## Normal-State Properties of the Uniform Resonating-Valence-Bond State

Naoto Nagaosa<sup>(a)</sup> and Patrick A. Lee

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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We study a version of the uniform resonating-valence-bond state in which fermions and spinless bosons are coupled by a gauge field. We show that above the Bose-Einstein temperature, the boson inverse lifetime due to scattering by the gauge field is of order kT, which suppresses the condensation temperature and leads to a linear T resistivity. The Hall number is proportional to the hole density and temperature dependent. The single-particle spectral weight exhibits a continuum plus a broadened peak. The "Fermi-surface" area satisfies Luttinger's theorem. Comparison with the anomalous normal-state properties of oxide superconductors is made.

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In the three years since the discovery of the copperoxide superconductors, a large body of experiments has been accumulated which begins to paint a rather clear picture of what the solution of this puzzle has to look like. The normal state exhibits a number of anomalous properties which suggest that it is not an ordinary Fermi liquid.<sup>2</sup> The resistivity is linear in T. The Hall number is positive and roughly proportional to x, the number of doped holes per copper. The optical conductivity  $\sigma(\omega)$ shows a Drude peak with spectral weight  $\approx x/m$ , where  $m \approx 2m_e$ . Neutron, muon-spin-rotation, and NMR experiments reveal  $S = \frac{1}{2}$  local moments on the copper sites with decreasing antiferromagnetic (AF) correlation as x increases.<sup>3</sup> These observations suggest a Mott-Hubbard insulator at x = 0, so that doping introduces a low density of carriers in a Hubbard band. Yet angularresolved photoemission data<sup>4</sup> indicate the existence of a Fermi surface, with an area consistent with band calculation and therefore Luttinger's theorem; i.e., the local moments should be counted as part of the Fermi-surface area which is proportional to 1-x. Taken together, these observations place severe restrictions on the theory. Much of the theoretical efforts have focused on strongly correlated models such as large-U Hubbard models or t-J models. A number of ground states that exhibit shortrange AF order have been suggested, but it has proven to be very difficult to decide which state is favored for finite doping. In this paper we set a less ambitious goal, and instead ask the question: Which of the suggested states exhibits normal-state properties consistent with experiments?

The photoemission experiment leads us to consider a resonating-valence-bond (RVB) state with a spinon Fermi surface<sup>2,5</sup> which obeys Luttinger's theorem. Later refinements, such as the flux phase, typically have a pointlike Fermi surface which acquires an area proportional to x upon doping. These states are believed to be favorable for small doping, but it is difficult to see how a Luttinger Fermi surface can emerge. We assume that the doping level of interest is sufficiently large to stabilize the uniform RVB state. The physical implication of this kind of state was studied earlier,<sup>6</sup> but the coupling to

gauge field was not considered. The gauge field was discussed in detail by Ioffe and Larkin,<sup>7</sup> and, more recently, Grilli and Kotliar<sup>8</sup> derived similar results using a slaveboson formulation of the large-N t-J model. We follow this formalism and decompose the electron operator  $c_{i\sigma}$  into  $f_{i\sigma}b_i^{\dagger}$ , subject to the constraint  $b_i^{\dagger}b_i + \sum_{\sigma} f_{i\sigma}^{\dagger}f_{i\sigma} = 1$ . The mean-field order parameter is  $\chi_{ij} = \langle f_{i\sigma}^{\dagger}f_{j\sigma} \rangle$  and is different from the Baskaran-Zou-Anderson (BZA) state<sup>5</sup> for  $x \neq 0$ . The low-energy behavior can be described by the effective Lagrangian<sup>7</sup>  $L = L_B + L_F$ , where

$$L_{F} = \sum_{i,\sigma} f_{i\sigma}^{\dagger}(\partial_{\tau} - a_{0} - \mu_{F}) f_{i\sigma} + \sum_{\langle ij \rangle,\sigma} J e^{ia_{ij}} f_{i\sigma}^{\dagger} f_{j\sigma} + \text{c.c.} ,$$

