Scaling Approach to Pinning: Charge-Density Waves and Giant Flux Creep in Superconductors

Thomas Nattermann[(a)]
Institut für Festkörperforschung, Kernforschungsanlage Jülich, D-5170 Jülich, Federal Republic of Germany
(Received 12 October 1989)

A general scaling approach to pinning and response in weakly disordered systems is developed that considers pinning at arbitrary high energy barriers. These are a consequence of a disordered $T=0$ renormalization-group fixed point which characterizes the condensed phase. Application of flux creep in superconductors yields a creep velocity $v(j) \propto \exp(-Cj^{-\mu})$, where μ is related to the roughness exponents ζ of the flux-line lattice. We argue that $\zeta=O(\log)$ and $\mu=(d-2)/2$ as for charge-density waves.

PACS numbers: 71.45.Lr, 74.60.Ge, 74.60.Jg, 75.50.Lk

The most prominent feature of condensed phases in solids is the rigidity of the order emerging below the condensation temperature T_c.[1] This rigidity can be decreased by the formation of topological defects of dimensionality D. Examples are domain walls (DW's) ($D=d-1$), flux lines (FL's) and dislocations ($D=1$), or hedgehog configurations ($D=0$). If there are competing interactions, defects may form lattices of their own as in the Shubnikov phase of type-II superconductors or in incommensurate magnetic structures. Other examples are mass-, spin-, or charge-density waves (CDW's) in the plane-wave limit. All these objects will collectively be referred to as "*defect structures*" (DS). The support of the DS is in the latter case the whole d-dimensional space ($D=d$).

For pure crystals the response to an external force $\mathbf{p}_{ex}$ which couples to the order parameter consists in the motion of the DS which transmits a local perturbation through the crystal. The defect motion can be slowed down by pinning at randomly distributed impurities.[2] In this paper we present a very general scaling approach to pinning and response in weakly disordered systems. In the first part we present the outline of the theory. The central quantity which describes the DS in a disordered medium is its *roughness*: For $D \leq 4$ and on sufficiently large length scales, $L \gg L_\Delta \propto n_i^{-1/(4-D)}$, fluctuation in the impurity density n_i adds up to overcome the elastic stiffness of DS. This leads to large-scale distortions $u \sim L^\zeta$ of the DS, $\zeta \geq 0$.[3] Different metastable configurations are separated by energy barriers of increasing height, $E_B \sim L^\psi$. Pinning forces are related to the slopes of these barriers. On small scales $L \ll L_\Delta$, there is no pinning and the response to p_{ex} can be expressed by a generalized susceptibility $\hat{\chi}$. In previous approaches to collective pinning[4-6] only a *single* energy barrier $T_\Delta \approx E_B(L_\Delta)$ has been considered, which leads to correct results if $T \approx 0$ and p_{ex} is close to the depinning threshold p_Δ. However, at finite temperatures this barrier can be jumped over and higher barriers become relevant.[7-10] For $T \ll T_\Delta$ this leads to a weak logarithmic time (or frequency) dependence of the pinning force and the susceptibility, with exponents related to ζ and ψ. A small driving force generates a creep motion of the DS whose velocity depends nonanalytically on p_{ex}. For $D \leq 2$ and $T_\Delta < T < T_c$ thermal fluctuations are already important on short time scales where p_Δ and $\hat{\chi}$ are renormalized by powers of T/T_Δ.

In the second part of this paper the results are applied to CDW's and type-II superconductors. In particular, we calculate the flux-creep velocity and the ac resistance of a FL lattice. The roughness of the lattice grows only logarithmically with the scale L, in strong contrast to previous results.[10] The application of our approach to single DW's or FL's will be published elsewhere.[11]

