Late-Time Dissipation of Primordial Baryon-Number Fluctuations and Nucleosynthesis

C. R. Alcock,[1] D. S. Dearborn,[1] G. M. Fuller,[1,2] G. J. Mathews,[1] and B. S. Meyer[1]
[1]*Institute of Geophysics and Planetary Physics, University of California, Lawrence Livermore National Laboratory, Livermore, California 94550*
[2]*Department of Physics and Center for Astrophysics and Space Science, University of California at San Diego, La Jolla, California 92093*
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Primordial baryon-number fluctuations can be damped at temperatures ≈ 20 keV when the photon mean free path becomes larger than the high-density-region length scale. This dissipation process may result in mixing of the high- and low-density material on a time scale comparable to or shorter than that of the universal expansion. The nucleosynthesis yields in inhomogeneous cosmologies can be altered by this process for any Ω_b: ^{7}Li can be reduced to an abundance consistent with observations of Population II halo stars and the abundances of ^{9}Be and ^{10}B can be reduced by several orders of magnitude.

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Recent studies have outlined how isothermal baryon-number fluctuations could be generated in a cosmic quark-hadron phase transition, how, subsequently, these fluctuations would be modified by neutron diffusion, and ultimately how primordial nucleosynthesis yields could be altered from those of the standard, homogeneous big bang.[1-11] In this Letter we point out that an important process, hydrodynamic expansion of the ions against Thomson drag, can dissipate baryon-number fluctuations and modify the nucleosynthesis yields of inhomogeneous cosmological models for any Ω_b (fraction of closure density contributed by baryons). We call this previously overlooked process hydrodynamic-Thomas-drag dissipation.

First we follow Ref. 2 and note that if isothermal baryon-number fluctuations are created by a cosmic QCD phase transition at high temperature ($T \geq 100$ MeV), they would not remain strictly isothermal for very long and would be expected to expand. If we idealize a baryon-number fluctuation as consisting of a high-density region of spatial scale l_h and a low-density region, then the initially isothermal nature of the radiation distribution implies that the photon contributions to the pressure in each region are equal. However, there is clearly a baryon-number pressure difference, the magnitude of which depends on the amplitude of the fluctuations, $R = n_h/n_l$, where n_h and n_l are the baryon-number densities in the high- and low-density zones, respectively. It is not yet possible to predict with confidence the value of R that emerges from the QCD transition and values from unity to 10^6 have been discussed.[5,6] In any case, the very large photon-to-baryon ratio observed in the Universe today ($\eta \approx 3.5\times 10^7 h_{100}^{-2}\Omega_b^{-1}$, where h_{100} is the Hubble constant in units of 100 km s^{-1}Mpc^{-1}) will ensure that radiation dominates the pressure in both the high- and low-density regions.

The photon mean free path is short compared to l_h immediately following the phase transition so that the high-density region can expand adiabatically until pressure equilibrium is attained. Since the radiation pressure is proportional to T^4 and η is so large, only a very small expansion of the high-density region is required to convert the fluctuation from isothermal to isobaric in nature. The result is a temperature gradient from the low-density to the high-density zones. The fractional change in temperature between these regions can be shown to be

$$\frac{\Delta T}{T} \approx \left(\frac{\pi^4}{30}\right)\left(\frac{1}{\zeta(3)}\right)\left(\frac{R-1}{f_V(R-1)+1}\right)\eta^{-1}$$
$$\approx 7.8\times 10^{-8}\left(\frac{R-1}{f_V(R-1)+1}\right)h_{100}^2\Omega_b\,, \qquad (1)$$

where $\zeta(3)$ is the Riemann zeta function and f_V is the volume fraction of the high-density region.[4] Again it is difficult to predict f_V but values between 0.5 and 10^{-4} have been discussed.[5,6,8] For $T > 1$ MeV, we note that neutrino pressure is important, and the temperature gradient will be smaller than as given in Eq. (1). For $T < 1$ MeV, neutrinos play no role in fluctuation dissipation.

