Experimental Violation of Bell's Inequality Based on Phase and Momentum

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Two-color photon pairs are selected by two double apertures placed to satisfy the phase-matching conditions at a down-conversion crystal. The different wavelengths are superposed at spatially separated points on a beamsplitter and coincident two-photon detections are measured. On adjusting phase plates in the beams before the beamsplitter an apparent nonlocal fourth-order interference effect is seen which violates Bell's inequality by several standard deviations.

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Although quantum mechanics is an extremely successful predictive theory, it does incorporate a rather counterintuitive world view which has led people to formulate alternative models. Bell's inequalities[1] were derived as conditions which an alternative class of local realistic theories must satisfy, and which are violated, in certain conditions, by quantum-mechanical models. A large number of experiments have been performed in order to verify the predictions of quantum mechanics. These have mainly been of three types: cascade-photon experiments,[2] high-energy experiments involving polarized protons or γ photons,[3] and more recently using the angle-correlated pair photons produced in parametric down conversion.[4] All of these experiments are based on superposition of spin or polarization states as first described by Bohm.[5] We demonstrate here that similar effects can be obtained when each photon in a correlated photon pair is split (or is created with two separate parts) and recombined with variable phase delay. It is only recently that spin-free tests of Bell's inequality have been suggested in the literature[6-10] and we believe that the work presented below is the first experimental demonstration of a violation of Bell's inequality based on phase and momentum, rather than spin.

We use parametric down conversion in an arrangement similar to that suggested independently by Horne and co-workers.[6] An outline of the experimental apparatus is shown in Fig. 1. Light from a krypton-ion laser operating at 413.4-nm wavelength is weakly focused in a crystal (CR) of deuterated potassium dihydrogen phosphate (KD*P) with crystal axis cut at 90° to the incident beam. As a result of crystal nonlinearity a small fraction of the vertically polarized incident light (photons) is down converted to pairs horizontally polarized photons satisfying energy conservation and propagating in directions set by momentum conservation within the birefringent crystal. In this case the symmetric pairs of 826.8-nm wavelength photons are emitted in directions subtending an angle of 28.6°. Conjugate colors a and b with wavelengths above and below 826.8 nm, respectively, are selected using double apertures (A) in each arm of the apparatus. Pairs of photons can now be detected in opposite beams of different wavelengths. Mirrors $M1$ and $M2$ reflect the beams onto a beamsplitter (BS) where recombinations of the different colors occurs at points separated by several millimeters. The four outputs from the beamsplitter are measured using photon counting detectors $D_{a,b;3,4}$. Photodetection coincidences between the four possible combinations of detectors viewing different colors are measured simultaneously using a modified photon correlator with a gate time $\Delta T = 10$ ns. The clocked nature of the correlator ensures that the four gates are exactly equal. The gross path-length difference δx between arms 1 and 2 of the interferometer can be changed by suitable adjustment of mirror assembly $M1$ which is configured as an "optical trombone" (see inset in Fig. 1). Two tiltable glass plates P_a and P_b allow independent adjustment of the relative phases, ϕ_a and ϕ_b, at recombination of colors a and b. As the colors overlap at different points on the beamsplitter one could, in principle, construct an apparatus of the

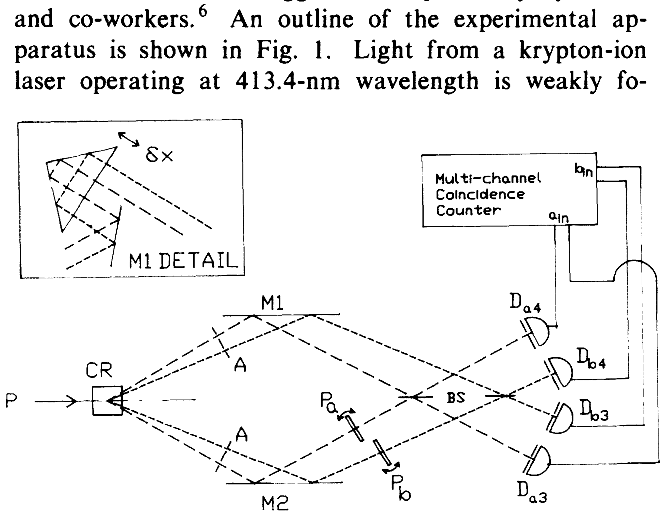

FIG. 1. Outline of the apparatus.

