

Vacuum Rabi Splitting as a Feature of Linear-Dispersion Theory: Analysis and Experimental Observations

Yifu Zhu, Daniel J. Gauthier, S. E. Morin, Qilin Wu, H. J. Carmichael, and T. W. Mossberg

Department of Physics and Chemical Physics Institute, University of Oregon, Eugene, Oregon 97403

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The spectral and temporal response of an optical cavity resonantly coupled to an ensemble of barium atoms has been investigated experimentally. The empty-cavity transmission resonances are found to split in the presence of the atoms and, under these conditions, the cavity's temporal response is found to be oscillatory. These effects may be viewed as a manifestation of a vacuum-field Rabi splitting, or as a simple consequence of the linear absorption and dispersion of the intracavity atoms.

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A composite atom-cavity system can be created by placing atoms inside an optical cavity. The behavior of such a coupled system can often be more complex and hence richer than that of either the atoms or the cavity considered separately. Elucidation of the properties of such atom-cavity systems is important since they play a vital role in the analysis of effects such as optical bistability, laser operation, and quantum fluctuations. Recently, it has been predicted that the insertion of a single atom into a cavity can lead to a splitting in the atomic fluorescence spectrum¹ and the empty-cavity transmission resonances^{2,3} when the atom is strongly coupled to the cavity. This splitting, termed the vacuum Rabi splitting, has attracted the attention of the quantum-optics community because it is considered to be an important manifestation of the quantum nature of the electromagnetic field.

In the optical regime, the experimental confirmation of the single-atom vacuum Rabi splitting has been precluded by the smallish size of the coupling between the atom and the cavity.⁴ Fortunately, it has been shown that cavity resonance splitting also occurs when many atoms are inserted into a cavity, and that the magnitude of the splitting increases with the square root of the number of atoms inserted.^{2,3} Recently, multiatom enhancement has been employed successfully in an effort to observe vacuum Rabi splittings.⁵⁻⁷

In this paper, we describe an experimental investigation of resonant, multiatom, vacuum Rabi splitting. Cavity throughput as a function of both frequency (continuous-wave input) and time (pulsed input) has been measured in the same system. We find that our experimental results are in excellent agreement with a *completely classical* model in which the cavity transmission function is derived using the standard concepts of multibeam interference⁸ applied to a cavity containing atoms displaying *linear* absorption and dispersion.⁹ This approach to calculating the cavity's transmission function is dramatically different from the one employed in QED descriptions of the problem. The success of a total-classical model indicates, contrary to popular belief,

that the vacuum Rabi splitting is not an inherently quantum phenomenon. In addition, our experimental results demonstrate that the multiatom vacuum Rabi splitting can be observed in cavities having an active volume (finesse) more than an order of magnitude larger (smaller) than previously demonstrated.

In our model of the coupled atom-cavity system, the cavity is taken to be a confocal cavity with length \mathcal{L}_c , and the atoms are taken to be classical Lorentz oscillators whose resonance frequency ν_0 is near a cavity resonance (transmission maxima) at frequency ν_m . The cavity mirrors have an intensity reflection (transmission) coefficient of R (T). The Lorentz oscillators have a full width at half maximum (FWHM) δ_H , an oscillator strength f_0 , a number density N , and are contained in a slab of length $L < \mathcal{L}_c$.

Using a standard multibeam interference analysis, it can be shown that the intensity transmission function of the coupled atom-cavity system is given by

$$T_c(\nu) = |t_c(\nu)|^2 = \frac{T^2 e^{-\alpha L}}{(1 - Re^{-\alpha L})^2 + 4Re^{-\alpha L} \sin^2(\epsilon/2)} \quad (1)$$

for a plane wave propagating along the cavity axis where

$$\epsilon(\nu) = 2\pi(\Delta - \Delta_m)/\Delta_{\text{FSR}} + 4\pi(n-1)L\nu/c$$

is the phase shift experienced by the field upon completion of a round-trip through the cavity, αL is the single-pass absorption, and $t_c(\nu)$ is the amplitude transmission function. Here, $\Delta = \nu - \nu_0$, $\Delta_m = \nu_m - \nu_0$, and $\Delta_{\text{FSR}} = c/2\mathcal{L}_c$ is the free spectral range of the empty cavity where c is the speed of light in vacuum. For an empty cavity with a large finesse $F = \pi\sqrt{R}/(1-R)$, the empty-cavity resonance width is given by $\delta_c = \Delta_{\text{FSR}}/F$. The frequency-dependent intensity-absorption coefficient and refractive index of the collection of Lorentz oscillators are given by

$$\alpha = \alpha_0 \frac{\delta_H}{4\Delta^2 + \delta_H^2}, \quad n = 1 - \alpha_0 \frac{c}{2\pi\nu} \frac{\Delta\delta_H}{4\Delta^2 + \delta_H^2}, \quad (2)$$

respectively, where $\alpha_0 = 2f_0 N e^2 / m c \delta_H$ is the line-center absorption coefficient, and e (m) is the electron charge (mass). It should be noted that only *linear*-absorption and refractive-index terms are included in Eq. (2). The inclusion of nonlinear, i.e., intensity-dependent, terms leads to effects such as optical bistability.¹⁰