This temperature gradient can drive neutrino and photon diffusion. Neutrino diffusion before weak decoupling ($T \approx 1$ MeV) can inflate the fluctuations, driving down the amplitude to $R \approx 10^3$–10^4 from higher values at the end of the phase transition.[12] In this Letter, we will concentrate on photon diffusion and inflation.

We can identify two limits in the subsequent expansion and dissipation of the fluctuations. In the high-temperature limit ($T \geq 50$ keV), the photon mean free path is short compared to the high-density length scale, $\lambda_\gamma < l_h$, and the fluctuations are inefficiently dissipated by photon diffusion driven by the temperature gradient in Eq. (1). The other limit to dissipation is reached when the temperature drops to $20 \leq T \leq 50$ keV where the photon mean free path becomes comparable to or larger than the high-density length scale, $\lambda_\gamma \geq l_h$. In this limit temperature gradients are completely erased and the dissipation is driven by any residual baryon pressure gradients left after neutron diffusion.

The photon mean free path is approximately $\lambda_\gamma \approx 1/n_e\sigma$, where n_e is the number density of electrons and positrons and σ is a characteristic cross section. For the purpose of the calculations here it is sufficient to use the Thomson cross section ($\sigma_T \approx 6.65\times10^{-25}$ cm^2). At temperatures greater than 50 keV, electron-positron pairs make contributions to, or dominate, the electron density, whereas at lower temperatures the electron density can be derived from the requirement for charge neutrality.

The high-density-region length scale l_h depends on the volume fraction f_V and the mean spacing of the fluctuations, l. Neither f_V nor l is well determined since they appear to depend on the detailed evolution of the QCD phase transition.[2,5] A range of l_h characteristic of recently discussed inhomogeneous models[3,5,6] is

$$l_h = (5\text{ km}) f_V^{1/3} l \left[\frac{20\text{ keV}}{T}\right] \approx (1\text{-}125\text{ km}) \left[\frac{20\text{ keV}}{T}\right], \tag{2a}$$

where the comoving mean spacing l is in meters at $T=100$ MeV. At low temperatures (≤ 50 keV) we can show that

$$\frac{\lambda_\gamma}{l_h} \approx 4(l f_V^{1/3})^{-1} \left[\frac{f_V(R_0-1)+1}{R_0}\right] \left[\frac{20\text{ keV}}{T}\right]^2 \Omega_b^{-1} \approx (0.05\text{-}5) \left[\frac{20\text{ keV}}{T}\right]^2, \tag{2b}$$

where l is as above and R_0 is the post-neutron-diffusion ($T<1$ MeV) density contrast which is related to R by

$$R_0 \approx R[1-Y_n(1-f_V)(1-1/R)] \times [1+Y_n f_V(R-1)]^{-1}, \tag{2c}$$

where the neutron fraction characteristic of neutron diffusion is $Y_n < \frac{1}{2}$.

In the first limit to the expansion, where $T \gtrsim 50$ keV and $\lambda_\gamma \ll l_h$, photons can diffuse down the temperature gradient in Eq. (1) into the high-density region, thereby increasing the pressure and causing expansion. If the radius of the high-density region is $r = l_h/2$ then the velocity of expansion $v = dr/dt$ is

$$v/c \approx (\lambda_\gamma/r) 3P_g/P_r, \tag{3}$$

where P_g is the pressure due to the baryons inside the fluctuation and P_r is the radiation pressure including contributions from neutrinos and muons at high enough temperatures. In general, this diffusive dissipation time scale is very slow compared with the universal-expansion time scale. For instance, at a temperature of 100 keV the time scale for the fluctuations to increase in radius by a factor of 2 is $\tau_{\text{dif}} \approx 1000$ s, whereas the Hubble time is $\tau_{\text{exp}} \approx 100$ s. Diffusive damping of the fluctuations is inefficient and fluctuation characteristics will change only slightly due to this process over the range $100 \gtrsim T \gtrsim 0.05$ MeV.

