Ion-Channel Laser

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A relativistic electron beam propagating through a plasma in the ion-focused regime exhibits an electromagnetic instability with peak growth rate near a resonant frequency $\omega \sim 2\gamma^2\omega_\beta$. Growth is enhanced by optical guiding in the ion channel, which acts as a dielectric waveguide, with fiber parameter $V \sim 2(I/I_A)^{1/2}$. A 1D theory for a laser making use of this instability is formulated, scaling laws are derived, and numerical examples are given. Possible experimental evidence is noted.

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The transport of relativistic electron beams (REB's) in plasmas has a long history,[1] and over the last ten years the mechanism of ion focusing has been developed and successfully employed in accelerator work.[2] In addition, the propagation in plasmas of short-pulse, low-emittance REB's has attracted interest in connection with the plasma lens,[3] the continuous plasma focus,[4] the plasma wakefield accelerator,[5] and the beat-wave accelerator.[6]

At the same time, coherent radiation from REB's has been the subject of extensive work, in connection with the free-electron laser (FEL),[7] the cyclotron autoresonant maser (CARM),[8] and other free-electron devices.[9]

In this Letter, we develop the theory of an ion-channel "free-electron" laser (ICL), consisting of a short-pulse, low-emittance REB injected into an unmagnetized, preformed plasma, less dense than the beam. The ICL makes use of ion focusing to transport the beam, and a resonance, akin to that of the planar wiggler FEL, to produce coherent radiation. Here, the wiggler is provided by the electrostatic field of the ion channel, analogous in some respects to the quadrupole FEL proposed by Levush *et al.*[10]

In contrast to other proposed plasma-loaded rf devices,[11] no external magnets are used, and, in principle, no external structures (waveguides, cavities) are required. However, in the microwave regime, the use of a waveguide can enhance the overlap of the beam and the rf signal, making the ICL comparable to the FEL as an rf power source.

The ICL consists of a tank of neutral gas, through which a plasma is produced, for example, by an ionizing laser pulse.[2] Within less than a recombination time, and with proper matching (as in a continuous plasma focus[4]), an REB is injected into the preformed plasma, propagating in the axial ($+z$) direction (Fig. 1).

As the beam head propagates through the plasma, it continuously expels plasma electrons from the beam volume, leaving fixed the relatively immobile ions to provide focusing for the remainder of the beam. These unneutralized ions occupy a cylindrical volume of radius $b \sim a(n_b/n_p)^{1/2}$ and this is the "ion channel." It is assumed that the transverse electric field due to this ion charge is much larger than the transverse force on the beam due to self-fields. This requires $n_p \gg n_b/\gamma^2$, where n_b is the beam density, n_p is the plasma density prior to channel formation, and γ is the Lorentz factor. We also assume $n_b > n_p$. For definiteness the beam density is assumed to be a step radial profile, with radius a.

It is assumed that the beam-current rise time τ_r is long compared to the plasma electron period so that large radial plasma oscillations are not excited as plasma electrons are ejected from the channel. This requires $\omega_e\tau_r \gg 1$, where ω_e is the plasma electron frequency, $\omega_e^2 = 4\pi n_p e^2/m$, $-e$ is the electron charge, and m is the electron mass. It is also assumed that the beam length τ is short compared to the time τ_i for the ions to collapse inward due to the radial electric field of the beam. This requires $\tau_i \sim 1/2\omega_i > \tau$, where ω_i is the ion plasma frequency, $\omega_i^2 = 4\pi n_p e^2/m_i$, and m_i is the ion mass.

In the ion channel, the zeroth-order transverse motion of a beam electron is that of a relativistic, 2D, simple

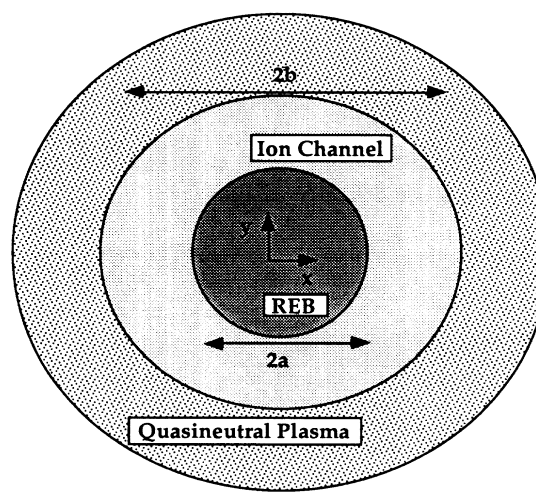

FIG. 1. An REB, propagating through an underdense plasma, expels plasma electrons from the beam volume and beyond to produce an "ion channel," which then focuses the beam, and causes it to radiate.