We find that linear absorption and dispersion introduced by the intracavity atoms alter the transmission function in that $T_c(\nu)$ may exhibit a structure which is completely different from that found in the empty-cavity case. The location and even the number of cavity resonances [maxima in $T_c(\nu)$] may change. In the analysis of these changes, we limit our attention to the case of low single-pass absorption ($\alpha_0 L \ll 1$), high-cavity finesse ($R \approx 1$), atom-cavity resonance ($\Delta_m = 0$), small-frequency differences ($\Delta \ll \Delta_{\text{FSR}}$), and comparable atomic and cavity resonance widths ($\delta_c \approx \delta_H$).

Neglecting the effects of absorption, the zeros of $\epsilon(\nu)$ determine the peaks in the cavity transmission function $T_c(\nu)$, and the slope of $\epsilon(\nu)$ at the zeros provides a measure of the resonance widths. In Fig. 1(a), we plot $\epsilon(\nu)$ for several different atomic number densities. In Fig. 1(b), the atomic absorption and dispersion functions of Eq. (2) are plotted. At zero density, the expression for $\epsilon(\nu)$ contains a single nonzero term corresponding to a straight line with a slope of $2\pi/\Delta_{\text{FSR}}$ [dashed line in Fig. 1(a)] indicating that the cavity has a single peak at $\Delta = 0$. For nonzero atomic densities, a second term, proportional to the atomic dispersion function, contributes to $\epsilon(\nu)$ as well. At low densities, the dispersion reduces the slope of $\epsilon(\nu)$ at $\Delta = 0$ thereby broadening the cavity transmission resonance. (Absorption also contributes to this broadening.) At higher atomic densities, $\epsilon(\nu)$ is so distorted by the dispersive term that it actually passes through zero three times. One zero, as in the empty-cavity case, occurs at $\Delta = 0$, and two new zeros, located symmetrically about $\Delta = 0$, appear [see Fig. 1(a)]. For atomic densities high enough to produce a three-zero structure in $\epsilon(\nu)$, the absorptive part of the atomic response destroys the central transmission peak and slightly shifts the remaining two peaks away from the zeros of the function $\epsilon(\nu)$ [see Fig. 1(c)].

A more detailed analysis reveals that the two peaks in the cavity transmission are approximately Lorentzian in shape,² occur at the frequencies $\Delta = \pm \Omega/2$, where

$$\Omega = \left[\frac{F \alpha_0 L}{\pi} \delta_H \delta_c - \frac{(\delta_H - \delta_c)^2}{4} \right]^{1/2}, \quad (3)$$

and have a height

$$h = \left[\frac{T^2}{(1-R)^2} \right] \left[\frac{\delta_c^2}{(\delta_H + \delta_c)^2} \right] \quad (4)$$

