Can a Noninvasive Measurement of Magnetic Flux be Performed with Superconducting Circuits?

C. D. Tesche
IBM Research Division, P.O. Box 218, Yorktown Heights, New York 10598

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A gedanken experiment is described in which the noninvasive determination of joint probability densities for the magnetic flux linking a superconducting quantum-interference device is determined using variable threshold superconducting switches followed by superconducting magnetometers. As a result, it is demonstrated that an important prediction of quantum mechanics can be tested in principle against those of macrorealistic theories using superconducting circuits.

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The rf SQUID (superconducting quantum-interference device) has been predicted to display quantum-interference effects between two macroscopically distinct magnetic-flux states for some values of the device parameters and bias point.[1-3] If macroscopic quantum-interference effects are observed in this device, the experimentally determined range of validity of the quantum theory would be extended and the contrast between the predictions of quantum mechanics and commonly held intuitions about the behavior of macroscopic objects sharpened.

In particular, Leggett and Garg have shown that the assumptions of macroscopic realism and noninvasive measurability, which are implicit in much of our thinking about the macroscopic world, yield constraints on the behavior of two-state systems which are incompatible with the predictions of quantum mechanics.[4] They find that correlation functions $K_{1,2}$ derived from the joint probability densities for the state of an rf SQUID at arbitrary times t_1 and t_2 are constrained to satisfy a set of inequalities which are similar to Bell's inequality for the Einstein-Podolsky-Rosen (EPR) experiment. These inequalities can be violated by a quantum-mechanical two-state system. This proposed experimental test has been criticized by Peres on the grounds that a noninvasive measurement of the flux state of an rf SQUID is impossible.[5] A more general argument has been given by Ballentine.[6] Implicit in both of these arguments is that the determination of the state of the system at time t_i occurs through a "measurement" of the state of the system at that instant, and that the system is not, in general, in an eigenstate of the variable to be measured. However, in the formulation presented by Leggett and Garg, they assert that the times t_i are the analog of polarization settings for the EPR experiment. Thus, the only requirement on the measurement system is that it must be capable of determining the probability that the system occupied specific states at times t_1 and t_2 without affecting the dynamics of the system at times $t < t_1, t_2$. Although it is possible to generate measurement schemes which do not satisfy this requirement, this in no way proves that the determination of the joint probability density is, in principle, impossible. Clearly if one demands that the state of the SQUID be "measured" at time t_1 as proposed by Peres and Ballantine, then this particular measurement scheme will destroy the coherent oscillations of the device at this time, and thus render impossible the determination of a joint probability density. Thus, the solution is to abandon the kind of scheme described by these authors, and invent instead a measurement scheme for the joint probability density which is analogous to that used in a conventional EPR experiment.

In this paper, I describe a gedanken experiment in which the joint probability densities of the rf SQUID are determined by a novel superconducting measurement circuit without violating the quantum-mechanical constraints on the measurement of the state of the rf SQUID. In order to anchor the discussion, specific models for the rf SQUID and for the superconducting measurement circuit are chosen which are experimentally realizable. First, recent experimental results[7] for the intrinsic resistance of Josephson junctions are used to demonstrate the feasibility of fabricating an rf SQUID which satisfies the constraints for coherent behavior. Next, the inevitable destruction of the coherent oscillations which occur in the continuous measurement of the state of the rf SQUID are illustrated by an analysis of the back-action of a conventional dc-SQUID magnetometer on the rf SQUID.[8] This illustrates the process described by Peres. Finally, a novel measurement system is described which can be used to determine the joint probability density of the rf SQUID.

An rf SQUID is a superconducting loop interrupted by a single Josephson tunnel junction [Fig. 1, circuit (a)]. The macroscopic variable is the net magnetic flux linking the loop. Coherent oscillations between two distinct flux states are predicted if the effect of intrinsic dissipation in the Josephson junction and environment is negligible.[1-3] Thus, two key elements in this experiment are the identification of the source of dissipation in the Josephson junction of the SQUID, and the fabrication of junctions with extremely low dissipation. Recently, careful