harmonic oscillator in the potential $U=m\omega_p^2(x^2+y^2)/4$. Electrons oscillate in x and y at the betatron frequency $\omega_\beta \sim \omega_p(mc/2p_z)^{1/2}$. Energy, axial momentum, and axial angular momentum are constants of the motion. In the center-of-momentum frame, electrons are oscillating with upshifted frequency $\omega_1 \sim \gamma\omega_\beta$ and radiate incoherently. In the laboratory frame, the frequency of radiation in the forward $(+z)$ direction is $\omega \sim 2\gamma\omega_1 \sim 2\gamma^2\omega_\beta$. We will show that coherent radiation, near the frequency ω, may be amplified via an induced correlation of longitudinal and betatron phase, which corresponds to a phase-space bunching of the beam.

We consider the motion in the ion channel of a single electron, subject to an electromagnetic wave linearly polarized in the y direction. Denote the vector potential $A_y=(mc^2/e)A\sin\zeta$, where $\zeta=k_zz-\omega t+\phi$. A and ϕ are eikonal amplitude and phase, respectively, and vary slowly in time on the ω^{-1} scale, and in z on the k_z^{-1} scale. The Hamiltonian is

$$H=\left[m^2c^4+p_z^2c^2+p_x^2c^2+\left(p_y+\frac{e}{c}A_y\right)^2c^2\right]^{1/2}+U\,, \quad (1)$$

where p_x, p_y, and p_z are the canonical momenta in x, y, and z, respectively. In deriving the equations of motion, we will neglect anharmonicity due to relativistic effects in the transverse motion, second-order terms in A, derivatives of the slowly varying eikonal quantities, and transverse gradients.

It is convenient to introduce variables q_z, q_x, θ_x, q_y, and θ_y, such that $p_z=mcq_z$, $p_y=mcq_y\sin\theta_y$, and $p_x=mcq_x\sin\theta_x$. For $A=0$, q_x and q_y are constants and $d\theta_{x,y}/dt=\omega_\beta$. We also define the phase variable $\psi=\theta_y+\zeta$, averaged over the betatron period, and the detuning parameter $\Delta\omega=k_zv_z-\omega+\omega_\beta$, where v_z is the betatron-averaged drift velocity in z, $v_z/c=1-(2+q_x^2+q_y^2)/4q_z^2$. We define $a_\beta^2=(q_x^2+q_y^2)/2$, analogous to the wiggler parameter in an FEL. For the round beam of Fig. 1, $a_\beta=q_zk_\beta a/2^{1/2}=2^{3/2}\epsilon_n/a$ and is initially the same for each particle. The rms normalized emittance is $\epsilon_n=0.25q_zk_\beta a^2$, where $k_\beta=\omega_\beta/c$.

With an average over the betatron period, the perturbed equations of motion derived from Eq. (1) take a form reminiscent of that found by Kroll, Morton, and Rosenbluth for the FEL:[12]

$$\begin{aligned}
\frac{d\psi}{dt}&=k_zv_z-\omega+\frac{d\theta_y}{dt}+\frac{d\phi}{dt}+\frac{1}{2}k_zc\frac{q_y}{q_z^2}A\cos\psi\,,\\
\frac{d\theta_y}{dt}&=\omega_\beta\left[1-\frac{1}{2q_y}A\cos\psi\right],\\
\frac{dq_z}{dt}&=-\frac{1}{2}k_zc\frac{q_y}{q_z}A\sin\psi\,,\\
\frac{dq_y}{dt}&=-\frac{1}{2}\left[\omega_\beta+\frac{1}{4}k_zc\frac{q_y^2}{q_z^2}\right]A\sin\psi\,,\\
\frac{dq_x}{dt}&=-\frac{1}{8}k_zc\frac{q_yq_x}{q_z^2}A\sin\psi\,.
\end{aligned} \quad (2)$$

It is assumed that electrons with $q_y<A$ contribute negligibly to amplification, and that $a_\beta<1$. It is also assumed that $\Delta\omega \ll \omega_\beta$, or $\omega-k_zv_z\sim\omega_\beta$. For a fast wave, with $a_\beta \ll 1$, this corresponds to $\omega\sim 2\gamma^2\omega_\beta$.