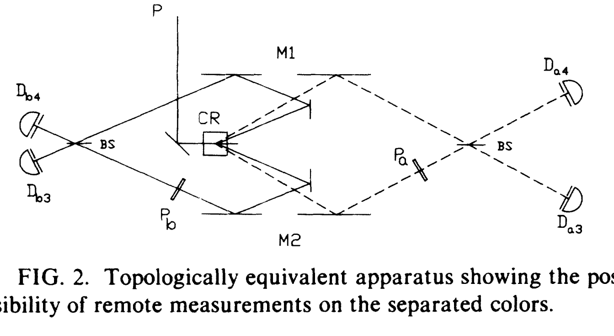

FIG. 2. Topologically equivalent apparatus showing the possibility of remote measurements on the separated colors.

form shown in Fig. 2 where the phase shifting and recombination of the separate colors occurs at remote locations. In the following it will be shown that given a photodetection in either of detectors D_{a3},D_{a4} the direction that the coincident b-color photon takes at its beamsplitter will be strongly dependent on the setting of the remote phase plate P_a. The extent of this dependence cannot be explained by any local realistic theory.

A measure of the distribution of coincidences between detectors on the same side and on opposite sides of the beamsplitter for a particular phase setting is given by the correlation coefficient[8,11]

$$E(\phi_a,\phi_b)=\frac{\bar{C}_{a3,b3}(\phi_a,\phi_b)+\bar{C}_{a4,b4}(\phi_a,\phi_b)-\bar{C}_{a3,b4}(\phi_a,\phi_b)-\bar{C}_{a4,b3}(\phi_a,\phi_b)}{\bar{C}_{a3,b3}(\phi_a,\phi_b)+\bar{C}_{a4,b4}(\phi_a,\phi_b)+\bar{C}_{a3,b4}(\phi_a,\phi_b)+\bar{C}_{a4,b3}(\phi_a,\phi_b)}, \tag{1}$$

where $\bar{C}_{ai,bj}(\phi_a,\phi_b)$ ($i=3,4$; $j=3,4$) is the mean coincidence rate between detectors D_{ai} and D_{bj} with phase angles set to ϕ_a,ϕ_b. The generalized Bell inequality states that the combination of four such measurements at various phase angles[1,12]

$$S=E(\phi_a,\phi_b)-E(\phi_a,\phi_b')+E(\phi_a',\phi_b)+E(\phi_a',\phi_b') \tag{2}$$

should fall within the bounds

$$-2\le S\le 2 \tag{3}$$

when a classical locality assumption is made. We show here that for suitable choice of phase angles that $S=2\sqrt{2}$ could be measured in theory and present an experimental measurement of S exceeding 2 by several standard deviations.

The experimentally measured coincidence rate is proportional to the integral of the instantaneous two-photon photodetection probability $P_{ai,bj}$ over the finite coincidence gate time

$$\bar{C}_{ai,bj}=\frac{1}{\Delta T}\int_{-\Delta T/2}^{\Delta T/2}P_{ai,bj}(\phi_a,\phi_b,\tau)d\tau, \tag{4}$$

where τ is a time difference. ΔT is usually long compared to any typical photon coherence time (of order femtoseconds) or any difference in propagation time from the crystal to detectors hence the limits of the integral can be written as $\pm\infty$. The two-photon detection probability is given by

$$P_{ai,bj}(\phi_a,\phi_b,\tau)=\eta_a\eta_b\langle\hat{A}_{ai}^{(-)}(\bar{t}_{ai})\hat{A}_{bj}^{(-)}(\bar{t}_{bj}+\tau)\times\hat{A}_{bj}^{(+)}(\bar{t}_{bj}+\tau)\hat{A}_{ai}^{(+)}(\bar{t}_{ai})\rangle, \tag{5}$$

where $\hat{A}^{(+)}(\bar{t})$ is the positive-frequency part of the electric-field operator (which can also be described as a photon annihilation operator) at reduced time $\bar{t}=t-X/c$ with X a propagation distance measured from the down-conversion crystal and $\eta_{a,b}$ are effective detector (plus collection) efficiencies at wavelengths a and b.

