Global Texture and the Microwave Background

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The metric perturbation produced by a collapsing "knot" of global texture in a flat background is calculated. This is used to calculate the energy shift for photons traversing such knots, leading to a prediction for the microwave anisotropy pattern produced in the global-texture scenario for large-scale structure formation. The metric is also used to calculated the velocity field induced in nonrelativistic matter, on scales well inside the horizon. This would produce an early generation of small gravitationally bound objects. At larger scales, the texture field would induce coherent velocity fields on a scale comparable to that seen in galaxy surveys.

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Global texture has recently been proposed as a mechanism for generating large-scale structure in an initially homogeneous universe.[1] The main assumption necessary is that a non-Abelian global symmetry be broken at a scale of order the grand-unified-theory (GUT) scale. This generically leads to the formation of a topological defect known as "texture," related to the third homotopy group π_3 of the vacuum manifold. Texture "knots" are regions where the Higgs-field winds around a three-sphere in a nontrivial way: This is generally the case in regions larger than the horizon. As knots come inside the horizon, they collapse at the speed of light, down to the inverse GUT scale (i.e., essentially to zero size) whereupon they unwind themselves. In the process they emit a spherical shell of outgoing massless (Goldstone boson) radiation. Our main concern will be the effect most likely to rule out (or confirm) the scenario, the imprint produced by texture on the microwave background.

The simplest models in which global texture is produced involve the breaking of a global SU(2). This might be a family symmetry,[2] or simply an extra global symmetry rotating two or more Higgs fields into one another. For example, in a SO(10) GUT one can impose an SU(2) symmetry between two **126** fields. The only parameter which is important in the texture scenario is η, the magnitude of the vacuum expectation value of the Higgs field. This is a nice feature: The theory is independent of scalar self-couplings or the shape of the Higgs potential. In simple GUT's, η is directly related to the unification scale, determined from low-energy physics. In these theories, the texture scenario has *no* free parameters and is thus highly predictive.

The simplest case occurs where a global SU(2) is broken by a complex doublet Φ (which we shall write as a four-component real field). The SU(2) symmetry breaks as the Universe cools through a temperature of order the GUT scale, and the initial conditions for Φ are specified by its being in thermal equilibrium, with the resulting familiar "random domain" picture.[3] As the Universe expands, the scale of spatial variation of Φ grows at the speed of light.[1]

Using a code developed by Press, Ryden, and Spergel,[4] we have simulated texture in expanding backgrounds. The results will be reported shortly.[5] In the simulations, we see the scale on which Φ varies growing with the horizon, as expected, and the number of knots per unit comoving volume n collapsing per unit conformal time τ as

$$dn/d\tau = c/\tau^4 , \tag{1}$$

with $c \approx 1$ in the matter era.[5] [Conformal time is defined from $dt = a(t)d\tau$, with $a(t)$ the scale factor.] This is the texture "scaling solution."

We also observe that the knots rapidly become quite spherical as they collapse, presumably because it is only the "S-wave" mode that is constrained to collapse by the topology, while other modes radiate away. In this paper we shall calculate the metric and microwave-background distortion produced by the exact spherical texture solution in flat spacetime, which we recently discovered:[6] This should be a reasonable approximation to the realistic case when the scale of interest is smaller than the horizon scale.

On large scales (much larger than the inverse GUT scale) the evolution of Φ is accurately described by the nonlinear σ model,[1]

$$\nabla^\mu \partial_\mu \Phi^a = -\frac{\partial^\mu \Phi \cdot \partial_\mu \Phi}{\eta^2} \Phi^a , \qquad \Phi^2 = \eta^2 , \tag{2}$$

where Φ^a, $a = 1$–4 are the four real components of Φ. With the spherically symmetric *Ansatz* $\Phi^a = \eta(\cos\chi, \sin\chi\sin\theta\cos\phi, \sin\chi\sin\theta\sin\phi, \sin\chi\cos\theta)$, where θ and ϕ are the usual polar angles, in flat spacetime (2) becomes

$$\ddot{\chi} - \frac{2}{r}\chi' - \chi'' = -\frac{\sin(2\chi)}{r^2} . \tag{3}$$

For a unit-winding-number knot, χ ranges from 0 at the origin to π at infinity. The exact spherical solution to (3) describing the collapse and unwinding of a texture which we shall use is[6]

$$\chi=\begin{cases}2\arctan(-r/t), & t<0,\\ 2\arctan(r/t)+\pi, & t>0,\ r<t,\\ 2\arctan(t/r)+\pi, & t>0,\ r>t.\end{cases} \tag{4}$$