By contrast, fluctuations can be substantially altered by expansion in the low-temperature limit where $\lambda_\gamma \gtrsim l_h$. In this limit the diffusive approximation for photons is not valid on scales of l_h. The material in the high-density regions now responds directly to the baryon pressure gradient. This proton-rich material can stream out into the low-density neutron-rich regions with a flux set by the balance of hydrodynamic pressure due to the baryon-number gradients, on the one hand, and Thomson drag from the photon fluid, on the other. The high-density material does behave as a fluid since ion-electron and ion-ion collisions are more effective at transferring momentum than are photon-electron and photon-ion collisions. This is due to the high degree of isotropy of the photon field. This isotropy is broken once the high-density material begins to move. If the velocity of the fluid is v, then the momentum transfer rate per electron from Thomson drag is[13]

$$F \approx -4P_r\sigma_T(v/c), \tag{4a}$$

and this must be balanced against a driving hydrodynamic force per electron,

$$F \approx -\nabla P/n_e. \tag{4b}$$

Equating these forces and noting that the pressure gradient is roughly $2P_g/l_h$ yields an instantaneous terminal velocity

$$v/c \approx \tfrac{1}{2}(\lambda_\gamma/l_h)P_g/P_r. \tag{4c}$$

We expect from Eq. (2b) that the dissipation of fluctuations by this process should become important when $T \approx 20$ keV. Taking account of the increase in λ_γ/l_h and decrease of P_g/P_r with time, Eq. (4c) can be integrated to give the increase of f_V with time,

$$f_V^{2/3} - (f_V^{2/3})_0 \approx 10^{-2} l^{-2} \left[\frac{20\text{ keV}}{T_0}\right] \Delta t, \tag{5}$$

where l is as in Eq. (2a), and Δt is the time elapsed in seconds since the onset of expansion at temperature T_0, when the volume fraction is $(f_V)_0$. There can be a substantial increase in the volume fraction of the high-density region on a time scale comparable to or shorter than the Hubble time ($\approx (2500$ s$)$ $[(20\text{ keV})/T_0]^2$) and the nucleosynthesis time scale. This expansion and dissipation of the fluctuations can result in enhanced diffusive coupling of baryons between high- and low-density regions.

It is not clear if the high- and low-density material will be completely mixed by the expansion itself. Simple estimates show that any interface between high- and low-density regimes will be Rayleigh-Taylor stable. Furthermore, the photons provide a high-viscosity environment so that the Reynold's number is low on the scale of l and any fluid flows will probably not be turbulent. Nevertheless, it is likely that the Universe will be homogenized by the expansion because of baryon diffusion. Once the high-density regions expand they provide a larger target for back diffusion of neutrons. An extreme limit is where there is no mixing at the interface between high- and low-density material and the low-density material is swept up and compressed. As the neutron-rich material is compressed, neutrons will diffuse through the interface on a time scale short compared to both the Hubble time and the nucleosynthesis time scale.[11] In effect this process of expansion and enhanced diffusion looks very much like the proposal for ^{7}Be destruction from neutron diffusion by Malaney and Fowler.[7]

We can get an idea of the effects of dissipation of fluctuations on primordial nucleosynthesis by homogenizing the Universe at a temperature T_m. This temperature will be chosen to be characteristic of the onset of expansion when $\lambda_\gamma \approx l_h$. At temperatures much higher than 20 keV we note that instantaneous mixing is not correct as the dissipation time scale is longer than the Hubble time, though some enhanced diffusion may occur and our calculations serve as upper limits to the effects of dissipation on nucleosynthesis. We have modeled the fluctuations with two spatial zones: a high-density proton-rich zone and a low-density neutron-rich zone. Nucleosynthesis and neutron diffusion are solved implicitly for these zones as described in Ref. 11. The process described in this paper is not included in any other work, including Ref. 11. Figure 1 shows the mass fractions of ^{3}He, ^{7}Li, D, ^{11}B, ^{9}Be, and ^{10}B to emerge from an inhomogeneous nucleosynthesis epoch as a function of the temperature at which the Universe is homogenized, T_m. As a particular example we take $R=10^6$ at the end of the QCD transition at $T=100$ MeV, $f_V^{1/3}=0.25$, $l=(100\text{ m})[(100\text{ MeV})/T]$, and $\Omega_b=1$. If the Universe is mixed at a temperature in excess of 50 keV then the abundance yields look very much like those of an initially homogeneous universe. If $T_m < 10$ keV then, although the Universe is mixed, the temperature is too low for significant nuclear reactions and we recover the usual results of inhomogeneous nucleosynthesis.[6,11]