Maxwell's equations take the form

$$\left[k_z\frac{\partial}{\partial z}+\frac{\omega}{c^2}\frac{\partial}{\partial t}+i\eta\frac{\omega_b^2}{2c^2}\left\langle\frac{1}{q_z}\right\rangle\right]Ae^{i\phi}=i\eta\frac{\omega_b^2}{2c^2}\left\langle\frac{q_y}{q_z}\exp(-i\chi)\right\rangle, \quad (3)$$

where $\chi=\psi-\phi$ and an average has been performed over the periods $2\pi/\omega$ and $2\pi/\omega_\beta$ and over all electrons at z,t as indicated by the angular brackets. The quantity ω_b is the beam-plasma frequency, $\omega_b^2=4\pi n_be^2/m$, and η is the overlap integral of the mode and beam radial profiles. Following Bonifacio, Pellegrini, and Narducci,[13] slippage is neglected and a change of coordinates is made from z,t to $s=t-z/v_z$ and t. As for an FEL, combining Eqs. (2) and (3) gives an equation for the eikonal alone. The solution for the eikonal is given by a superposition of three terms varying as $\exp(\Gamma t)$, corresponding to the three roots of the cubic gain equation, which is, in the limit of zero detuning, $\Gamma^3=i(2\rho\omega_\beta)^3$. In the fast-wave, small-a_β limit, with $\eta=1$, $\rho\sim(I/32\gamma I_A)^{1/3}$. $I=\pi a^2n_bec$ is the peak beam current and $I_A=mc^3/e\sim17$ kA is the Alfvén current. Growth is cubic for short times, and for longer times may be characterized by an exponential gain length $L_{\text{gain}}=c/\text{Re}(\Gamma)$, or $L_{\text{gain}}\sim0.3\lambda_\beta(\gamma I_A/I)^{1/3}$, taking $a_\beta<1$.

The essential mechanism for this instability is bunching in ψ, due to the axial component of the Lorentz force (the "ponderomotive force"), much as in an FEL. However, this bunching is reduced due to the relativistic mass effect and due to a resonant damping of the transverse motion. In the slow-wave limit $(k_zc\ll\omega)$, the relativistic mass effect dominates bunching, which occurs in the opposite sense as for axial bunching. Such a transition from axial to azimuthal bunching was examined by Chu and Hirshfield[14] for the cyclotron maser instability.

We have considered a step radial profile; however, a more realistic beam profile will have an intrinsic spread in detuning. To prevent damping of the instability, one expects that the detuning spread should be small compared to the growth rate. Indeed, a Maxwell-Vlasov treatment[15] imposes the approximate conditions on the spreads in momenta: $\Delta a_\beta^2<2\rho$ and $\Delta p_z/p_z<\rho$. Beam self-fields also produce a detuning spread, and impose the condition on the plasma density $n_p>n_b/4\rho\gamma^2$.

Neglecting optical guiding[16] and diffraction, the approximation $\eta\sim1$ is adequate when the gain length is short compared to the Rayleigh length, $L_{\text{gain}}\ll L_{\text{Ray}}=\pi a^2/\lambda$, where $\lambda=2\pi c/\omega$. However, diffraction is typically important and, in this case, the effect of the channel wall must be included. Neglecting collisions of plasma electrons, the channel serves as a cylindrically symmetric, dielectric waveguide, with step discontinuity in the dielectric constant: $\epsilon=1$ for $r<b$ and $\epsilon=1-\omega_p^2/\omega^2$ for $r>b$.

