Comment on "Theory of Electronic Diamagnetism in Two-Dimensional Lattices"

In a recent Letter, Hasegawa *et al.*[1] consider the kinetic energy of noninteracting spinless electrons at fixed density in two dimensions (see also Ref. 2). The energy in zero magnetic field is equal to that in a nonzero field that fills an integral number of Landau levels. Hasegawa *et al.* assert that any periodic potential lifts this degeneracy in favor of the nonzero field that fills one Landau level, and support this assertion with numerical calculations on three lattices. In this Comment we show that for many weak potentials, the lowest-energy state is in fact at zero magnetic field. Other potentials favor one or more than one filled Landau levels. This problem is of interest in the mean-field theory of the t-J model.[3,4]

We calculate the leading response to a weak potential $\lambda V(\mathbf{r})$, which need not be periodic. A slowly varying potential clearly favors zero field, because a filled Landau level is "incompressible" and cannot adjust its density to take advantage of regions of low $\lambda V(\mathbf{r})$. To order λ^2 each Fourier component acts independently. A potential $\lambda V(x) = \lambda \cos(kx)$ shifts the energy in zero field by[5]

$$\Delta E_0 = -\frac{\lambda^2 m}{8\pi\hbar^2}\left\{1-\theta(\tilde{k}-2)\left[1-\left(\frac{2}{\tilde{k}}\right)^2\right]^{1/2}\right\},$$

where $\tilde{k} \equiv k/q_F$ and q_F is the Fermi wave vector. From perturbation theory, the shift in energy of a completely filled lowest Landau level is

$$\Delta E_B = -(\lambda^2 m/4\pi\hbar^2)e^{-\tilde{k}^2}\sum_{n=1}^{\infty}(\tilde{k}^2)^n/nn!,$$

where $l_B^2 = 2/q_F^2$ for a filled ground Landau level.

The energy shift for fields that completely fill any number of Landau levels can be similarly calculated. In Fig. 1 we plot the energy shift versus $\tilde{k}$ for zero field, a field B_1 that at the same density fills the lowest Landau level, and a field B_2 that fills the two lowest Landau levels. Among the three, the lowest-energy state is determined by the wavelength of the weak periodic potential. The field B_1 is favored for $1.06 < \tilde{k} < 1.38$ and $2.82 < \tilde{k} < \infty$, and B_2 for $0.57 < \tilde{k} < 0.64$, $1.38 < \tilde{k} < 1.65$, and $2.36 < \tilde{k} < 2.82$. All other wavelengths favor $B = 0$. The vertical lines in Fig. 1 indicate the wavelengths of the dominant components of a weak periodic potential with triangular (left) and square (right) symmetry, with two local minima per particle. Triangular symmetry favors $B = 0$, and square narrowly favors B_2.

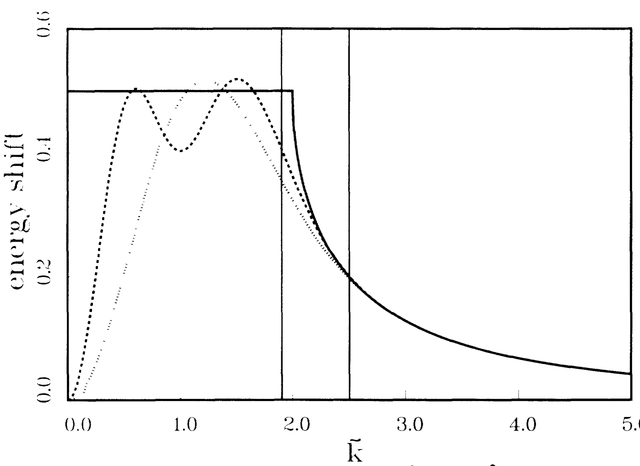

FIG. 1. Energy shift (in units of $-\lambda^2 m/4\pi\hbar^2$) caused by a weak periodic potential to the ground state of two-dimensional noninteracting electrons in zero magnetic field (solid curve), and magnetic fields that fill the ground Landau level (dotted curve) and the two lowest Landau levels (dashed curve).

The lattice limit, which is examined in detail in Ref. 1, is approached as λ becomes large. If a magnetic field is present for a strong periodic potential, the diamagnetic energy of the tight-binding orbitals dominates the energy, as the hopping matrix element between sites decreases exponentially with increasing λ. The zero-field state is thus favored by an effect not included in the calculations presented in Ref. 1.

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V. Nikos Nicopoulos and S. A. Trugman
Theoretical Division, MS B262
Los Alamos National Laboratory
Los Alamos, New Mexico 87545

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