To be specific, we consider a DS whose deformations are described by an n-component distortion field $\mathbf{u}(\vec{x})$. $\mathbf{u}(\vec{x})$ depends on the coordinates of D-dimensional subspace of R^d, $D \leq d$. The Hamiltonian of the system is assumed to be

$$\mathcal{H}=\int d^D x\left[\tfrac{1}{2}\hat{\Gamma}(\nabla\cdot\mathbf{u})^2+V(\vec{x},\mathbf{u})-\mathbf{p}_{ex}\cdot\mathbf{u}\right], \qquad (1)$$

where $\hat{\Gamma}$ denotes an elastic stiffness constant and $V(\vec{x},\mathbf{u})$ includes the interaction with the disorder and is assumed to result from randomly distributed impurities of energy v. Equation (1) describes DW's and their lattices, as well as CDW's, if $n=1$ and FL's and their lattices if $n=d-1$. For CDW's u denotes a phase. In the following we will assume that $\mathbf{u}(\vec{x})$ is a single-valued function of $\vec{x}$; i.e., we neglect overhangs of DW's and FL's, and, probably more important, vortices in CDW's and dislocations in FL lattices, for the sake of simplicity. We come back to this point at the end of the paper.

First we consider the case where $\mathbf{p}_{ex}=0$. The central quantity is the *roughness* $w(L\vec{n})=\langle[u(\vec{x}+L\vec{n})-u(\vec{x})]^2\rangle^{1/2}$. Here $\langle\,\rangle$ denotes the average over thermal fluctuations and the randomness. There is a parameter range where the system has a *rough phase* such that for large L

$$w(L\vec{n}) \approx \xi_\perp[L(\vec{n})/L_\parallel]^\zeta, \quad 0 \leq \zeta < 1, \qquad (2)$$

with $L(\vec{n}) \approx L \gg L_\parallel$ and a positive roughness exponent ζ

(for $\zeta \geq 1$ the model is no longer well defined on large scales). $\xi_\perp$ and $L_\parallel$ set the scale for w and L. In particular, $L_\parallel$ plays the role of a correlation length in the case of CDW's[6] or for FL or DW lattices,[4,9] respectively. Throughout this paper we will assume only the case of *weak disorder* such that $L_\parallel \gg \xi_\perp$. Since the roughness results from a competition between the first two terms in (1), the spread ΔE of energies of the rough DS in a volume L^D is $\Delta E \approx \hat{\Gamma} L^{D-2} w^2(L)$ or[12]

$$\Delta E(L) \approx T^*(L/L_\parallel)^\chi, \quad T^* = \hat{\Gamma}\xi_\perp^2 L_\parallel^{D-2}, \qquad \chi = 2\zeta + D - 2. \tag{3}$$

The low-frequency dynamics of the system is governed by the free energy barriers E_B between different metastable states which are close in energy to the ground state. We make the natural conjecture that typical barriers scale as $E_B(L) \approx B(L/L_\parallel)^\psi$. As long as the distribution of $E_B(L)$ has no long power-law tail we can obtain the correct long-time behavior of our system by considering only typical barriers.[12] Let us consider two neighboring local minima, separated by an unstable maximum on the scale L. All three states are saddle points $\mathbf{u}_s(\vec{x})$ which fulfill the Euler equation corresponding to (1). With $\mathbf{u}_s(\vec{x})$ the absolute values of the elastic and the random potential term in (1) scale in the same way, but with a plus and minus sign of the random term in typical maxima and minima, respectively. Hence

$$E_B \approx 2\left|\int dx^d V(\vec{x}, \mathbf{u}_s)\right| \approx \Gamma \mathbf{u}_s^2 L^{D-2}.$$

With $0 \leq u_s(L) \leq AL$ we get $D-2 \leq \psi \leq D$. In the long-time limit, where we are close to equilibrium, $u_s(L) \approx w(L)$ and hence $\psi = \chi$ and $B \approx T^*$, which we will use in the following. Next we consider the conclusions which follow from the scaling *Ansatz* for $E_B(L)$.

The slope of the barriers $E_B(L)/w(L) \approx p(L)L^D$ determines a *pinning force density* $p(L) = \hat{\Gamma}\xi_\perp L_\parallel^{-\zeta} L^{\zeta-2}$. Since $\zeta < 1$, the largest pinning force density p^* results from the smallest scale, $L \simeq L_\parallel$, on which barriers exist. On smaller scales a weak external force $p_{\rm ex} \ll p^*$ leads to distortion $\mathbf{u}(\mathbf{p}_{\rm ex}) \approx \hat{\chi}(\mathbf{L})\mathbf{p}_{\rm ex}$ which follows from $\Gamma \mathbf{u}^2 L^{-2} \approx \mathbf{u}\mathbf{p}_{\rm ex}$, i.e., $\hat{\chi}(L) = \hat{\Gamma}^{-1}L^2$.