Such a waveguide will always have at least one guided mode, the hybrid "HE_{11} mode";[17] we proceed to compute the overlap between this mode and the beam. The transverse vector potential is $A_y=(mc^2/e)AJ_0(k_pr)\sin\zeta$, for $r<b$, where $r^2=x^2+y^2$. The total power is $P_{\text{tot}}=P_0(\omega k_z/c)b^2A^2\Lambda$, where $P_0=m^2c^5/e^2=8.71$ GW,

$$\Lambda=\frac{\mu^2}{32}\left[VJ_1(V)\exp\left(\frac{1}{V}\frac{J_0(V)}{J_1(V)}\right)\right]^2, \tag{4}$$

$\mu=\exp(\gamma_E)$, and $\gamma_E\sim0.5772$ is Euler's constant. Λ is a dimensionless mode area, and J_0 and J_1 are the zeroth- and first-order Bessel functions. $V=k_pb=2(I/I_A)^{1/2}$ is the waveguide parameter and $V\le1$ ($I\le4$ kA) is assumed. In this case, the field varies little transversely across the beam. The dispersion relation is $\omega^2\sim c^2k_z^2+\omega_p^2$. The power flowing through the beam volume is $P_b=P_0(\omega k_z/c)a^2A^2/8$ and the overlap integral is $\eta=P_b/P_{\text{tot}}=a_\beta^2/2\gamma V^2\Lambda$. The Pierce parameter with dielectric guiding is then $\rho'=\rho\eta^{1/3}$, and the gain length is

$$L'_{\text{gain}}=L_{\text{gain}}/\eta^{1/3}\sim0.6\lambda_\beta\gamma^{2/3}\Lambda^{1/3}/a_\beta^{2/3}\,.$$

The factor $\Lambda^{1/3}$ ranges from 7×10^{10} for $I=0.2$ kA to 2.4 for $I\sim2$ kA.

Scattering with the neutral atoms and ions of the gas will increase the emittance and this has been studied by Montague and Schnell.[18] Extending their result, and assuming scattering with neutrals dominates, the increase in normalized emittance in one betatron wavelength is $\Delta\epsilon_n=4\pi r_eZ^2\ln(\theta_{\max}/\theta_{\min})/f$, where f is the ionization fraction, r_e is the classical electron radius, and $\theta_{\max}/\theta_{\min}\sim5.26\times10^4/(AZ)^{1/3}$. Z is the atomic number, and A is the atomic weight. For the examples below we will take $Z\sim50$, $A\sim100$, and $f\sim10\%$, corresponding to $\Delta\epsilon_n\sim10^{-6}$ cm rad.

Most beam-plasma instabilities will be rather benign for typical parameters; however, growth of the ion-hose instability[19] is not always negligible. In the rigid-beam model, we find that growth varies as $\exp(z/L_h)^{1/3}$ with

$$L_h\sim\frac{(2^8/3^{9/2})(I_A/I)\epsilon_n}{(\omega_i\tau)^2}\,.$$

The efficiency ϵ may be estimated from the power at the onset of nonlinearity and particle trapping; this gives $\epsilon\sim\rho$, and an output power $P_{\text{out}}\sim\epsilon P_{\text{beam}}$, where $P_{\text{beam}}\sim mc^2\gamma I$ is the initial beam power. Numerical studies indicate that the efficiency may be increased significantly by tapering the plasma density in z, near saturation.

These scaling laws have been applied to four numerical examples for which parameters are given in Table I. The results have been checked with a many-particle simulation based on Eqs. (2) and (3). The first example was also checked with a simulation following the full equations of motion derived from Eq. (1). We note that, in the first example (microwave regime), it would be most natural to confine the radiation in a conducting waveguide. Operating in the TE_{01} mode, the overlap factor is $\eta\sim2\pi a^2/W$, where W is the cross-sectional area of the waveguide. In this case, the gain length and efficiency will be comparable to that for an FEL.

TABLE I. Examples of ion-channel laser scalings.

	Microwave	Submillimeter	Infrared	X ray
λ (cm)	2	5×10^{-2}	1×10^{-3}	1×10^{-6}
E (MeV)	2	4	10	100
I (kA)	4	4	4	4
ϵ_n (cm rad)	3×10^{-1}	1×10^{-2}	5×10^{-4}	3×10^{-5}
n_p (cm^{-3})	6×10^{10}	8×10^{12}	1×10^{15}	2×10^{19}
L'_{gain} (cm)	70	16	4	0.2
λ_β (cm)	4×10^{1}	5	6×10^{-1}	2×10^{-2}
τ_i (ns)	2×10^{1}	1	0.1	1×10^{-3}
a_β	0.6	0.5	0.4	2
a (cm)	1	7×10^{-2}	3×10^{-3}	4×10^{-5}
ρ' (%)	5	3	1	1
P_{out} (GW)	0.4	0.5	0.6	3
N_h	3	6	11	14