The σ-model solution is matched through the singularity by the requirement that there be zero winding number after collapse; the value of χ at the origin is then dictated by spherical symmetry. The stress-energy tensor calculated from (4) is

$$\begin{aligned}T_{00}&=2\frac{r^2+3t^2}{(r^2+t^2)^2}\eta^2,\\ T_{0i}&=-x^i\frac{4t}{(r^2+t^2)^2}\eta^2,\\ T_{ij}&=2\delta_{ij}\frac{r^2-t^2}{(r^2+t^2)^2}\eta^2.\end{aligned} \tag{5}$$

This solution has a mass which diverges at large r: $M_{<r}\equiv4\pi\int_0^r r^2dr\,T^{00}\approx8\pi\eta^2r$ (with exact equality at the instant of collapse, $t=0$). We will have to cut this off at a scale corresponding to the size of a realistic knot, of order t when it collapses.

To calculate the metric perturbation produced by (5), it is adequate for our purposes to work in the weak-field approximation since $G\eta^2\ll1$. It is convenient to work in the gravitational Coulomb gauge, specified by the conditions $h_{0i,i}=\frac{1}{2}h_{ii,0}$, and $h_{ik,k}=\frac{1}{2}h_{kk,i}$. [Our conventions are $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$, $\eta_{\mu\nu}=\mathrm{diag}(-1,1,1,1)$. Greek indices run from 0 to 3; Latin ones from 1 to 3.]

In this gauge, and using spherical symmetry, $h_{\mu\nu}$ may be completely determined from equations which involve no time derivatives. Thus one is left with only radial integrals, which can be performed analytically.[7] Parametrizing $h_{\mu\nu}$ by four functions of r and t, $h_{00}=H$, $h_{0i}=Ix^i$, and $h_{ij}=Jx^ix^j+K\delta^{ij}$, one solves for H, I, and h_{kk} from Einstein's equations, and then determines J and K from h_{kk} using the second gauge condition above. The result, with the boundary condition that $h_{\mu\nu}$ vanish at $r=0$, is that

$$\begin{aligned}H_C&=-2\epsilon\left\{\ln\left(1+\frac{r^2}{t^2}\right)+3\left[\frac{t}{r}\arctan\left(\frac{r}{t}\right)-1\right]\right\}, \quad I_C=-2\epsilon t\left[-\frac{1}{r^2}+\frac{t}{r^3}\arctan\left(\frac{r}{t}\right)+\frac{1}{3t^2}\right],\\ J_C&=-\epsilon\left[\frac{1}{3r^2}-\frac{t^2}{r^4}+\frac{t^3}{r^5}\arctan\left(\frac{r}{t}\right)\right], \quad K_C=\epsilon\left[-\frac{2}{3}\ln\left(1+\frac{r^2}{t^2}\right)+\frac{1}{9}-\frac{t^2}{3r^2}+\frac{t^3}{3r^3}\arctan\left(\frac{r}{t}\right)\right],\end{aligned} \tag{6}$$

where $\epsilon\equiv16\pi G\eta^2$, and the subscript C indicates "Coulomb" gauge. These coordinates have the nice property that at finite radius and for large negative or positive times the metric is the Minkowski metric, so particle and photon trajectories are easy to interpret.

Unfortunately, H_C, I_C, and K_C are singular at $t=0$. This is a coordinate singularity, as may be seen by changing coordinates to $x'^{\mu}=x^{\mu}+\xi^{\mu}$, with $\xi_0=2\epsilon(t\times\ln|t/r_c|-t)$, $\xi_i=\frac{2}{3}\epsilon x_i\ln|t/r_c|$, in which the metric is given by $h'_{\mu\nu}=h_{\mu\nu}-\xi_{\mu,\nu}-\xi_{\nu,\mu}$. Then

$$\begin{aligned}H&=-2\epsilon\left\{\ln\left(\frac{r^2+t^2}{r_c^2}\right)+3\left[\frac{t}{r}\arctan\left(\frac{r}{t}\right)-1\right]\right\},\\ I&=-2\epsilon t\left[-\frac{1}{r^2}+\frac{t}{r^3}\arctan\left(\frac{r}{t}\right)\right],\\ J&=-\epsilon\left[\frac{1}{3r^2}-\frac{t^2}{r^4}+\frac{t^3}{r^5}\arctan\left(\frac{r}{t}\right)\right],\\ K&=\epsilon\left[-\frac{2}{3}\ln\left(\frac{r^2+t^2}{r_c^2}\right)+\frac{1}{9}-\frac{t^2}{3r^2}+\frac{t^3}{3r^3}\arctan\left(\frac{r}{t}\right)\right].\end{aligned} \tag{7}$$