At *finite* temperatures energy barriers $E_B(L)$ are jumped over in a time $t(L) \simeq t_0 \exp[E_B(L)/T]$. t_0^{-1} is a microscopic attempt frequency. Thus, in a time t, barriers on scales $L < L_t \simeq L_\parallel[1 + (T/T^*)\ln(t/t_0)]^{1/\chi}$ are ineffective for pinning.[7-9] Using L_t in $p(L)$ and $\hat{\chi}(L)$ we get a time- (or frequency-) dependent pinning force density

$$p(t) \approx p^*[1 + (T/T^*)\ln(t/t_0)]^{(\zeta-2)/\chi}, \tag{4}$$

$p^* = \hat{\Gamma}\xi_\perp/L_\parallel^2$, and susceptibility

$$\hat{\chi}(\omega) \approx \hat{\chi}^*[1 + (T/T^*)\ln(1/\omega t_0)]^{2/\chi}, \tag{5}$$

where $\hat{\chi}^* = \xi_\perp/p^*$. For $p_{\rm ex} \ll p^*$, the DS is free on scales $L > L_p \approx L_\parallel(p^*/p_{\rm ex})^{1/(2-\zeta)}$ [compare $p(L)$]. Pinning on smaller scales can be overcome only by thermal jumping, which leads to a *creep* of the DS with a velocity $v \approx w(L_p)/t(L_p)$, i.e.,

$$v(p_{\rm ex}) \approx \frac{\xi_\perp}{t_0}\exp\left[-\frac{T^*}{T}\left(\frac{p^*}{p_{\rm ex}}\right)^{\chi/(2-\zeta)}\right] \tag{6}$$

and $p_{\rm ex} \ll p^*$. We now have to specify $L_\parallel$, $\xi_\perp$, ζ, and χ. In the trivial case of purely *thermal roughness* $w(L) \equiv w_T(L)$, where $V \equiv 0$, $\zeta \equiv \zeta_T = (2-D)/2$, $\chi = 0$, and $L_\parallel \equiv L_T = (\hat{\Gamma}\xi^2/T)^{1/(2-D)}$, which implies $T^* = T$, $p^* \approx T/\xi_\perp L_T^D$, and a Curie-like susceptibility $\chi^* \approx L_T^D \xi_\perp^2/T$. It is obvious to choose as $\xi_\perp$ the relevant correlation length of the problem. In most of the cases this is the widths ξ of the FL's or DW's, which diverge at the condensation temperature T_c.[13] Then $\hat{\Gamma}\xi^D \approx T_c$ and $L_\parallel \propto \xi$ since $\hat{\Gamma}$ scales like a free-energy density $\hat{\Gamma} \sim \xi^{-(2-\alpha)/\nu + (d-D)}$.