and a width (FWHM) of

$$\delta'_c = (\delta_H + \delta_c)/2. \quad (5)$$

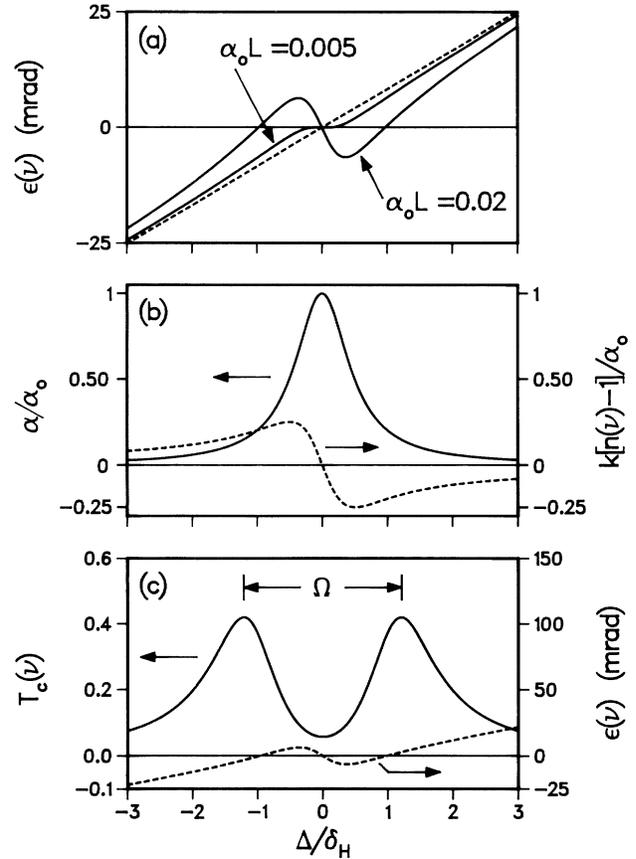


FIG. 1. (a) Phase shift experienced by the field upon completion of a round trip through the cavity for various values of the line-center, single-pass absorption as a function of the detuning of the probe-laser frequency from the atomic resonance frequency normalized to the atomic resonance width. The cavity resonance is tuned to the atomic resonance ($\Delta_m = 0$), $\Delta_{\text{FSR}}/\delta_H = 750$, $\delta_c/\delta_H = 1.5$, and $F = 500$. The dashed line is the empty-cavity phase shift. (b) Normalized absorption (solid line) and change in refractive index (dashed line) produced by the collection of Lorentz oscillators. The magnitude of the probe wave vector is denoted by k . (c) Cavity transmission function and phase shift as functions of the normalized frequency. The values of the parameters used to generate this plot are the same as in (a) with $\alpha_0 L = 0.02$.

Note that the width of the peaks is the average of the uncoupled atom and cavity widths.⁷ The splitting will be resolved when $\Omega \gg (\delta_H + \delta_c)/2$; that is, when

$$\alpha_0 L \gg \pi/F. \quad (6)$$

The high-atomic-density resonance splitting Ω and width δ'_c predicted above with our totally classical model are identical to the corresponding vacuum-field Rabi-splitting results obtained by Carmichael *et al.*¹¹ We may thus conclude that the multiatom vacuum Rabi splitting may also be regarded as a feature of linear-dispersion theory. While this same point can be inferred from the

demonstrated success of mean-field, Maxwell-Bloch-type calculations in the treatment of vacuum Rabi splittings,^{11,12} the completely classical nature of vacuum Rabi splittings has not been emphasized.

In all our experiments we measured the transmission of a weak continuous-wave (cw) or pulsed-probe laser through a 1-cm confocal optical cavity. The probe beam had a cw linewidth of ≈ 2 MHz, was carefully aligned to propagate along the cavity axis, and was focused to a 90- μm spot at the center of the cavity where it intersected at normal incidence a collimated beam of natural barium (72% ^{138}Ba), having a 20-MHz residual Doppler width and $a \approx 1$ mm diameter. The cavity was adjusted to be nearly resonant with the 554-nm ^{138}Ba $6s^2^1S_0$ - $6s6p^1P_1$ transition (19-MHz natural width). Approximately 2 pW of the probe power was coupled into the cavity, resulting in a maximum intracavity intensity of $\approx 15 \mu\text{W cm}^{-2}$ which is far below the 15-mW cm^{-2} saturation intensity of the barium transition. Absolute single-pass absorption of the atomic beam was determined by direct measurements at high atomic number densities. Absorption at lower atomic densities was assumed to scale linearly with the atomic fluorescence signal produced by an auxiliary laser beam. The atomic-beam density was varied by adjusting the temperature of the beam source oven, and was as high as 1.5×10^9 atoms cm^{-3} , corresponding to a maximum line-center single-pass absorption equal to 2.2 cm^{-1} .

Measured and calculated cavity transmission functions are shown in Fig. 2. In the calculations of Figs.

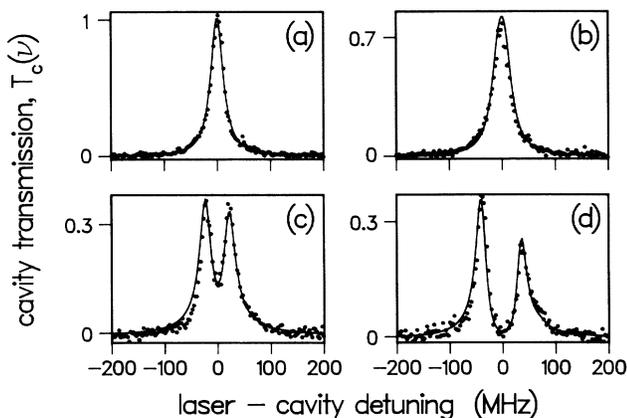


FIG. 2. Experimentally measured (solid circles) and theoretically predicted (solid line) cavity transmission functions. The values of the parameters for the cavity are $\Delta_{\text{FSR}} = 15$ GHz, $\delta_c = 30$ MHz, and $F = 500$. The cavity resonance is precisely set to the ^{138}Ba $6s^2^1S_0$ - $6s6p^1P_1$ transition ($\delta_H = 19$ MHz). The value of the line-center, single-pass absorption $\alpha_0 L$ was set to (a) 0, (b) 0.0013, (c) 0.008, and (d) 0.055. The measured cavity transmission functions have been scaled in the vertical direction so that the measured empty-cavity transmission is equal to 1.