In selecting these examples, the most severe constraint was found to be the condition $\Delta a_\beta^2<2\rho$, which is marginally satisfied in the first three examples. To exhibit the consequences of this constraint, the fourth example was designed with a large a_β. It should be emphasized, however, that such an x-ray laser could not be realized without a sharp distribution in transverse energy, corresponding to a step radial profile, or perhaps a spinning beam. In the first three examples, the plasma densities required are not out of the ordinary. For the fourth example, the plasma density is high; however, it need only be maintained over a few centimeters. The number of ion-hose e folds at saturation N_h becomes severe at shorter wavelengths; however, it can be reduced by further shortening of the REB pulse length τ.

Ion neutralization of the beam on the ω_i^{-1} time scale imposes a significant constraint on beam length and this motivates the study of the analogous instability of a magnetically self-focused beam ($n_b<n_p$). This regime is also of special interest in that experimental evidence has already been found of coherent radiation from intense electron beams injected into overdense, unmagnetized plasmas.[20,21] Explanations offered for the high microwave power levels observed have included streaming instabilities, strong turbulence, and virtual cathode oscillations. Kato, Benford, and Tzach[20] remark on the possibility of an FEL analogy based on jitter motion in "large-amplitude electrostatic waves generated by instability"; however, to date, no satisfactory theory has been set down to explain the measured power levels. We propose the ICL instability as a possible mechanism.

Adopting the theory set down above, estimates may be made by identifying $k_\beta \sim (2I_{\rm net}/\gamma I_A)^{1/2}/a$ and $a_\beta \sim (\gamma I_{\rm net}/I_A)^{1/2}$. The experimental results are characterized by efficiencies of a few percent or less and a broadband spectrum extending far above ω_p. Such efficiencies are lower than predicted by the scalings given here, probably due to nonlinear focusing and spreads in momenta. The spectrum may be understood from the result for the resonant frequency: $\omega - k_z v_z \sim \omega_\beta$. An electron with small transverse energy has $a_\beta \sim 0$, and is resonant with $\omega \sim 2\gamma^2\omega_\beta$, while electrons with large transverse energies are resonant with $\omega \sim 2\gamma^2\omega_\beta/a_\beta^2$.

It also seems likely that the ion-channel-laser instability will appear naturally, in astrophysical circumstances, and we are studying its applicability to solar bursts.[20,22]

In conclusion, we have presented the concept of the ion-channel laser, together with the first theory. Three novel physical mechanisms were noted: the electromagnetic instability itself, dielectric guiding by the ion-channel, and resonant damping of the transverse motion. An advantage of the ICL over the FEL is the short betatron wavelength achievable in a plasma. To reach a given wavelength with the ICL, a lower beam energy may be used than in an FEL, resulting in a higher efficiency.

A disadvantage of the ICL is that the "pump field" is unstable. Fortunately, ion motion represents a cumulative, electrostatic instability, with zero group velocity, while the ICL instability is electromagnetic, propagating with the beam. Thus, a reduction in τ lowers the growth of ion instabilities, while not reducing the peak laser power, or efficiency.

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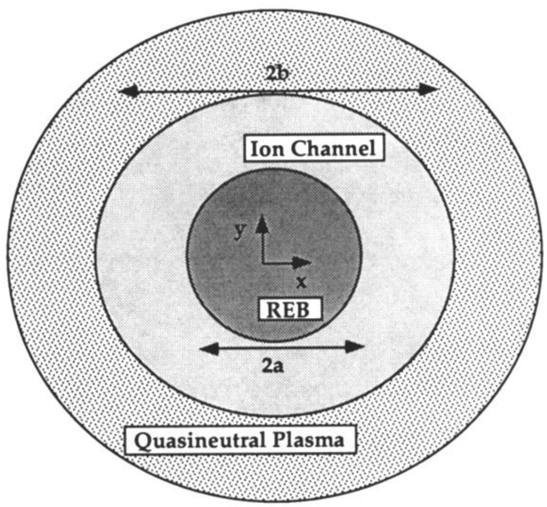

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