Next we consider the case of *quenched random impurities* at very low temperature $T \ll T_\Delta$. In order to find $L_\parallel \equiv L_\Delta$ we consider a deformation $u \approx \xi_\Delta \approx \xi$ on the scale $L \approx L_\Delta$. The elastic energy scales like $\hat{\Gamma}\xi^2 L_\Delta^{D-2}$, which has to be compared with the energy gain $-v[N(L_\Delta)]^{1/2}$ from the *fluctuation* of the impurity number $N(L_\Delta) = n_i \xi^{d-D} L_\Delta^D$ in the volume L_Δ^D. Thus, the DS is rough for $D \leq 4$ and $L_\Delta = \xi(\hat{\Gamma}\xi^D/\Delta)^{2/(4-D)}$, where $\Delta = v(n_i\xi^d)^{1/2}$ and we assume $N(L_\Delta) \gg 1$. Equations (3)-(5) give $T^* \approx T_\Delta = \hat{\Gamma}\xi^2 L_\Delta^{D-2}$, $p^* \approx p_\Delta = \hat{\Gamma}\xi L_\Delta^{-2}$, etc. T_Δ corresponds to the height of the smallest energy barriers in the system and characterizes the disordered system in a way similar to the way the Debye temperature characterizes a harmonic solid. In particular, the specific heat c_v at low T behaves as $c_v \approx T/T_\Delta$.[14] Close to T_c, $L_\Delta \propto \xi$ and $T_\Delta \propto \xi^\theta$, where $\theta \geq 0$ is the "violation of hyperscaling" exponent describing the irrelevance of temperature, e.g., in random-field systems.[15] The exponent $\zeta \equiv \zeta_\Delta$ for quenched randomness is only known in special cases.[3] ζ_Δ depends on D, n, and on the character of the impurities, i.e., whether they are of random-bond (RB) or random-field (RF) type. In a simple (Flory) argument one compares the scaling behavior of the elastic and the random energy on the same scale L.[11,16] Clearly the exponent ζ_F obtained in this way is, in general, not exact since $V(\vec{x}, \mathbf{u})$ gets renormalized from disorder fluctuations on smaller scales.

The results for $L_\parallel$, T^*, p^*, etc., are so far valid for $T \ll T_\Delta$. In the opposite case $T \gg T_\Delta$, $w(L)$ is given by $w_T(L)$ for $(L/L_T)^{\zeta_T} > (L/L_\Delta)^{\zeta_F}$ (there is no renormalization of v on these scales), i.e., for $L < L_\parallel = L_\Delta(T/T_\Delta)^{1/\phi}$, $\phi = 2(\zeta_F - \zeta_T)$. For $\phi > 0$ and $L > L_\parallel$, $w(L)$ has the form (2) with $\xi_\perp = w_T(L_\parallel)$. To get results which interpolate between low and high T we introduce the factor $\tilde{\Theta}(T) \approx (1 + T/T_\Delta)^{1/\phi}$. Then from (2) and (3) we get

$$L_\parallel = L_\Delta\tilde{\Theta}, \quad \xi_\perp = \xi_\Delta\tilde{\Theta}^{\zeta_F}, \quad T^* = T_\Delta\tilde{\Theta}^\phi, \tag{7}$$

which give $p^* = p_\Delta \tilde{\Theta}^{\zeta_F - 2}$ and $\hat{\chi}^* = \hat{\chi}_\Delta \tilde{\Theta}^2$. Thus, the pinning pressure is diminished and the susceptibility is enhanced considerably by thermal fluctuations, in particular, for small ϕ. The amplitudes of $w(L)$ are given here in the general case for the first time and decrease with temperature as $T^{-(\zeta-\zeta_F)}$ for $T > T_\Delta$. We note that for $D > 2$, where there is no thermal roughness, $\phi = 2\zeta_F$ and T_Δ has to be replaced by $\hat{\Gamma}\xi^D \approx T_c$ in the definition of $\tilde{\Theta}$. $\phi(D_c) = 0$ defines a critical dimensionality D_c. For $\phi < 0$ weak disorder is perturbatively irrelevant. This opens the possibility for a *depinning transition* at $T \simeq T_\Delta$ between a low- and a high-T phase with $\zeta = \zeta_\Delta$ ($> \zeta_T$) and $\zeta = \zeta_T$, respectively. If χ_Δ vanishes, the transition temperature goes to zero. Equations (3)-(7) are the basic results of our paper.

In the following we consider two examples.

(i) Charge-density waves $n=1$ and $D=d$.— Here u is a phase, thus $\xi \approx \pi$. $\hat{\Gamma} = v_F$ is the Fermi velocity and $V(\vec{x},u) = v\cos[u - \alpha(\vec{x})]$ with $\alpha(\vec{x})$ a random phase. $L_\Delta \approx (v_F/v n_i^{1/2})^{2/(4-d)}$ is the correlation length of the CDW, $p_\Delta \approx v_F/L_\Delta^2$, and $\hat{\chi}_\Delta \propto v_F^{-1} L_\Delta^{-2}$, in agreement with previous results.[6] A prediction for $w(L)$ at $T=0$ can be obtained from a Flory-type argument: In the first step we define a trial Hamiltonian $\mathcal{H}_0$ by replacing $V(\vec{x},u)$ in (1) by $\frac{1}{2} r(\vec{x}) u^2$. From a variational treatment for the free energy follows

$$r(\vec{x}) = \langle \partial^2 V(\vec{x},u)/\partial u^2 \rangle_0 = -v[\cos\alpha(x)] e^{-(1/2)\langle u^2 \rangle_0}.$$