$$L_{B} = \sum_{i} b_{i}^{\dagger}(\partial_{\tau} - a_{0} - \mu_{B}) b_{i} + \sum_{\langle ii \rangle} \tilde{\iota} e^{ia_{ij}} b_{i}^{\dagger} b_{j} + \text{c.c.} ,$$
(1)

and  $a_{ij}$  is the gauge field associated with the phase of  $\chi_{ij}$ . The chemical potentials  $\mu_F, \mu_B$  enforce  $\langle b_i^{\dagger} b_i \rangle = x$  and  $\sum_{\sigma} \langle f_{i\sigma}^{\dagger} f_{i\sigma} \rangle = 1 - x$ . In the large-N mean-field theory  $\tilde{t} = t$ , and b exhibits Bose-Einstein (BE) condensation below a characteristic temperature  $T_{BE}^{(0)} \approx 4\pi \tilde{t}x$ . The ground state is then a Fermi liquid and the theory resembles the heavy-fermion theory except that the bandwidth is given by  $8J.^8$  In this paper we analyze Eq. (1) with N=2 above the BE temperature. As explained later, we intend to identify the BE temperature with the superconducting  $T_c$ . However,  $T_{BE}^{(0)}$  is very large and we must discuss how this temperature scale can be reduced. Because of strong coupling to spinon excitations, we expect a reduction from  $\tilde{t} = t$  to  $\tilde{t} = J^{9}$ . However,  $T_{BE}^{(0)}$  is still of order 1000 K. In this paper we shall argue that inelastic scattering of the bosons by the gauge field brings  $T_c$  $=T_{BE}$  down to the observed range of 100 K. It is in this sense that we expect the effective Lagrangian Eq. (1) above the BE temperature to describe the normal-state properties.

We treat Eq. (1) in the continuum limit and introduce  $a_{\mu}(\mathbf{r})$  such that  $a_{ij} = (r_{i\mu} - r_{j\mu})a_{\mu}((\mathbf{r}_i + \mathbf{r}_j)/2)$  and the boson Hamiltonian takes the familiar form  $(2m)^{-1}b^{\dagger} \times (i\partial_{\mu} - a_{\mu})^2 b$ . The gauge-field propagator  $D_{\mu\nu}(\mathbf{r}, \tau) = \langle T_{\tau}a_{\mu}(\mathbf{r}, \tau)a_{\nu}^{\dagger}(0) \rangle$  is obtained by integrating out quadratic fermion and boson fluctuations,  $^{7}D_{\mu\nu} = (\Pi_{JJ}^{F} + \Pi_{JJ}^{B})^{-1}$ , where  $\Pi_{JJ}^{B(F)}$  is the boson (fermion) current-

current correlation function. The important low-lying excitation comes from the transverse part  $(\delta_{\mu\nu} - q_{\mu}q_{\nu}/q^2)D^T$ , where  $D^T(q,\omega_n) \approx (|\omega_n|/q + \chi_d q^2)^{-1}$  which is valid for  $|\omega_n| < q$  and has the familiar form of the transverse response function in metals.  $\chi_d = \chi_F + \chi_B$  is the sum of the Landau diamagnetic susceptibility for fermions and bosons; i.e.,  $\chi_F \approx (1-x)/m_F$  and for  $T > T_{\text{BE}}$ ,  $\chi_B \approx 2\pi x \tilde{t}^2/T \ll \chi_F$ . However, below  $T_{\text{BE}}$ ,  $\chi_B q^2$  must be replaced by  $x\tilde{t}$  due to the Anderson-Higgs mechanism.<sup>8</sup> This is why our results, which rely on the existence of the soft overdamped mode  $\omega \approx i\chi_d q^3$ , are limited to  $T > T_{\text{BE}}$ .

