Two-Body Nuclear Density Determined from Quasielastic Electron Scattering in the Three-Nucleon System

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The proton-proton two-body density in the three-nucleon system is extracted from quasielastic-electron-scattering data. The two-body density is determined in momentum space using the integrated longitudinal response functions and assuming that the nonrelativistic Coulomb sum rule is valid. The range of the measurement is for momenta between 200 and 550 MeV/c. The data are compared to a proton-proton density calculated using Faddeev wave functions. The data and the calculation both show a "second maximum" but the data exceed the calculation by about an order of magnitude.

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One of the most important and long sought after observables in nuclear physics is the two-nucleon density in the nuclear medium. At best, only indirect information about two-nucleon densities has thus far been gathered. A relatively direct determination of the proton-proton density in the three-nucleon system is presented here.

The proton-proton density is extracted from the longitudinal quasielastic-electron-scattering response by integrating over all energy transfers. The quantity so formed is the familiar Coulomb sum. A sketch of its derivation[1] will make the connection to the proton-proton density clear. The Coulomb sum is defined here to be

$$\tilde{\mathcal{C}}(q)\equiv\frac{1}{Z}\int_0^\infty\frac{R_L(q,\omega)}{[G_E^p(q^2)]^2}d\omega\,,$$

where Z is the nuclear charge, R_L is the longitudinal nuclear response, G_E^p is the proton charge form factor,[2] and q, ω, and Q are the momentum, energy, and four-momentum transferred by the electron, respectively (related by $Q^2=q^2-\omega^2$). The *inelastic* Coulomb sum is the quantity most often formed from the data and is simply the total sum less the contribution from the elastic scattering,

$$\mathcal{C}\equiv\tilde{\mathcal{C}}-\frac{1}{Z}\frac{[ZF_c(q^2)]^2}{[G_E^p(q^2)]^2}\,,$$

where both F_c, the nuclear charge form factor, and G_E^p are normalized to 1 at $Q^2=0$. Rewriting the Coulomb sum $\tilde{\mathcal{C}}$ using closure (i.e., summing over all final nuclear states) gives

$$\mathcal{C}=\frac{1}{Z}\left(\frac{\langle i\,|\sum_{j,k}^{A}\hat{\rho}_j^\dagger(q)\hat{\rho}_k(q)\,|\,i\rangle}{[G_E^p(q^2)]^2}-\frac{[ZF_c(q^2)]^2}{[G_E^p(q^2)]^2}\right),$$

assuming that the photon couples only to one-body currents via $\hat{\rho}(q)$ and using the definition of $R_L=|\langle f(\omega)\,|\,\hat{\rho}(q)\,|\,i\rangle|^2$, where i and $f(\omega)$ denote the initial and final nuclear states. The sum can then be broken down into two pieces, $j=k$ and $j\neq k$. The $j=k$ piece of the matrix element simply gives $Z[G_E^p(q^2)]^2$; therefore

$$\mathcal{C}=1-Z\frac{[F_c(q^2)]^2}{[G_E^p(q^2)]^2}+\frac{1}{Z}\rho_{pp}(q)\,,$$

where $\rho_{pp}(q)$ is defined to be

$$\rho_{pp}\equiv\frac{\langle i\,|\sum_{j\neq k}^{A}\hat{\rho}_j^\dagger(q)\hat{\rho}_k(q)\,|\,i\rangle}{[G_E^p(q^2)]^2}\,,$$

i.e., the *point* proton-proton density (ignoring the neutron contribution which is addressed subsequently). With this definition $\rho_{pp}(q)$ is normalized to $Z(Z-1)$. It is therefore clear that the proton-proton density can be determined using only data and the assumption that the nonrelativistic Coulomb sum rule is valid.

The Coulomb sum has been determined from data on ^{3}H and ^{3}He taken at the MIT Bates Linear Accelerator Center in the following manner:[3]

$$\mathcal{C}=\frac{1}{Z}\int_{\text{thresh}}^\infty\frac{R_L(q,\omega)}{[G_E^p(q^2)]^2\left(\dfrac{1+Q^2/4M_p^2}{1+Q^2/2M_p^2}\right)}d\omega\,.$$

The data are integrated from the inelastic threshold to infinity by adding an exponential tail fitted to the last few data points. The contribution from the region beyond the range of the data is largest (30%) at a momentum transfer of 200 MeV but decreases to be $\lesssim$5% above 300 MeV. Examples of the tails are shown in Fig. 1. The extra factor multiplying G_E^p is a relativistic correction suggested by deForest.[4] At a momentum transfer of 550 MeV this correction is about 7%. In all cases the value used for G_E^p is that corresponding to the momentum transfer at the quasielastic peak. The data used to define F_c for both ^{3}H and ^{3}He are those taken in the same experiment.[5] The random and systematic uncertainties in these data are about 2% each.

