.)

We conclude that generalizing the extended-inflation concept to include higher-order couplings is not only natural, but greatly expands the range of models and parameters which can lead to successful inflation. In the process, we have developed a novel approach to induced gravity, and we have discovered a simple, essentially model-independent prediction for the bubble-size distribution, Eq. (9), which should now be analyzed for its potential effect on large-scale structure. The robustness of this approach makes inflation seem significantly more plausible. To the extent to which inflation is an attractive approach for resolving cosmological problems, the success of this scheme represents, perhaps, a compelling argument for modifications to the conventional Einstein theory of gravity even though direct evidence for such modifications may be difficult to detect.

This work was supported in part by the Monell Foundation and by the DOE under Grants No. DOE-EY-76-C-02-3071 (P.J.S.) and by No. DE-AC02-ER03075 (F.S.A.).

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