The model is invariant under the local gauge transformation  $f_{\sigma} \rightarrow f_{\sigma} e^{i\theta(\mathbf{r},\tau)}$ ,  $b \rightarrow b e^{i\theta(\mathbf{r},\tau)}$ , and  $a_{\mu} \rightarrow a_{\mu} + \partial_{\mu}\theta$ . We emphasize that coupling to the gauge field serves to enforce the local constraint  $b_i^{\dagger}b_i + \sum_{\sigma} f_{i\sigma}^{\dagger}f_{i\sigma} = 1$ . Note that b and f are not by themselves gauge invariant, and should not be interpreted as physical excitations such as holons and spinons. For example,  $G_B = -\langle T_{\tau}b(\mathbf{r},\tau) \rangle \times b^{\dagger}(0) \rangle$  and  $G_F = -\langle T_{\tau}f_{\sigma}(\mathbf{r},\tau)f_{\sigma}^{\dagger}(0) \rangle$  are not gauge invariant. The gauge-invariant objects we are interested in are

$$\Pi_B = \langle T_\tau b^{\dagger}(\mathbf{r},\tau)b(\mathbf{r},\tau)b^{\dagger}(0)b(0)\rangle$$

 $\Pi_F = \langle T_\tau f_\sigma^{\dagger}(\mathbf{r},\tau) f_\sigma(\mathbf{r},\tau) f_\sigma^{\dagger}(0) f_\sigma(0) \rangle,$ 

and the physical electron Green's function

$$G_{\sigma}(\mathbf{r},\tau) = -\langle T_{\tau}f_{\sigma}(\mathbf{r},\tau)b^{\dagger}(\mathbf{r},\tau)f_{\sigma}^{\dagger}(0)b(0)\rangle$$

We first consider  $\Pi_{B}$ .<sup>10</sup> For  $T > T_{BE}$ , the transport rate  $\tau_{tr}^{-1}$  of a Boltzmann gas due to the scattering by a fluctuating transverse gauge field is given by the thermal average of  $\tau_{k}^{-1}$ , where

$$\tau_{\mathbf{k}}^{-1} = \int d\mathbf{q} \, d\omega (q/k)^2 (\hat{\mathbf{q}} \times \mathbf{k}/m)^2 (e^{\beta \omega} - 1)^{-1} \\ \times \mathrm{Im} D^T (\mathbf{q}, \omega) \delta(\omega_k - \omega_{k+q} + \omega)$$
(2)

and  $\omega_k = k^{2}/2m$ . Noting that important contributions come from low-frequency, gauge-field fluctuations  $\omega \approx \chi_d q^3$  and  $q < \lambda_T^{-1} \equiv (Tm/2\pi)^{1/2}$ , i.e.,  $\omega < (Tm)^{3/2} \times m_F^{-1}$ , Eq. (2) is evaluated to give  $\tau_{tr}^{-1} \approx k_B T \times (\chi_d m)^{-1}$ . A previous calculation <sup>6,11</sup> of the boson lifetime due to scattering by a Fermi particle-hole-pair excitation yields  $\tau_{tr}^{-1} \propto T^{3/2}$  and we conclude that the gauge-field contributions dominate. Our result can also be obtained diagrammatically, but it is essential to keep both self-energy and vertex corrections. The self-energy itself is without the factor  $q^2/k^2$  in Eq. (2), and turns out to be infrared divergent.  $G_B$  is not gauge invariant and therefore its self-energy does not have physical meaning. To emphasize this point, we employ a spacetime Feynman-path formulation. In a gauge field, each path  $\mathbf{r}_1(\tau_1)$  is weighted by  $e^{i\Phi}$ , where

$$\Phi[\mathbf{r}_1] = \int_0^1 d\tau_1 [\mathbf{a}(\mathbf{r}_1(\tau_1), \tau_1) \cdot \dot{\mathbf{r}}_1(\tau_1) + a_0(\mathbf{r}_1(\tau_1), \tau_1)].$$