$\langle\,\rangle_0$ denotes the thermal average with $\mathcal{H}_0$. For $T \to 0$, $u(\vec{x})$ takes the ground-state $u_G(\vec{x})$, i.e., $\langle u^2(\vec{x})\rangle_0 \approx u_G^2(\vec{x})$. On the scale L, $u_G(\vec{x}) \approx u(L)$, and hence the potential energy scales as $\Delta L^{d/2} e^{-u^2/2}$. Comparison with the elastic energy and subsequent summation over contribution on scales $L' \leq L$ yields $w(L) \propto (4-d) \times \ln(L/L_\Delta)$ for $d<4$, with L_Δ from (7).[16] Thus, $\zeta_\Delta = O(\log)$, $\chi_\Delta = \phi = d-2$, and the CDW is frozen in for $d > 2$. A small external field $p_{\mathrm{ex}} \propto E$ leads to a creep motion according to (6). For $d=2$ where thermal and disorder fluctuations predict a logarithmic roughness there is a depinning transition for the CDW at a transition temperature $T_c = 4\pi\hat{\Gamma}$ which is independent of Δ.[17] For $d<2$, $\zeta_\Delta = 0 < \zeta_T$ and hence there is only the high-T phase with small T_Δ for small disorder. The CDW is weakly pinned.[18]

(ii) FL lattices (FLL) in type-II superconductors.—[10] In this case our results apply if we choose $\hat{\Gamma} = \mu\gamma_A$, with the anisotropy constant $\gamma_A = (K/\mu)^{1/2}$, where K and μ are the tilt and shear modulus of the FLL. The bulk modulus is assumed to be infinite for simplicity.[4] $L(\vec{n}) = L(n_\perp^2 + \gamma_A^2 n_z^2)^{1/2}$, the definition of Δ obtains an additional factor $\gamma_A^{1/2}$ and $\xi_\perp \approx \xi$.[13] From (2)-(7) we get, in $d=3$, $L_\Delta \approx \mu^{3/2} K^{1/2} \xi^4 / n_i v^2$, $T_\Delta \approx \mu^2 K \xi^6 / v^2 n_i$, and the critical current $j_\Delta(T=0) \approx \mu\xi/L_\Delta^2 B$ in agreement with conventional approaches.[4,10] B denotes the magnetic field and $p_{\mathrm{ex}} = jB$. It is important to consider the discreteness of the FLL if one treats the interaction with the randomness $\sum_n \int dz\, V_0[\mathbf{y}_n^0 + \mathbf{u}_n(z), z]$. Here the sum is over all FL's with the reference position $\mathbf{y}_n^0$. We go over to the continuum using[19]

$$\sum_n f(n) = \int dy\, f(y) + 2\sum_{k=1}^{\infty} \int dy \cos(2\pi k y) f(y).$$

The first term on the right-hand side is the continuum approximation which gives no contribution since $V_0[\mathbf{y} + \mathbf{u}(\mathbf{y})]$ becomes independent of $\mathbf{u}$ on large scales. Taking only the $k=1$ part in the second term we get for FL's of distance l

$$V(\vec{x},\mathbf{u}) \approx \frac{2}{l^2} V_0(\vec{x}) \left[\sum_{\alpha=1}^{n} \cos\left(\frac{2\pi}{l}\right)[u_\alpha(\vec{x}) - x_\alpha] + \cdots \right], \tag{8}$$

where we used the transformation $\mathbf{y} + \mathbf{u}(\mathbf{y}) = \mathbf{x}$, $\vec{x} = (\mathbf{x}, z)$, and assumed a square lattice for simplicity. Thus, the FLL is in the universality class of CDW's giving only a logarithmic roughness $\zeta_\Delta = 0$ for $d<4$, in contrast to earlier results.[4,10] Hence, the FLL undergoes a depinning transition in $d=2$,[20] but not for $d>2$ where it forms a vortex glass with a $T=0$ fixed point, $\chi_\Delta = d-2 > 0$.[16,21]