The (point) proton-proton densities are shown in Fig. 2 for both ^{3}H and ^{3}He. These data are determined as described above except that the small contribution from the neutrons has been subtracted out using the calcula-

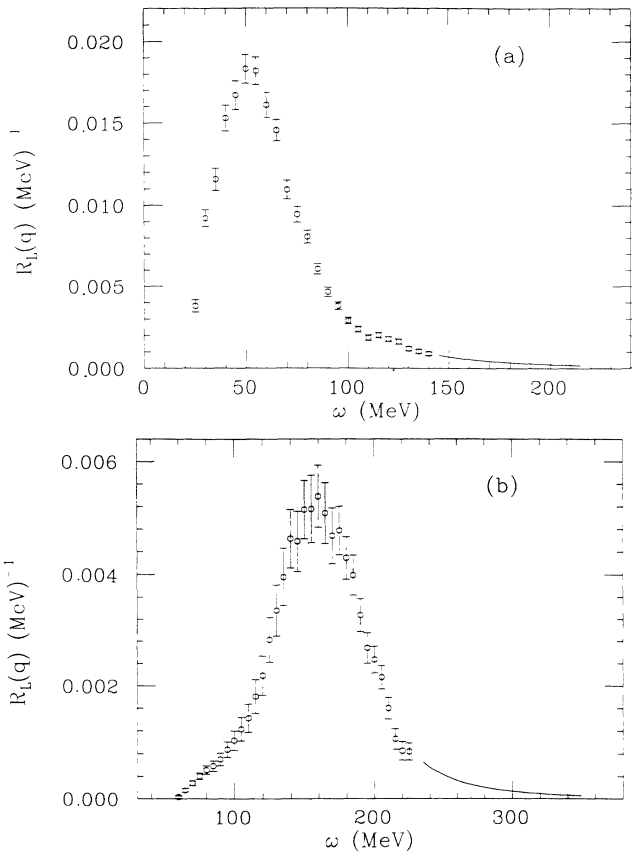

FIG. 1. Longitudinal response for ^{3}He, including fitted tail, at (a) $q = 300$ MeV and (b) $q = 550$ MeV. Random uncertainties are shown.

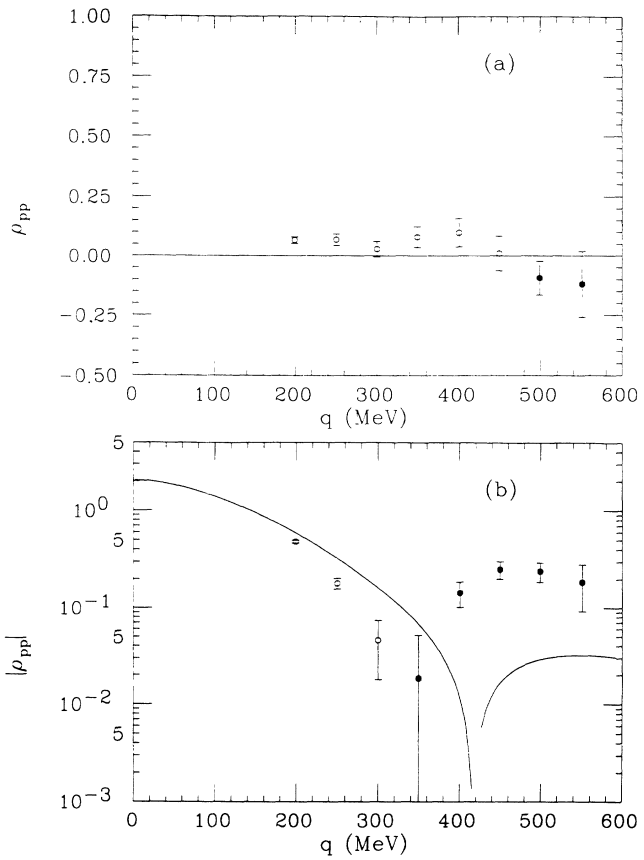

FIG. 2. (a) Proton-proton density for ^{3}H using MIT-Bates data (Ref. 3). Total uncertainties are shown. Solid symbols indicate negative ρ_{pp}. (b) As in (a) except for ^{3}He. Solid line: calculation of Ref. 6 with the Argonne v_{14} NN potential and Urbana model-VII three-nucleon potential.

tion of Schiavilla *et al.*[6] Total uncertainties are shown; they are dominated by systematic uncertainties in the extraction of R_L. Note that the proton-proton density for ^{3}H should be identically zero given the assumptions of the discussion above. The corresponding density for ^{3}He displays an interesting form reminiscent of one-body densities (elastic form factors); i.e., it is an oscillating function of q.

The point proton-proton density can be determined from three-body Faddeev wave functions in a straightforward way. The calculation shown in Fig. 2 is that of Schiavilla *et al.*[6,7] It uses the Argonne v_{14} NN potential and the Urbana model-VII three-nucleon interaction. The data suggest that the minimum in the calculation is at too high a momentum transfer, and that there is more strength in the region of the second maximum than is calculated. This in turn suggests that the coordinate-space density at the origin is smaller than calculated (since this density is simply the integral of the momentum-space density which is reduced by the large negative lobe) and therefore that there is some short-range repulsion missing in the model.