Since the important contributions come from  $\omega < (Tm)^{3/2}m_F^{-1} \ll T$ , it is a good approximation to consider a static but spatially varying "magnetic" field  $h = \nabla \times \mathbf{a}$ .  $\Pi_B(\mathbf{r}, \tau)$  is given by the sum over all Feynman

paths, the projection of which onto real space is shown in Fig. 1(a), multiplied by  $e^{i\phi}$ , where  $\phi = \oint a \cdot dl$  is the flux through the area A enclosed by the closed loop in Fig. 1(a), and averaged over **a**. To estimate **h** we note that

$$\langle |h_q|^2 \rangle = \int d\omega (e^{\beta\omega} - 1)^{-1} q^2 \operatorname{Im} D^T(q, \omega) \approx T/\chi_d$$

for  $q < q_0$ , where  $q_0 \approx (T/\chi_d)^{1/3}$ . Thus we envision a spatially random h which varies on a scale of  $q_0^{-1}$ with  $\langle h^2 \rangle \approx q_0^2 T/\chi_d$ . We can write  $\langle e^{i\phi} \rangle = \exp(-\frac{1}{2} \langle \phi^2 \rangle)$ and  $\langle \phi^2 \rangle = N \langle \phi_i^2 \rangle$ , where  $N = Aq_0^{-2}$  and  $\langle \phi_i^2 \rangle = \langle h^2 \rangle q_0^{-4}$ . Thus we conclude that  $\Pi_B = \Pi_B^0 \langle e^{i\phi} \rangle$ , where  $\Pi_B^0 = \tau^{-2}$  $\times \exp(-2mr^2/\tau)$  is the noninteracting polarizability and  $\langle e^{i\phi} \rangle \approx \exp[-(T/\chi_d)r(\tau/m)^{1/2}]$ ; we have set  $A \approx r(\tau/m)^{1/2}$ . For  $T > T_{\text{BE}}$ ,  $\chi_d \approx \chi_F$  is a constant. Noting that the typical  $r \approx (\tau/m)^{1/2}$ , the exponential decay can be interpreted as a mean free path of  $T^{-1/2}$  or a transport time  $\tau_{\text{tr}} \sim T^{-1}\chi_d m$ , in agreement with Eq. (2).

We note that  $T_{BE}^{(0)}$  occurs when the de Broglie wavelength  $\lambda_T$  becomes comparable to the average particle spacing. The incoherent effects introduce an inelastic mean free path  $(T/m)^{-1/2}$  which is comparable to  $\lambda_T$ and we expect a reduction in  $T_{BE}$ . This can be described by introducing a gauge-invariant propagator  $\tilde{G}_B(\mathbf{r},\tau)$  $=\langle G_B(\mathbf{r},\tau, \{\mathbf{a}\})\exp(i\Phi[\mathbf{r}_0])\rangle$ , where  $\mathbf{r}_0(\tau_1)=\mathbf{r}(\tau_1/\tau)$  is the straight-line path in space time. By considering a path-integral representation of  $\tilde{G}_B(\mathbf{r},0)$  we obtain in the Boltzmann regime  $\tilde{G}_B(\mathbf{r},0) = \tilde{G}_B(\mathbf{r},\beta) \sim x \exp[(-\lambda_T^{-2} + T/\chi_d)r^2]$ , so that the correlation length  $\xi = (\lambda_T^{-2} + T/\chi_d)^{-1/2}$  is reduced from  $\lambda_T$  by inelastic scattering. Since  $\chi_B$  can be thought of as fluctuating droplets of perfect diamagnets with radius  $\xi$ , we estimate  $\chi_B \approx \tilde{t}x\xi^2$  $\equiv \tilde{t}T_{BE}/T$ , where

$$T_{\rm BE} = T_{\rm BE}^{(0)} (\xi/\lambda_T)^2 \approx (1 + m\chi_d^{-1})^{-1} T_{\rm BE}^{(0)}$$

For  $T < T_{BE}$ ,  $\chi_d = \chi_F + \chi_B$  becomes dominated by  $\chi_B$  and  $\xi$  can be shown to grow exponentially so that we may interpret  $T_{BE}$  as the reduced crossover temperature.