According to (4), the critical current is reduced to $j_c(\omega) = j_\Delta[1 + (T/T_\Delta)\ln(1/\omega t_0)]^{-1/\mu}$ at finite temperatures, where $\mu = \chi/(2-\zeta) = (d-2)/2$. Note that $j_c(1/t) \sim \delta M(t)$ describes the decay of a metastable current (or magnetization).[20,22,23] From (6) we get for the *nonlinear* dc resistivity to a current $j \ll j_\Delta$,

$$\frac{\rho(j)}{\rho_\Delta} \approx \frac{v(j)}{v_0} \approx \exp\left[-\frac{T_\Delta}{T}\left(\frac{j_\Delta}{j}\right)^{\mu} \right], \tag{9}$$

$\rho_\Delta \approx B v_0 / j_\Delta$, i.e., the linear dc resistivity vanishes.[20] For an ac current (5) gives for the *linear* resistivity $\rho_l(\omega) \propto \rho_\Delta \omega t_0 [(T/T_\Delta)\ln(1/\omega t_0)]^{2/\chi}$, $\omega t_0 < \exp(-T_\Delta/T)$.

Since in a time t the FLL reaches equilibrium only on scales L_t [see below (3)], one has to expect memory effects at $t \approx 2t_1$, if one starts from a nonequilibrium state of higher FL density and increases the field after a waiting time t_1.[24]

The present scaling approach to pinning extends and unifies previous theories which are included as limiting cases. Its most important ingredient, the existence of energy barriers of arbitrary height, is a consequence of the existence of a disordered $T=0$ renormalization-group fixed point which describes the condensed phase. Defects like vortices or dislocations, which have been neglected here, occur at sufficiently large length scales even at low T and for weak disorder.[16,25] As long as they occur as bound pairs or closed loops, our conclusions are changed only quantitatively, e.g., by reducing the stiffness $\hat{\Gamma}$. Defect dissociation, however, may lead to new phases with finite barriers only.

I thank E. H. Brandt, G. Eilenberger, M. Feigel'man, K. Fischer, K. Kehr, W. Renz, and H. Ullmaier for useful discussions. This research was supported by a grant from the German Israeli Foundation for Scientific Research and Development.

---

[a]Permanent address: Fakultät für Physik und Astronomie, Ruhr-Universität, D-4630 Bochum, Federal Republic of Germany.

[1]P. W. Anderson, *Basic Notions in Condensed Matter Physics* (Benjamin, Melo Park, 1984).

[2]We neglect here pinning due to the underlying crystal lattice; i.e., we assume that FL's or DW's are sufficiently broad.

[3]For a review on roughness of isolated DW's or FL's, see, e.g., M. E. Fisher, J. Chem. Soc. Faraday Trans. **82**, 1569 (1986); T. Nattermann and J. Villain, Phase Trans. **11**, 5 (1988).

[4]For superconductors, see, e.g., M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975), and references therein; A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979).

[5]R. Bruinsma and G. Aeppli, Phys. Rev. Lett. **52**, 1547 (1984).

[6]H. Fukuyama and P. A. Lee, Phys. Rev. B **17**, 535 (1978); P. A. Lee and T. M. Rice, Phys. Rev. B **19**, 3970 (1979).

[7]M. V. Feigelman, Zh. Eksp. Teor. Fiz. **85**, 1851 (1983) [Sov. Phys. JETP **58**, 1076 (1983)]; J. Villain, Phys. Rev. Lett. **52**, 1543 (1984).

[8]D. A. Huse and C L. Henley, Phys. Rev. Lett. **54**, 2708 (1985).

[9]T. Nattermann, J. Phys. C **18**, 5683 (1985); **18**, 6681 (1985); Phys. Status Solidi (b) **129**, 153 (1985); T. Nattermann and I. Vilfan, Phys. Rev. Lett. **61**, 223 (1988).