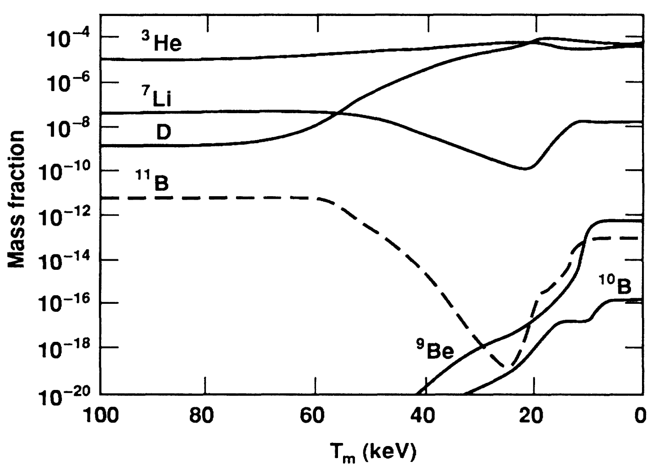

FIG. 1. Indicated elemental mass fractions emerging from primordial nucleosynthesis calculations in an inhomogeneous cosmology with $R=10^6$, $l=(100\text{ m})[(100\text{ MeV})/T]$, $f_V^{1/3}=0.25$, and $\Omega_b=1$ as a function of the temperature at which the Universe is homogenized, T_m. Note that ^{10}B is synthesized as ^{10}Be and ^{10}B, while ^{11}B is synthesized as ^{11}C and ^{11}B.

In the temperature range $50 \geq T \geq 10$ keV, where we predict expansion, several of the light-element abundances may be substantially altered. In the large-R and small-f_V limit, characteristic of the best fits of inhomogeneous models to the light-element abundance for high Ω_b, most of the ^{7}Li is made as ^{7}Be in the high-density regions. In the rapid-dissipation and neutron-diffusion epoch ^{7}Be is destroyed by the very fast ^{7}Be$(n,p)^7$Li reaction and, subsequently, ^{7}Li is destroyed by ^{7}Li$(p,{}^4\text{He})^4$He down to very low levels, $n(^7\text{Li})/n(\text{H}) \approx 10^{-10}$, which is consistent with the lowest estimates for the primordial Li abundance.[14]

Beryllium has been discussed as a possible indicator of inhomogeneity or constraint on inhomogeneous models of nucleosynthesis,[15,16] since ^{9}Be is produced at a mass fraction of 10^{-16} in the homogeneous big bang but at substantially larger values in inhomogeneous models with large Rf_V. Recent observations indicate that it may be possible to put stringent limits on primordial Be.[17] Most of the production of ^{9}Be in the inhomogeneous models is in the low-density neutron-rich zones via the ^{7}Li$(^3\text{H},n)^2$Be reaction. On mixing in the range $10 \leq T_m \leq 50$ keV, ^{9}Be is effectively destroyed by ^{9}Be$(p,\alpha)^6$Li down to an abundance at or below the homogeneous big-bang value.

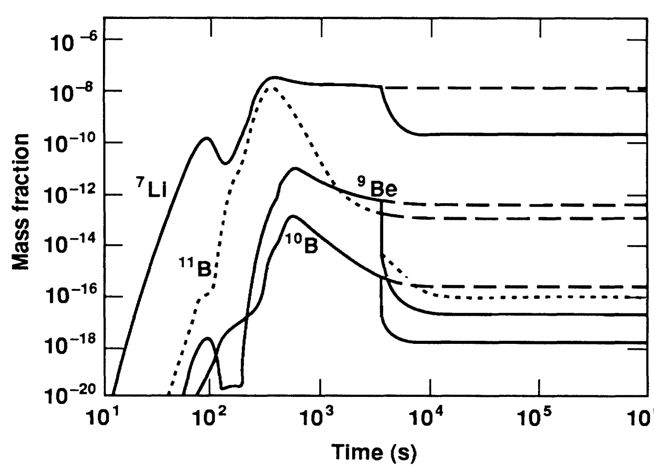

FIG. 2. Time evolution of indicated elemental mass fractions in an inhomogeneous universe with parameters as in Fig. 1, but for mixing at $T=20$ keV. Dashed lines indicate evolution in the absence of mixing.