The electric-field operators beyond the beamsplitter can be expressed as superpositions of the operators at the beamsplitter inputs[13-15]

$$\hat{A}_{a3}^{(+)}(t)=i\sqrt{R}A_{a1}^{(+)}(\bar{t})+\sqrt{T}A_{a2}^{(+)}(\bar{t}+\phi_a/\omega_a+\delta t),$$
$$\hat{A}_{a4}^{(+)}(t)=\sqrt{T}A_{a1}^{(+)}(\bar{t})+i\sqrt{R}A_{a2}^{(+)}(\bar{t}+\phi_a/\omega_a+\delta t), \tag{6}$$

and similar with a replaced by b. R and T are the intensity reflection and transmission coefficients of the beamsplitter and the imaginary component implies a $\pi/2$ phase change on reflection valid for symmetric beamsplitters. Gross path-length differences to the beamsplitter are assumed corrected for by suitable adjustment of mirror assembly $M1$ leaving only a small variable path difference $\delta x=c\delta t$ and phase-time differences $\phi_{a,b}/\omega_{a,b}$, where ω_a and ω_b are the angular frequencies of colors a and b (assuming a quasimonochromatic approximation). Any path-length differences beyond the beamsplitter are ignored being physically insignificant for the reasons given above.

$P_{ai,bj}$ can be expressed as a set of fourfold products of the input field operators which in the low-flux limit considered here can be represented by a set of twofold products involving only pair photon amplitude terms of the type $\langle\hat{A}_{a1,2}^{(+)}\hat{A}_{b2,1}^{(+)}\rangle$ (Ref. 16) (and complex conjugate) reflecting the fact that before the beamsplitter coincidences are only seen between opposite beams of different colors. Propagation delays are incorporated by writing pair products of the electric-field operators as integrals over modes with

$$\hat{A}_a^{(+)}(\bar{t}+U)\hat{A}_b^{(+)}(\bar{t}+U')=\hat{A}_a^{(+)}(\bar{t})\hat{A}_b^{(+)}(\bar{t})\int f(\omega)e^{iU(\omega_a+\omega)}e^{iU'(\omega_b-\omega)}d\omega. \tag{7}$$

$f(\omega)$ is a normalized spectral function set by aperture geometry and pump beam divergence at the crystal.[15] Energy conservation is implied here by the use of a single difference frequency ω such that $(\omega_a+\omega)+(\omega_b-\omega)=\omega_0$, where ω_0 is the angular frequency of the pump beam. For simplicity we approximate $f(\omega)$ by a Gaussian spectral function with $1/\sqrt{e}$ bandwidth c/σ. Combining Eqs. (4)–(7) then leads to[10,17]

$$\bar{C}_{ai,bj}=\bar{C}_0(R^2+T^2)\left[1+(-1)^{i+j}V\cos\left(\phi_a-\phi_b+(\omega_a-\omega_b)\frac{\delta x}{c}\right)\exp\left(-\frac{(\delta x')^2}{\sigma^2}\right)\right], \tag{8}$$

where

$$\bar{C}_0=\frac{\eta_a\eta_b}{\Delta T}\int_{-\infty}^{\infty}|\langle\hat{A}_{a1,2}^{(+)}(\bar{t})\hat{A}_{b2,1}^{(+)}(\bar{t}+\tau)\rangle|^2d\tau \tag{9}$$

is the coincidence rate measured between two opposite beams before the beamsplitters; the magnitude of which is related to the nonlinear coefficient of the crystal, pump-laser power, and detailed phase-matching integrals within the crystals.[16] A visibility $V=2RT/(R^2+T^2)$ is defined which is unity when a 50/50 beamsplitter ($R=T=0.5$) is used and $\delta x'=\delta x-c(\phi_a/\omega_a-\phi_b/\omega_b)$. The coincidence rate as a function of gross path-length difference δx shows a cosinusoidal oscillation within a Gaussian envelope of width σ. Normally the photon "coherence" length σ covers many optical cycles while the phase delays are less than one cycle and the approximation $\delta x \simeq \delta x'$ can be made. Near zero path-length difference ($\delta x \ll \sigma$) the exponential term becomes unity. Equation (8) is then formally equivalent to the polarization correlation for a pair of photons in an s state. A maximum coincidence rate at detectors ai,bj is obtained by tuning the phase delays to ensure the argument of the cosine term in Eq. (8) is π. This also implies a maximum coincidence rate between the other two detectors aj,bi. More significantly, one can redistribute the coincidences in detector pairs $a3,b4$ and $a3,b3$, adjusting the phase plate P_a, apparently remotely altering the direction that the *coincident* b-color photons take at the beamsplitter.

