

Comment on "Ionization of Rydberg Atoms by a Circularly Polarized Microwave Field"

In a recent Letter,¹ Fu, Scholz, Hetteima, and Gallagher report the observation of the ionization of Na atoms in Rydberg states with principal quantum number n , by a circularly polarized microwave field with frequency $\omega = 8.5$ GHz. They find that the threshold field E is approximately $E = 1/16n^4$ (in atomic units) as in the static case, and explain this result by analyzing the system in a frame of reference rotating with frequency ω , where the Hamiltonian becomes time independent. They conclude that "in essence, a static field has the same effect whether its coordinate system is rotating or not," apparently in agreement with their observation. However, I would like to point out that this result is expected from simple semiclassical arguments *only* if the frequency ω is much smaller than the Kepler frequency $\omega_K = 1/n^3$ of the Rydberg atom, and not (as argued by the authors) because $\hbar\omega$ is much smaller than the binding energy. This is a more stringent requirement, i.e., $\omega \ll 1/n^3$, for the validity of the classical approximation. In their experiment $\omega n^3 \lesssim 0.2$.

The effect of rotation on the ionization threshold can be readily estimated by extending the well-known saddle-point argument² for ionization in a static field. In the rotating frame all classical electron orbits near the nucleus with energies less than W are bound, where W is the value of the effective potential $V = -1/r - Ey - \frac{1}{2}\omega^2 r^2$ at its saddle point. One finds that

$$E = W^2/g(s), \quad (1)$$

where s is a scale-invariant variable, $s = \omega^2/E^{3/2}$, and

$$g(s) = [1/f(s) + f(s) + \frac{1}{2}sf(s)^2]^2.$$

The function $f(s)$ is obtained by solving the cubic equation $sf^3 + f^2 - 1 = 0$. For small s , $g(s) = 4 + 2s + \frac{1}{4}s^2 + \dots$, while for large s , $g(s) = \frac{9}{4}s^{2/3} + 3 + \dots$.

Neglecting the shift of the energy levels due to the effect of the rotating field, an approximation which is surprisingly well satisfied by alkali Rydberg atoms in a static field, one substitutes $W = -1/2n^2$ in Eq. (1) to obtain the dependence of E and ω on s and n ,

$$En^4 = 1/4g(s), \quad \omega n^3 = s^{1/2}/[4g(s)]^{3/4}. \quad (2)$$

From Eq. (2), one finds the relation between En^4 and ωn^3 as shown in Fig. 1. For small values of En^4 , Eq. (2) is expected to breakdown, because for vanishing E the saddle-point criteria is no longer valid.

Evidently this relation does not agree with the experimental data of Fu *et al.*¹ which gives En^4 approximately a constant equal to the static limit $\frac{1}{16}$ up to $\omega n^3 \sim 0.2$. However, this discrepancy does not imply that the dy-

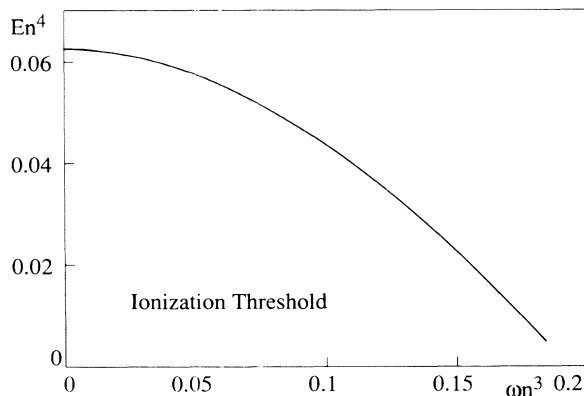


FIG. 1. The dependence of the scaled ionization threshold field En^4 as a function of the scaled frequency ωn^3 .

namics of the electron cannot be described by the classical approximation. In a strict sense Eq. (2) is only a lower bound for the threshold field which is not violated by these experimental results. For example, in two dimensions the classical dynamics of an electron for a Coulomb potential and a static electric field in a rotating frame is surprisingly regular³ (only very small regions of phase space are chaotic) even though there are no constants of the motion other than the energy. As in the case of a nonrotating frame, where the generalized Lenz vector is an additional constant of the motion, there also exists here bound states that are above the saddle-point threshold, which would require larger threshold fields for ionization. Further work is needed to understand the experimental results which cannot be explained theoretically as claimed in Ref. 1.

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¹Panming Fu, T. J. Scholz, J. M. Hetteima, and T. J. Gallagher, Phys. Rev. Lett. **64**, 511 (1990).

²R. F. Stebbings and F. B. Dunning, *Rydberg States of Atoms and Molecules* (Cambridge Univ. Press, Cambridge, 1983).

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