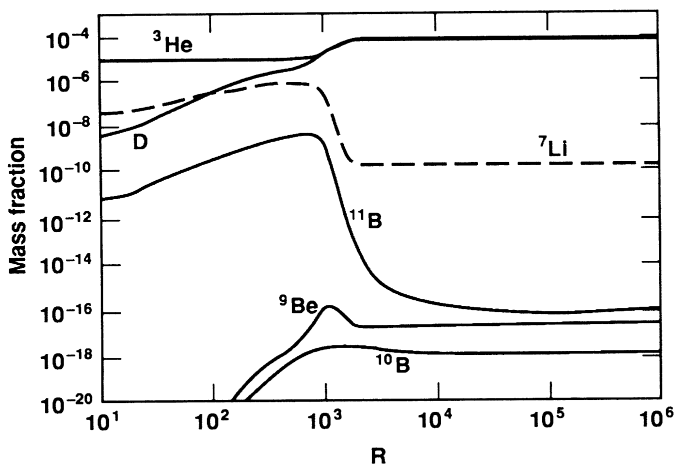

FIG. 3. Elemental mass fractions from inhomogeneous primordial nucleosynthesis with $\Omega_b=1$, $l=(100\text{ m})[(100\text{ MeV})/T]$, $f_V^{1/3}=0.25$, and mixing at $T_m=20$ keV for a range of fluctuation amplitude R (given at $T=100$ MeV).

Beryllium and lithium may still be interesting indicators of inhomogeneity. Since ^{9}Be and ^{7}Li are made in different regions, their primordial abundances taken together give an indication of inhomogeneity and the degree to which the Universe is completely mixed. Low ^{7}Li and high ^{9}Be might, for instance, indicate an inhomogeneous universe in which expansion and enhanced diffusion destroyed ^{7}Li but in which the ions in the low-density zones were not completely mixed in the expansion thus allowing ^{9}Be to escape destruction. In a similar fashion, ^{11}B and ^{10}B could be made in substantial amounts in the low-density neutron-rich zones[15] by $^7\text{Li}(\alpha,\gamma)^{11}\text{B}$ and $^9\text{Be}(n,\gamma)^{10}\text{Be}(e^-\nu_e)^{10}\text{B}$ and we find these species effectively destroyed by mixing. The primordial ^{10,11}B abundance would be an indicator of inhomogeneity and mixing in a manner similar to ^{9}Be.

The abundance of ^{7}Li, ^{9}Be, and ^{10,11}B is shown as a function of time in Fig. 2. In this calculation $T_m=20$ keV and the dashed lines indicate what would happen without mixing. We note that the principal limit to Ω_b in inhomogeneous cosmologies comes from the abundance of ^{4}He.[6,11] In the rapid expansion and mixing scenario discussed here ^{4}He might increase by less than 1% thus pushing down the limit on Ω_b. There is not a larger increase in ^{4}He on mixing because the temperature at mixing is too low to allow much ^{2}H to be burned to ^{4}He. There is also a slight decrease in the abundance of ^{2}H.

The results presented give an indication of the modifications to light-element abundances due to rapid dissipation of fluctuations for any Ω_b and a range of R and f_V. Clearly, the expansion and mixing depend on the baryon-number contrast at $T\approx 20$ keV and that, in turn, depends on the initial R. The effect on light-element abundances of mixing at $T_m=20$ keV for $\Omega_b=1$ for a range of R is shown in Fig. 3. The expansion has the effects in light-element abundances indicated above for $R\geq 10^3$. Detailed coupled hydrodynamics and nucleosynthesis calculations are required in order to pin down the effects of expansion and dissipation of baryon-number fluctuations in the early Universe. Any attempt to constrain R and l from inferred primordial abundances of the light elements must take this effect into account.

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