It should be noted that the agreement between the Coulomb sum data and theory appears more satisfactory[3] than that between the proton-proton densities would indicate. This is the result of a calculated charge form factor which is increasingly too large[5] at higher momentum transfers and which compensates the small proton-proton density.

Even though the proton-proton density is closely related to data, a considerable amount of analysis is required, particularly of the inelastic data, to reach the result.[8] Therefore, several checks have been made to assess the reliability of the result.

First, the overall normalization of the inelastic data is supported by elastic form factors measured at the same time. In both cases the elastic data agree with the world average within the uncertainties noted above. In addition, a uniform increase of 10%, for example, in the inelastic data would bring the proton-proton density for ^{3}He into better agreement with the calculation (reducing the data at the second maximum by about a factor of 3); however, it would raise the ^{3}H density by about 0.1 in Fig. 2, strongly disagreeing with the expectation that this density should be zero.

Second, and most importantly, the inelastic data on

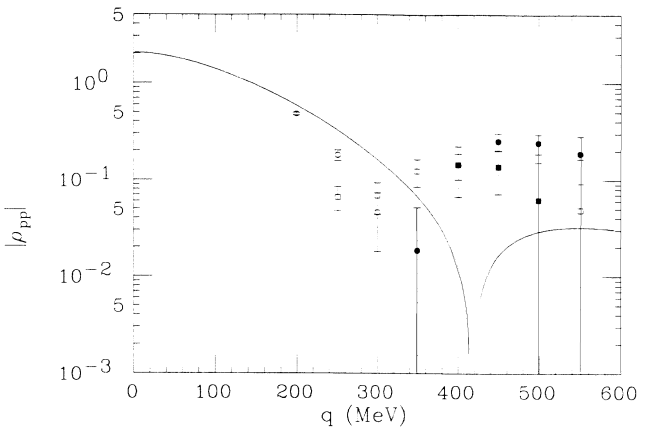

FIG. 3. As in Fig. 2(b) except that circles denote ρ_{pp} from the MIT-Bates data and squares denote ρ_{pp} from the Saclay data (Ref. 9).

^{3}He from the Saclay group[9] may be analyzed in the same manner.[10] The results, displayed in Fig. 3, are essentially consistent with those from the MIT-Bates measurement.

Third, the extrapolation procedure used to determine the large energy-loss contribution to the Coulomb sum is supported by a second, distinct analysis method. Schiavilla, Pandharipande, and Fabrocini[11] fit a tail which is a sum of two exponentials to the data and to the energy and energy-squared weighted sum rules.[12] This method gives results consistent with the present analysis for the proton-proton density from the ^{3}H data and from both ^{3}He data sets.

The proton-proton density has been extracted from inelastic-electron-scattering data for ^{3}H and ^{3}He. As expected, this density is approximately consistent with zero for ^{3}H. The ^{3}He density displays an oscillating form in momentum space and has a much larger second maximum than does a calculation employing exact ground-state wave functions. This larger second maximum in momentum space implies a reduced coordinate-space density at the origin as compared with the calculation.

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[2]The proton form factor is written here as a function of q^2 for notational consistency. The Coulomb sum is formed from the data using the proton form factor corresponding to the elastic scattering; i.e., it is evaluated at $q^2=Q^2+Q^4/4M_p^2$.

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[4]T. deForest, Nucl. Phys. **A414**, 347 (1984); see also T. W. Donnelly, E. L. Kronenberg, and J. W. Van Orden, Nucl. Phys. **A494**, 365 (1989).

[5]D. Beck *et al.*, Phys. Rev. Lett. **59**, 1537 (1987).

[6]R. Schiavilla *et al.*, Nucl. Phys. **A473**, 317 (1987). The largest corrections for the neutron contribution are 18% (decrease) for ^{3}H at 550 MeV and 12% (increase) for ^{3}He at 350 MeV; the typical corrections are $\lesssim 5\%$.

[7]The authors of Ref. 6 use the wave functions of the Iowa-Los Alamos group; see C. R. Chen *et al.*, Phys. Rev. C **33**, 1740 (1986).

[8]Specifically, the raw inelastic data are radiatively unfolded and then used for interpolation to determine R_L at constant momentum transfer for the purposes of calculating the Coulomb sum.

[9]C. Marchand *et al.*, Phys. Lett. **153B**, 29 (1985).

[10]K. Dow, Ph.D. thesis, Massachusetts Institute of Technology, 1987.

[11]R. Schiavilla, V. R. Pandharipande, and A. Fabrocini, Phys. Rev. C **40**, 1484 (1989).

[12]Note that these authors do not include the relativistic correction for G_E^p in the calculation of the sum rules. Consequently, the contribution to the sums from the region of extrapolation is larger than in the present analysis.