The four coincidence rates at a particular phase setting can be combined to estimate the correlation coefficient $E(\phi_a,\phi_b)$ as in Eq. (1). Choosing phase angles $\phi_a=0$, $\phi_a'=\pi/2$, $\phi_b=\pi/4$, $\phi_b'=3\pi/4$, and combining four measurements of $E(\phi_a,\phi_b)$ as in Eq. (2) leads to a measured $S=2\sqrt{2}V'$. We include any reduction in visibility due to misalignment in a lumped visibility factor V'. It is clear that $V'\geq 0.71$ is required to violate the inequality in Eq. (3). To ensure maximum visibility requires $R\simeq T\simeq 0.5$ and $\delta x \ll \sigma$. Any misalignment of the apertures such that the bimodal spectrum (centered on ω_a,ω_b) selected in arm 1 of the apparatus does not exactly match that in arm 2 can produce a background coincidence rate which will also reduce V'.[10]

It is also important to note that the mean count rate in each detector remains constant independent of the phase, reflecting the fact that the beams in opposite arms of the apparatus are incoherent with respect to one another and second-order interference effects are absent. An analogous experiment using classical light sources involves two separate sources with a randomly varying relative phase difference $\phi_R(t)$ to provide the light in the $M1$ and $M2$ arms of the apparatus. The intensity detected instantaneously at detector D_{ai} can be expressed as

$$I_{ai}(t)=I_a\{1+(-1)^i\cos[\phi_a+\phi_R(t)]\}, \qquad (10)$$

where I_a is the intensity of each a beam and $V=1$ is assumed. A similar expression holds for detectors D_{bj}. The classical normalized intensity correlation function $\langle I_{ai}I_{bj}\rangle$ is equivalent, in the low-flux limit, to the coincidence measure and when the sampling time is short compared to the phase fluctuation time

$$\bar{C}(ai,bj)=\langle I_{ai}I_{bj}\rangle = I_aI_b\left[1+\frac{(-1)^{i+j}}{2}\cos(\phi_b-\phi_a)\right], \qquad (11)$$

which has maximum visibility 50% implying $S\leq 2$ as expected. Similar results are predicted by more general hidden-variable theories.[18]

The coincidence rate measured between detectors $a3$ and $b4$ as a function of gross path-length difference δx with phase shifts ϕ_a and ϕ_b near zero is shown in Fig. 3. The Gaussian envelope over a cosinusoidal oscillation predicted by Eq. (8) is clearly seen in the figure. From a least-squares fit to the data based on Eq. (8) we obtain values of $V'=0.82$, $\sigma=30$ μm, and $(\omega_a-\omega_b)/2\pi=1.05\times10^{13}$ Hz. The beamsplitter used in this experiment had $R\simeq0.45$, $T\simeq0.55$ hence most of the reduction in visibility is due to misalignment of the apparatus (this is not an easy experiment to set up). The single photodetection rates in the detectors were all around 10^5 s^{-1} which given the maximum coincidence rate of about 500 s^{-1} implies effective detector efficiencies of order 0.5%.[19] The accidental coincidence rate of about 100 s^{-1} was calculated and subtracted from all coincidence rate measurements. The limited efficiencies are due primarily to optical losses and phase-matching limitations in the finite-depth down-conversion crystal. The detectors are silicon avalanche photodiodes operated in "Geiger" mode[20] with quantum efficiencies greater than 10%.[19]

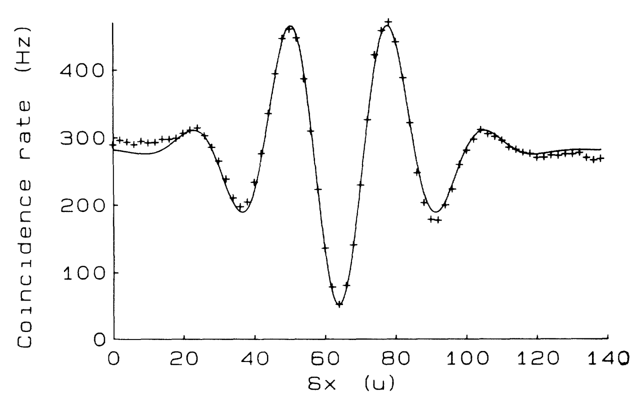

FIG. 3. Coincidence rate as a function of gross path-length difference δx between arms 1 and 2 of the apparatus. Experimental points + are fitted (solid line) by a function of the form given in Eq. (11) with phase angles ϕ_a,ϕ_b set to zero.