Next we consider  $\Pi_F$ . The analogous problem of a



FIG. 1. Typical Feynman paths, projected onto the twodimensional plane, which contribute to (a) the boson polarization  $\Pi_B$ , (b) the fermion polarization  $\Pi_F$ , and (c) the electron Green's function  $G_{\sigma}$ . Dashed and solid lines refer to boson and fermion paths. The circle with radius  $q_0^{-1}$  represents the scale of the fluctuating gauge-field flux.

transverse electromagnetic field coupled to electrons has been considered by Reizer.<sup>12</sup> Its extension<sup>13</sup> to compute the transport time due to staggered gauge field produced  $\tau_F^{-1} \sim (T/\chi_d)^{4/3} E_F$  in 2D. We find the same result for the present problem. We see in Fig. 1(b) that the important Feynman paths are restricted to a tube of radius  $k_F^{-1}$  around the classical straight-line path so that  $N = rq_0$ ,  $\langle \phi_i^2 \rangle = \langle h^2 \rangle (q_0 k_F)^{-2} = T k_F^{-2} / \chi_d$ , and  $\langle e^{i\theta} \rangle$  $\sim \exp[-(T/\chi_d)^{4/3} k_F r]$  leading to a mean free path in agreement with  $v_F \tau_F$  obtained by diagrams. Just as for bosons, the self-energy is infrared divergent and it is crucial to consider gauge-invariant quantities.

Next we consider the electron spectral function  $\operatorname{Im} G_{\sigma}(\mathbf{k}, \Omega)$ . The zeroth-order calculation<sup>6</sup> is just a convolution of the noninteracting  $G_F$  and  $G_B$ . We find a continuum background which begins at  $\Omega = -|\mu_B| - (|\mathbf{k}| - k_F)^2/2m_F$  and extends to -4J with an area of (1-x)/2, where  $\mu_B \approx -k_B T \ln(mT/2\pi x)$ . In addition, there is a peak centered at  $\Omega = |\mathbf{k}|^2/2m_F - \mu_F$  with a width equal to  $(Tm)^{1/2}m_F^{-1}$  and an area of x. This peak sharpens to become the quasiparticle  $\delta$  function below  $T_{\text{BE}}$ . The origin of the width is that the momentum of the fermion is broadened by the typical boson momentum  $\sim (Tm)^{1/2}$ . We note that the total area is (1+x)/2 instead of unity. This is because some spectral weight has been pushed to infinity as  $U \rightarrow \infty$ .

We next ask how this picture is modified by the coupling to the gauge field. We divide the gauge field into two regimes,  $\omega < T$  and  $\omega > T$ . For  $\omega < T$  we make the quasistatic approximation as before the compute  $G_{\sigma}$  using the path-integral method in an explicitly gaugeinvariant manner. As shown in Fig. 1(c), the boson path exhibits a random walk while the fermion path is restricted to nearly a straight line (the Gorkov approximation). We obtain  $G = G_0 \langle e^{i\phi} \rangle \approx G_0 \exp[-(T/\chi_d)r(\tau/t)]$ m)<sup>1/2</sup>]. Near the quasiparticle peak, we estimate the lifetime by replacing  $r = v_F \tau$  in the exponent, and we obtain  $\tau_{in}^{-1} \approx (T/\chi_d)^{2/3} (mm_F^2)^{-1/3}$ . This is less than the  $T^{1/2}$  width due to momentum broadening and is therefore negligible. For  $\omega > T$  we have to use diagrammatic methods. The dominant contribution is from the selfenergy correction to the fermion, which is  $J(\Omega/\chi_d)^{2/3}$ . We check that vertex corrections give rise to only logarithmic corrections. The fermion and boson have very different velocities and sample different frequency domains of the fluctuating gauge fields so that the cancellation between self-energy and vertex in the quasistatic limit no longer applies.

