Comment on "Quantum Noise in the Parametric Oscillator: From Squeezed States to Coherent-State Superpositions"

In their recent paper,[1] Wolinsky and Carmichael described the quantum degenerate parametric oscillator by using stochastic equations in the positive-P representation. From this we have now obtained the average time the oscillator takes to switch between its two above-threshold stable states in the small-g^2 limit, the limit of large threshold photon numbers. This has been calculated previously using the Wigner representation.[2] However, the third-order derivative terms that were neglected in that calculation can have a significant effect on the switching time.[3] The positive-P representation requires no such approximations.

Wolinsky and Carmichael obtained two real Ito stochastic differential equations (SDE's) on a bounded manifold in the variables x and y. These represent the real parts of the scaled independent complex variables α and $\alpha^\dagger$ describing the subharmonic mode of the oscillator. The variables x and y can be transformed into new variables u and v which are defined by

$$u=\sin^{-1}\left(\frac{x}{\sqrt{\lambda}}\right)+\sin^{-1}\left(\frac{y}{\sqrt{\lambda}}\right),$$
$$v=\sin^{-1}\left(\frac{x}{\sqrt{\lambda}}\right)-\sin^{-1}\left(\frac{y}{\sqrt{\lambda}}\right). \tag{1}$$

This gives a corresponding Fokker-Planck equation with constant diffusion, which has the following solution in the limit as $\tau\to\infty$:

$$P(u,v)=N\exp[-V(u,v)/g^2], \tag{2}$$

where

$$V(u,v)=-2\sigma\ln|\cos(u)+\cos(v)| +\lambda\cos(u)-\lambda\cos(v). \tag{3}$$

The potential $V(u,v)$ has two minima. Here $\sigma=1-\frac{1}{2}g^2$, where g is a dimensionless coupling parameter, and λ describes the driving amplitude with $\lambda>1$ above threshold. By applying a multidimensional Kramers method[4] to this, an expression for the mean time a stochastic trajectory will take to travel from one of the potential wells to the other is obtained. For a subharmonic decay rate of γ_a it is

$$T=\frac{\pi}{\gamma_a}\left(\frac{\lambda+\sigma}{\lambda(\lambda-\sigma)^2}\right)^{1/2}\exp\left\{\frac{2}{g^2}\left[\lambda-\sigma-\sigma\ln\left(\frac{\lambda}{\sigma}\right)\right]\right\}. \tag{4}$$

Although the manifold as described by Wolinsky and Carmichael is bounded, above threshold it has regions in which it becomes unstable to thermal and phase fluctuations. However, in these zero-temperature-limit calculations such effects are ignored. The practical implications of this approximation will be treated elsewhere.

We finally note that observation of the phase switching could be easily accomplished with a local oscillator detector as used in squeezing measurements[5] on the degenerate parametric oscillator[6] below threshold. The switching could be observed as a random telegraph signal in the detected current with an average residence time of T in each state. Our result indicates much more rapid switching well above threshold than obtained with the approximate Wigner theory.[2]

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