Here we choose r_c to be a scale of order the inverse GUT scale, where the σ-model approximation breaks down. The metric (7) diverges logarithmically at large r and large t. In the domain of interest, however, $h_{\mu\nu}$ is small and the weak-field approximation valid. The large-r divergence has an interesting and simple geometrical interpretation. The spatial hypersurfaces have a metric

$$\left[1-\frac{4}{3}\epsilon\ln\left(\frac{r}{r_c}\right)\right](dr^2+r^2d\Omega^2). \tag{8}$$

Changing variables to $d\tilde{r}=[1-\frac{2}{3}\epsilon\ln(r/r_c)]dr$, i.e., $\tilde{r}=r[1+\frac{2}{3}\epsilon-\frac{2}{3}\epsilon\ln(r/r_c)]$, this yields (to order ϵ)

$$d\tilde{r}^2+\tilde{r}^2(1-\tfrac{4}{3}\epsilon)d\Omega^2, \tag{9}$$

which is just flat space minus a solid angle $8\pi\epsilon/3=128\pi^2G\eta^2/3$. This geometry is exact at $t=0$, reminiscent of the result for a *static* global monopole.[8] Our result for the missing solid angle is just $\frac{4}{3}$ of theirs.

We wish to integrate the geodesic equation for photons traversing a knot metric—the usual "Sachs-Wolfe" calculation.[9] The unperturbed photon trajectory is taken to be $\bar{x}^{\mu}=n^{\mu}\lambda$, $\bar{p}^{\mu}=E\,d\bar{x}/d\lambda=En^{\mu}$, with E the energy, $n^0=1$, $\mathbf{n}^2=1$, and λ the affine parameter. The perturbation $p^{\mu}=\bar{p}^{\mu}+\delta p^{\mu}$ is calculated from the geodesic equation as

$$\left(\frac{\delta p^0}{\bar{p}^0}\right)_i^f=-\frac{1}{2}\int_{\lambda_i}^{\lambda_f}h_{\mu\nu,0}n^{\mu}n^{\nu}d\lambda+(h_{0\mu}n^{\mu})_i^f. \tag{10}$$

To interpret (10), it is important to ask a physical (i.e., coordinate-independent) question. We imagine that the photon is actually emitted from a massive particle nearly at rest in the coordinates (7), with four-momentum $k^\mu(i)=\bar{k}^\mu+\delta k^\mu(i)$, where $\bar{k}^\mu=(m,\mathbf{0})$ and m is its rest mass. The mass-shell condition for the particle implies that $\delta k^0=-\frac{1}{2}h_{00}m$. The photon energy is given by $E=-k\cdot p(i)/m$, which leads to $\delta p^0(i)/E = h_{0\nu}(i)n^\nu+\delta k^\nu(i)n_\nu/m$. Similarly, we imagine that the photon is finally detected by another particle with four-momentum $k^\mu(f)=\bar{k}^\mu+\delta k^\mu(f)$. The final energy of the photon measured by the particle is $E(f)\equiv E+\delta E = -k\cdot p(f)/m$, which leads to $\delta E/E=\delta p^0(f)/E-h_{0\nu}\times(f)n^\nu-\delta k^\nu(f)n_\nu/m$. Thus (10) leads to

$$\left(\frac{\delta E}{E}\right)_i^f=-\frac{1}{2}\int_{\lambda_i}^{\lambda_f}h_{\mu\nu,0}n^\mu n^\nu d\lambda+\frac{1}{2}(h_{00})_i^f-\left(\frac{\delta\mathbf{k}\cdot\mathbf{n}}{m}\right)_i^f. \tag{11}$$

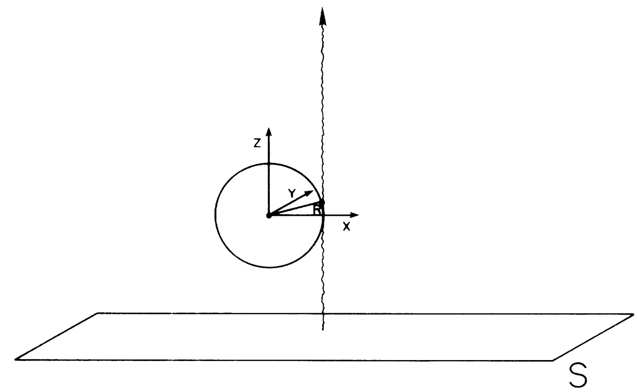

FIG. 1. The geometry for calculating the photon redshift or blueshift produced by a collapsing knot. The knot is at the origin: S, the surface of last scattering, is the plane $z=z_i<0$. The impact parameter is R, and the time the photon wave front passes $z=0$, relative to the time at which the texture collapses, is t_0. We are at $z=\infty$.