On sitting at the minimum position corresponding to $\delta x=0$ and tilting either phase plate one modulates the correlation coefficient E [Eq. (1)]. Phase plate thickness varies quadratically with tilt angle θ when the tilt (away from orthogonal to the beam θ_0) is small. Figure 4 shows a plot E versus the square of tilt angle of phase plate P_a showing the expected cosinusoidal variation. A least-squares fit to the data estimates the visibility $V'=0.78$ and provides a calibration of the phase plate allowing settings nominally corresponding to $\phi_a=0$, $\phi_a'=\pi/2$ to be chosen. A similar measurement made using phase plate P_b with $\phi_a=0$ fixed the phase delays $\phi_b=\pi/4$, $\phi_b'=3\pi/4$.

Having calibrated the angles several sets of measure-

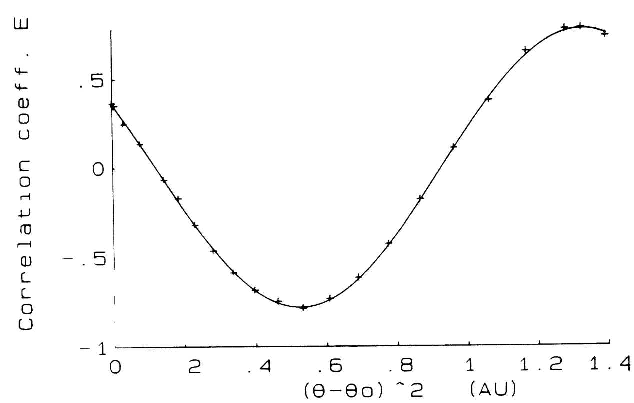

FIG. 4. Normalized correlation $E_{\text{expt}}(\phi_a,\phi_b)$ as a function of the square of phase plate tilt $\theta-\theta_0$ measured in arbitrary units. Experimental points + are fitted (solid line) by a function of the form given in Eq. (11) with δ_x set to zero. This acts as a calibration of the phase angle ϕ_a.

ments were made to eliminate S [Eq. (2)]. The base coincidence rate $\bar{C}_0$ was set to around 550 s^{-1} for the 30-s duration measurements, hence errors due to quantum noise were less than 1%. Detector efficiencies were equal to within 10% reducing any systematic errors from this source to less than 0.3%. To avoid biasing due to correlated drift (away from $\delta x=0$), pairs of measurement sets were taken reversing the order of the four phase settings in the second set. We obtain a value of

$$S=2.21\pm0.022$$

from five such measurement pairs. The error conservatively includes the small drift-induced variation seen on reversing the order of measurement. Clearly this violates Bell's inequality [Eq. (3)] by 10 standard deviations. Taking the average value $V'=0.80\pm0.02$ obtained from the measurements shown in Figs. 3 and 4 we would expect to measure $S=2.26\pm0.06$. The lower measured value reflects the uncertainties in the choice of phase angles and any misalignment occurring in the time (some hours) between the measurements shown in Figs. 3 and 4 and our measurements of S.

Although in this work the remote beams can be differentiated by color, this is not an absolute requirement in this type of experiment. The separated beams could, in principle, be obtained by use of partially reflecting beamsplitters,[7] although this would imply a reduction in the maximum attainable detection efficiency. A simpler apparatus avoiding this limitation has been suggested by Franson[9] where each photon of a conjugate pair is passed through a Mach-Zender interferometer. The coincidence rates measured in our experiment are an order of magnitude higher than previous work due to the angular correlation of the parametric down-conversion photon pairs and the high efficiency of the solid-state photon counting detectors in the near infrared. In principle, detection efficiencies approaching 100% could be achieved in this type of experiment and it may be possible to demonstrate a violation of the strong form of Bell's equation.[12,21]

In conclusion, we have demonstrated for the first time a violation of Bell's inequality based on phase and momentum, rather than spin.

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