2(b)-2(d), we have accounted for the residual atomic-beam Doppler width and the lower abundance, blueshifted barium isotopes. The empty-cavity transmission function [Fig. 2(a)] displays a linewidth of ≈ 30 MHz (FWHM) corresponding to a cavity finesse of ≈ 500 . Therefore, a splitting should appear [see Eq. (6)] when $\alpha_0 L > 0.006$. For $\alpha_0 L = 0.0013$ [Fig. 2(b)], only a broadening of the transmission peak is found. With $\alpha_0 L = 0.008$ [corresponding to ≈ 300 atoms in the cavity, Fig. 2(c)], a well-resolved mode splitting is observed. For $\alpha_0 L = 0.055$ [Fig. 2(d)], an even larger splitting is observed. The asymmetry in the peak heights is attributable to barium isotopes other than ^{138}Ba . Predicted and observed transmission functions are in excellent agreement.

From the perspective of quantum optics, the vacuum Rabi splitting may be seen to follow from the exchange of excitation back and forth between the atoms and the cavity field. In the transient regime, this exchange is manifest as a temporal oscillation on the light transmitted through the cavity. From the classical perspective, the atom-cavity system is a simple linear system, and the time- and frequency-domain responses of the system are connected via Fourier transform. If we denote the Fourier transform of a pulse incident in one end of (after transmission through) the cavity by $E_{\text{inc}}(\nu)$ [$E_{\text{out}}(\nu)$], we can immediately write

$$E_{\text{out}}(\nu) = t_c(\nu) E_{\text{inc}}(\nu). \quad (7)$$

If the atomic density is large enough to make $t_c(\nu)$ double peaked and if the input pulse is sufficiently short, inverse Fourier transformation of $E_{\text{out}}(\nu)$ reveals that the output pulse will contain an oscillation with a period of $1/\Omega$ and have a duration on the order of $1/\pi(\delta_H + \delta_c)$. This behavior is in agreement with that expected from the perspective of quantum optics.

The transient response of our atom-cavity system was investigated by injecting a pulse (acousto-optically sliced from the cw probe laser) in one end of the cavity and measuring the temporal evolution of the light emitted out through the other end. The temporal evolution of the input [output] pulse intensity is shown in Fig. 3(a) [3(b)]. The data shown in Fig. 3(b) were recorded with the single-pass atomic absorption $\alpha_0 L$ set to 0.22 and with a small atom-cavity detuning Δ_m equal to 25 MHz introduced in order to minimize the effect of the less abundant Ba isotopes on the measurement. Under these conditions, a 150-MHz frequency-domain cavity-resonance splitting was observed. To fit the data with our model, we used Eq. (7) and assumed that the spectrum of the incident pulse was given by the transform of the square root of the measured intensity profile. The predicted output pulse is shown as the solid line in Fig. 3(b). The data are somewhat noisy; however, the expected oscillations are clearly evident.

In summary, we have shown that the steady-state and

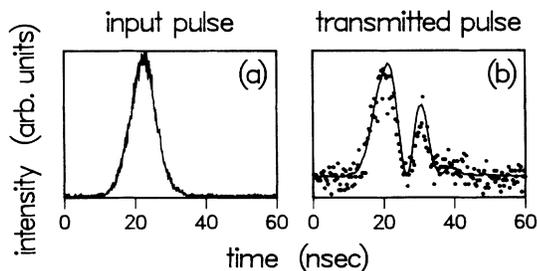


FIG. 3. Temporal behavior of the coupled atom-cavity system. (a) Experimentally measured intensity of the pulse input to the cavity as a function of time. (b) Experimentally measured (solid circles) and theoretically predicted (solid line) evolution of the pulse transmitted from the cavity. The cavity resonance was tuned 25 MHz to the high-frequency side of the $^{138}\text{Ba } 6s^2\ ^1S_0\text{-}6s6p\ ^1P_1$ transition and $a_0L = 0.22$.

transient transmission characteristics of a cavity coupled to a collection of atoms can be understood using an entirely classical, steady-state multibeam interference analysis of an optical cavity. In particular, it is found that the empty-cavity resonances can be split due to the effects of linear absorption and dispersion and that this mode splitting is identical to the vacuum-field Rabi splitting predicted by a fully quantum electrodynamical formalism. This of course implies that the mere observation of oscillatory transmission and/or mode splitting is no more intrinsically quantum than the observation of linear absorption and dispersion.

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