The three terms are a path-dependent redshift, a part from the local gravitational potential, and a Doppler term. It is simple to check that under a coordinate change $\delta h_{\mu\nu}=-\xi_{\mu,\nu}-\xi_{\nu,\mu}$, $\delta k^i=\xi^i_{,0}m$, $\delta E/E$ is invariant as it should be.

Ignoring the Doppler term (we show this is small later), we perform the integral [in the nonsingular metric (7)][7] to yield, in the geometry of Fig. 1,

$$\frac{\delta E}{E}=-\frac{\epsilon}{2}\left[\arctan\left(\frac{r}{t_0+z}\right)\left(\frac{z^3}{r^3}+2\frac{t_0}{r}+\frac{t_0^2z}{r^3}-2\frac{t_0R^2}{r^3}\right)+\frac{R^2-t_0z}{R^2+z^2}-\frac{t_0}{(2R^2+t_0^2)^{1/2}}\arctan\left(\frac{2z+t_0}{(2R^2+t_0^2)^{1/2}}\right)\right]_i^f. \tag{12}$$

Here R is the impact parameter for the photon, and t_0 is the time (relative to the time at which the knot collapses, $t=0$) at which the photon passes $z=0$. Photons crossing $z=0$ *before* the texture collapses are redshifted, while those crossing *after* collapse are blueshifted. A heuristic explanation for this is given by considering a spacetime diagram of the knot collapsing (Fig. 2). The former photons *fall into* a cloud of collapsing texture; the latter ones *climb out*. In the limit where $z_i=-\infty$ and $z_f=+\infty$, the result is

$$\frac{\delta E}{E}=\frac{\epsilon\pi}{2}\frac{t_0}{(2R^2+t_0^2)^{1/2}}. \tag{13}$$

Note that this only applies for R less than the scale of the knot—as we discussed earlier (13) is cut off on a scale of order t in the realistic case. On scales of order the knot size, effects of the expansion of the Universe should be included, which could slightly modify (13). Equation (13) shows that t_0 is the scale which determines the size of the red or blue disk produced in the microwave background, probably surrounded by a "compensating" ring of the opposite color. However, the peak amplitude is always a fixed constant, $\epsilon\pi/2$.

To summarize, texture knots produce blue or red disks on the microwave sky with maximal fractional temperature deviation $\epsilon\pi/2=8\pi^2G\eta^2$. This is π times the "naive Newtonian potential" $GM_{<r}/r=8\pi G\eta^2$. The distribution of $\delta T/T$ is quite non-Gaussian and the fixed maximum is a signal peculiar to texture.

The distribution of these red and blue disks on the microwave sky may be calculated from the scaling solution (1). Assuming $\Omega=1$, the background metric is conformally flat in comoving coordinates and conformal time τ. In the matter era the comoving radius of the horizon is $\tau=3t/a(t)$, with $a(t)$ the scale factor. The angle subtended by the horizon at early times is $\tau_i/\tau_0=(1+Z_i)^{-1/2}$, where the subscript i refers to the early time and the subscript 0 to today. At conformal time τ,

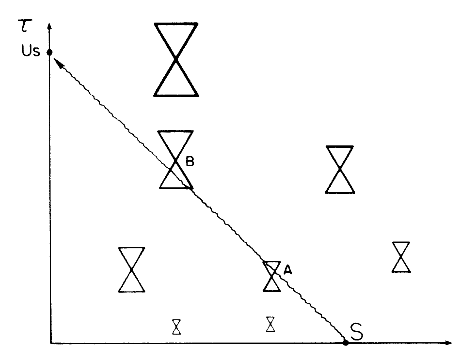

FIG. 2. Spacetime diagram showing photon from last-scattering surface traversing space with collapsing knots (depicted as "lightcones"). If the photon traverses a knot as at A, it falls into a knot and is blueshifted. If the situation is as at B, the photon climbs out of the knot and is redshifted.