[10]L. B. Joffe and V. M. Vinokur, J. Phys. C **20**, 6149 (1987); after the main results of the present paper had been derived, we obtained a paper by M. V. Feigelman *et al.* [Phys. Rev. Lett. **63**, 2303 (1989)] where similar ideas have been applied to type-II superconductors. In their approach $V(\vec{x},\mathbf{u})$ is assumed to be random in both arguments, which is only valid if the relevant distortions $\mathbf{u}$ are small compared to FL lattice spacing l, i.e., for not too small currents. Correspondingly, their $\mu \approx \frac{7}{9}$ is larger than ours, see also Ref. 20.

[11]T. Nattermann, Y. Shapir, and I. Vilfan (to be published); T. Nattermann and W. Renz (to be published).

[12]In the renormalization-group framework $\chi = 2\zeta + D - 2 > 0$ indicates that the large-scale behavior is controlled by a $T=0$ fixed point. The height of the typical barriers and hence ψ depends on which regime of the relaxation process is considered. We are here interested only in its late stage which is dominated by transition between states close in energy to the ground state. In the terminology of R. B. Littlewood and R. Rammal, Phys. Rev. B **38**, 2675 (1988), we consider regime IV of the relaxation process.

[13]If we consider FL or DW lattices close to transition, where its lattice spacing l diverges, it is more convenient to choose $\xi_\perp \approx l$. L_Δ remains valid if we include an appropriate factor of l/ξ in the definition of Δ.

[14]P. W. Anderson, B. I. Halperin, and C. M. Varma, Philos. Mag. **25**, 1 (1972).

[15]J. Villain, J. Phys. (Paris) **46**, 1843 (1985); D. S. Fisher, Phys. Rev. Lett. **56**, 416 (1986).

[16]This result is in agreement with more detailed renormalization-group calculations; see, e.g., J. Villain and J. F. Fernandez, Z. Phys. B **54**, 139 (1984), and references therein, who find, for $2 \le d < 4$, $\zeta = 0$ and $\zeta = \frac{1}{2}$ for $d=1$. For $d=1$ correlations are lost for $L > L_\Delta$ and a random-walk argument gives $\zeta = \frac{1}{2}$, in agreement with a recent numerical study by P. B. Littlewood, Phys. Rev. B **33**, 6694 (1988).

[17]Note that at the thermal fixed point, the random potential scales as $\Delta L^{d/2} \exp(-w\hat{f}/2)$, which gives $\Delta L^{1-T/4\pi\hat{\Gamma}}$ in $d=2$.

[18]M. V. Feigelman, Zh. Eksp. Teor. Fiz. **79**, 1095 (1980) [Sov. Phys. JETP **52**, 555 (1980)].

[19]P. Bak and V. L. Pokrovsky, Phys. Rev. Lett. **47**, 958 (1981).

[20]This is in agreement with a (scaling) result of M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989), but there the exponent μ remains undetermined. Recently, Eq. (9) has been proved experimentally in the low-T behavior of Y-Ba-Cu-O, with $\mu \approx 0.4 \pm 0.2$, in good agreement with our result $\mu = \frac{1}{2}$; see R. H. Koch *et al.*, Phys. Rev. Lett. **63**, 1511 (1989).

[21]For increasing T and Δ terms with $k > 1$ have to be considered in the continuum version of the random potential which may lead to a depinning transition even for $d > 2$.

[22]Expansion of our results in cases where $j_c, j_{ex} \approx j_\Delta$, i.e., for low T or short times, reproduces the results of the conventional theory (Ref. 4).

[23]Note that $-d\ln M(t)/d\ln t \approx \mu^{-1} T [T_\Delta + T \ln(t/t_0)]^{-1}$ remains finite, in contrast to approaches which use a single energy barrier only; see C. W. Hagen and R. Griesen, Phys. Rev. Lett. **62**, 2857 (1989).

[24]C. Rossel, Y. Maeno, and I. Morgenstern, Phys. Rev. Lett. **62**, 681 (1989).

[25]I thank D. A. Huse for a corresponding remark.