To summarize,  $\text{Im}G_{\sigma}(\mathbf{k}, \Omega)$  consists of a continuum for  $\Omega < -|\mu_B| - (|\mathbf{k}| - k_F)^2/2m_F$  and a peak at  $\Omega = |\mathbf{k}|^2/2m_F - \mu_F$ . The peak is severely broadened with a width equal to  $\max[\Omega^{2/3}J^{1/3}, (Tm)^{1/2}m_F^{-1}]$ , which leads to asymmetric line shape with high-energy tail  $\sim \Omega^{-2/3}$ . Thus the Landau criterion that the quasiparticle width should be less than its energy is violated. On the other hand, the location of the peak in **k** space is determined by the "spinon" Fermi surface which satisfies Luttinger's theorem. The dispersion of this peak is characterized by a bandwidth of 8J. All these features are consistent with the photoemission data. According to our view the observed continuum "background" is intrinsic and in fact contains the bulk of the spectral weight. Another interesting prediction is that for  $|\mathbf{k}| > k_F$ , when the peak has moved through the Fermi surface, the continuum remains with a threshold which recedes from the Fermi energy as  $|\mathbf{k}| - k_F$  increases. This feature may already have been observed<sup>4</sup> but further studies will be helpful. We also predict that the intrinsic continuum background is much reduced for inverse photoemission (bremsstrahlung isochromat spectroscopy) or for photoemission in electron-doped materials.

The tunneling density of states is readily obtained using  $\Gamma(\Omega) = \int d\mathbf{k} \operatorname{Im} G_{\sigma}(\mathbf{k}, \Omega)$  and we find that  $\Gamma(\Omega) \approx x + |\Omega|/J$  for  $\Omega < 0,^{6}$  but contrary to Ref. 6,  $\Gamma(\Omega) \approx x$  for  $\Omega > 0$ . The asymmetry between particle and hole in tunneling and in photoemission is not surprising in the large-U model. An added electron can only enter a vacant site, so that for  $\Omega > 0$ ,  $\operatorname{Im} G_{\sigma}$  and  $\Gamma$  are proportional to x. On the other hand, it is always possible to remove an electron so that  $\Gamma$  is of order unity for  $\Omega < 0$ .

We are now ready to discuss various transport properties. We begin with the conductivity where the A field couples to  $c_{\sigma} = f_{\sigma} b^{\dagger}$ . We can couple A to  $f_{\sigma}$  or to  $b^{\dagger}$ with the standard minimal coupling, but not to both. We choose the former. Ioffe and Larkin<sup>7</sup> showed by integrating out fluctuations in the a field that the resistivity is the sum of the boson and fermion resistivity. In our case  $\sigma_B \sim x e^2 T^{-1} / m \ll \sigma_F \sim T^{4/3}$  so that  $\sigma \sim \sigma_B$ , thus explaining the long-standing puzzle of the linear resistivity. Below  $T_{BE}$ ,  $\sigma_B$  diverges and we recover the Fermi-liquid result. We have generalized this composition rule to other transport properties using a diagrammatic technique.<sup>8</sup> Here we outline the physical basis of these results. The local number conservation requires the fermion current  $\mathbf{J}_F$  to be opposed by a boson backflow  $J_B$ . Since  $J_F = \sigma_F(\mathbf{E} + \boldsymbol{\epsilon})$  and  $J_B = \sigma_B \boldsymbol{\epsilon}$ , where  $\epsilon = -\nabla a_0 - \dot{a}$ , the constraint  $J_F + J_B = 0$  produces a to partially screen out **A**. Since the physical current is  $J_F = -J_B$ , we obtain  $\sigma^{-1} = \sigma_F^{-1} + \sigma_B^{-1}$ . For the Hall effect, the magnetic field produces a fermion diamagnetic current which must be canceled by a boson current and we obtain  $R_H = (R_H^F \chi_B + R_H^B \chi_F)/(\chi_B + \chi_F)$ . For  $T < T_{BE}, \chi_B \rightarrow \infty$  and we have the Fermi-liquid result  $R_H = R_H^F = -(1-x)^{-1}$ . However, for  $T > T_{BE}$ , using  $\chi_B \sim T_{BE}/mT$ ,  $\chi_F \sim (1-x)/m_F$ , and  $R_H^B \sim x^{-1}$ , we conclude that the boson  $R_{H}^{B}$  dominates so that upon expanding for  $T > T_{BE}$ , we obtain  $R_H \approx x^{-1} - \gamma J/T$ , where  $\gamma \approx 1$ . Note that there is a temperature-dependent correction term which is independent of x, and leads to an enhancement of the Hall number near  $T_{BE}$ . Experimentally, the Hall number is typically a factor of 2 larger than the hole density determined by chemical means. Unfortunately, the temperature dependence goes in the opposite direction from our prediction. Nevertheless, it will be interesting to measure  $R_H$  at very high temperatures such as 600 K, where the resistivity is known to be linear and see if  $R_H$  will go towards  $x^{-1}$ .