the comoving radius of the spherical front of photons traveling from the last scattering surface to us is $r(\tau)=\tau_0-\tau$. Consider adding the contribution of knots which collapse during each increment of conformal time $d\tau$. The number of knots per unit comoving volume collapsing in the interval $d\tau$ is $dn=cd\tau/\tau^4$. The typical comoving scale of a knot is $\approx\kappa\tau$ with $\kappa\approx\frac{1}{3}$.[5] A knot collapsing at τ at distance r from us only perturbs the microwave background with of order the maximal effect if $-\kappa\tau<r-r(\tau)<\kappa\tau$. For $\tau\ll\tau_0$, the angle subtended by the disk produced is given by $\theta\approx[r-r(\tau)]/r$. Thus the number of knots collapsing in $d\tau$ producing red disks subtending an angle in the range $d\theta$ is given by

$$dn=4\pi c\frac{d\tau}{\tau^4}r^2dr\approx4\pi c\frac{d\tau}{\tau^4}\tau_0^3d\theta,\quad -\frac{\kappa\tau}{\tau_0}<\theta<+\frac{\kappa\tau}{\tau_0}. \tag{14}$$

Integrating over τ, we find the number of red disks of angular radius θ to $\theta+d\theta$,

$$dn=\frac{4\pi c}{3}\kappa^3\frac{d\theta}{\theta^3}. \tag{15}$$

It follows that the total solid angle subtended by all red disks is of order $4\pi^2c\kappa^3[\ln(1+Z_{\rm ls})]/6$, where $Z_{\rm ls}$ is the redshift at last scattering, ≈1000. Thus about $4c\kappa^3\approx\frac{1}{10}$ of the sky is covered by red and an equal area by blue disks. If the Universe is reionized by nonlinear structure formation $Z_{\rm ls}$ could be as low as 30, and the fraction covered halved. The temperature distribution is thus highly non-Gaussian, and a strategy of large-area coverage is required.

Finally, we wish to normalize ϵ by demanding that the perturbation produced in the matter by the texture be enough to form the large-scale structure observed today. The small-scale structure near the collapse is found by solving the geodesic equation for particles initially at rest. In Coulomb gauge, for fixed radius and at early times the metric is clearly flat. Thus particles may be taken as initially stationary. The velocity perturbation $\delta v^i=\delta k^i/m$ is given by

$$\begin{aligned}\delta v^i(\mathbf{x},t)&=\int_{-\infty}^{\infty}dt'(-h_{0i,0}+\tfrac{1}{2}h_{00,i})(\mathbf{x},t')\\&=-(h_{0i})+\tfrac{1}{2}\int_{-\infty}^{\infty}dt'h_{00,i}(\mathbf{x},t')\\&=-\epsilon\frac{\pi}{2}\frac{x^i}{r}.\end{aligned} \tag{16}$$

Thus particles get a velocity kick towards the center, of magnitude $\epsilon\pi/2$, independent of their distance from the knot center. This effect, put in the expanding-universe context, should be a reasonable approximation out to scales $r\approx t$, the size of the knot. It means that regions in the immediate vicinity of a knot would form nonlinear structures quite early in the matter era.

A rough normalization may be obtained by supposing that (16) gives the right order of magnitude for the velocity field. In linear theory, the velocity perturbation grows as $a(t)^{1/2}$. Growing perturbations produced at equal matter-radiation density and using $Z_{\rm eq}\approx2500h^2$ (taking $\Omega=1$), we demand that $\delta v\approx600$ km s^{-1} today (the velocity of our galaxy relative to the microwave background). This determines $\epsilon\pi/2=8\pi^2G\eta^2\approx10^{-5}\times h^{-1}$. This is therefore the expected magnitude of the peak temperature fluctuations produced in the microwave background, in an $\Omega=1$ universe with texture and cold dark matter. The velocity field of this magnitude induced by texture will certainly be coherent on a scale comparable to the comoving Hubble radius at equal matter-radiation density, when perturbations started to grow. This scale corresponds to $13h^{-1}$ Mpc, comparable to the coherence scale seen in the large-scale velocity field, and in the deep redshift surveys.

The magnitude of the temperature fluctuations calculated here should be within reach of the Cosmic Background Explorer (COBE) satellite which is currently in operation. The direct effect we have calculated should dominate on the large angular scales COBE is sensitive to ($>7°$). The Sachs-Wolfe and Doppler effects on scales larger than the horizon at last scattering are small in this theory due to the isocurvature nature of the perturbations on superhorizon scales.

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