Similar considerations lead us to conclude that the thermopower  $S = S_F + S_B$  and the thermoconductivity  $K = K_F + K_B$ . Recall that  $S_F \sim -(k_B/e)k_BT/E_F$  and  $S_B \sim (k_B/e)[1 - \ln(2\pi x/mk_BT)]$ ; we expect  $S_B$  to dominate and S to be near  $k_B/e$ . Experimentally S for  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  is indeed large and positive and independent of magnetic field, as we would predict. We expect K to be dominated by  $K_F$ , so that the Wiedemann-Franz law will not be obeyed. Experimentally, K seems to be complicated by phonon effects.

Spin fluctuations couple only to  $f_{\sigma}$  and no composition law is required. With a spin Fermi surface we expect a Pauli spin susceptibility  $\chi_{spin}$ , as observed in the Bi compounds and O<sub>7</sub> Y-Ba-Cu-O. The spin-spin correlation will have a very short correlation length, but will be incommensurate due to nesting of the f Fermi surface when  $x \neq 0$ . The NMR relaxation rate  $1/T_1$  will be enhanced and we may expect  $1/T_1T \approx J^{-1}$  on the copper site. Whether this enhancement will be reduced on the oxygen site due to form-factor effects requires a quantitative analysis which we have not undertaken.

We conclude by outlining a scenario whereby  $T_{BE}$  can be identified with the superconducting  $T_c$  rather than the onset of a Fermi-liquid state. At half filling the order parameter  $D_{ij} = \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \rangle$  is equivalent to  $\chi_{ij}$  by SU(2) symmetry. For  $x \neq 0$  at the mean-field level, it is possible that at a temperature  $T_D^{(0)}$  below the onset of  $\chi_{ii}$ ,  $D_{ii}$  becomes nonzero. However, the gauge field produces random magnetic fluxes and is pair breaking for  $D_{ii}$ , so that we expect a significant suppression of the transition to  $T_D < T_D^{(0)}$ . As long as both  $T_D$  and  $T_{BE}$  are below min $(T_D^{(0)}, T_{BE}^{(0)})$ ,  $\langle b \rangle$  and  $D_{ij}$  will become nonzero simultaneously below  $T_c = \max(T_D, T_{BE})$ , so that the gauge-invariant (under a) superconducting order parameter  $\Delta_{ij} = \langle c_{i\uparrow} c_{j\downarrow} - c_{i\downarrow} c_{j\uparrow} \rangle \approx \langle b^{\dagger} \rangle^2 D_{ij}$  becomes nonzero. This is because the diamagnetic response  $\chi_F$  or  $\chi_B$ diverges below  $T_D$  or  $T_{BE}$ , respectively, which in turn stiffens up the gauge field that was responsible for the suppression of  $T_{BE}$  or  $T_D$  in the first place. More generally, we believe that the short inelastic lifetime due to scattering by the gauge field plays an important role in suppressing  $T_c$ , so that we expect a large  $2\Delta_0/kT_c$  ratio and a rapid growth of the gap just below  $T_c$ .

To conclude, we find that holes are strongly scattered by a fluctuating gauge field which corresponds physically to fluctuations in the chirality parameter  $\langle S_1 \cdot S_2 \times S_3 \rangle$ .<sup>14</sup> Superconductivity coincides with the onset of coherence among the holes. We have found at least one model which explains qualitatively most of the anomalous features of the oxide superconductors. However, since the strong scattering by gauge field seems quite generic to RVB models, the uniqueness of this model is not known at this point.

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<sup>(a)</sup>Permanent address: Department of Applied Physics, University of Tokyo, Tokyo 113, Japan.

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