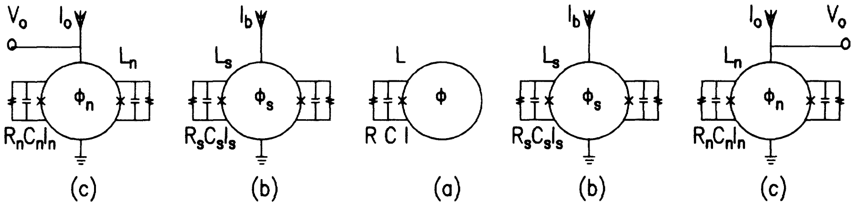

FIG. 1. The superconducting measurement circuit for the noninvasive determination of the joint probability density for the rf SQUID. (a) The state of the rf SQUID is determined by the combined action of (b) two hysteretic dc-SQUID switches and (c) two dissipative dc-SQUID magnetometers.

measurements of the intrinsic subgap dissipation in high-quality Nb-PbInAu Josephson junctions have been performed by Kirtley *et al.*[7] The dissipation at low frequencies has been determined from switching distributions for the return of the junction to the zero-voltage state. The intrinsic junction resistance is dominated by ideal thermally activated quasiparticle tunneling and increases exponentially with decreasing temperature. At temperatures below 2 K for a junction critical current of 600 nA, a lower bound on the intrinsic resistance of 1 MΩ was observed. Direct measurement of the return branch of the I-V characteristic gave a subgap dc resistance in excess of 10^{10} Ω.[7] However, the crossover temperature $T^{\times}$ for the onset of coherent oscillations in the rf SQUID has been shown to scale inversely with the junction resistance.[1-3] Thus, values of the resistance R in excess of MΩ are compatible with $T^{\times}$ above a few mK. In this paper, an rf SQUID with junction critical current $I=2.54$ μA, capacitance $C=0.17$ pF, and SQUID-loop inductance $L=150$ pH, coherent oscillation frequency of 1.35 MHz, and crossover temperature $T^{\times}=3.3\times10^{-8}$ Ry will be used as an example of a SQUID which may undergo coherent oscillations.

In order to develop an analog to the EPR experiment, we need to have the equivalent of polarizers and particle detectors for this rf SQUID. The state of the rf SQUID can be determined by measuring the magnetic field generated by the circulating current around the SQUID loop. The most sensitive magnetometer now available is a dc SQUID. This device consists of a superconducting loop interrupted by two Josephson junctions. The dc SQUID can be operated as a linear flux-to-voltage transducer with gain provided that the junctions are shunted by a resistance of on the order of a few ohms ($\beta_c=2\pi I_n R_n^2 C_n/\phi_0<1$). These shunts eliminate hysteresis in the dc-SQUID characteristics. As a result, the nonhysteretic dc SQUID can also be operated as a phase-insensitive linear amplifier, and thus must generate broadband noise which is coupled back into the source.[9] Since this noise is intrinsic to the dc SQUID, there is also a minimum back-action noise coupled into the source when the SQUID is operated as a magnetometer. In fact, if a dc-SQUID magnetometer is used to detect the state of an rf SQUID during a measurement interval which is short compared to the coherent oscillation period, the broadband noise generated by the magnetometer will be sufficient to destroy the coherent state.[8] This is the source of the destruction of the coherent oscillation which is described in more general terms in the paper by Peres.[5] As a result, a nonhysteretic dc SQUID cannot be considered to be the analog of a polarizer in the EPR experiment. However, the magnetometer is an excellent choice for the analog of a particle detector. In particular, the reverse transfer function of the dc-SQUID magnetometer is extremely small. Thus, the back-action of the magnetometer can be made to dominate all other noise sources in a chain of macroscopic amplifiers and detectors. For this reason, it is the only amplifier explicitly included in the measurement circuit of this paper.

The analog to a polarizer for the rf SQUID must have the property of leaving one flux state of the SQUID unaffected. In addition, the coherent oscillations of the rf SQUID must continue during and after the action of the "polarizer." This implies that the back-action of the SQUID polarizer must be nondissipative. The source of the back-action flux noise in the dc-SQUID magnetometer is the dissipation in the resistive shunts across the Josephson junctions. The flux noise can be greatly reduced by removing the junction shunts. In general, the action of the broadband flux noise on the rf SQUID can be described by an equivalent broadband noise source at temperature $T_e=a^2\gamma R L_{\mathrm{dc}} T/R_{\mathrm{dc}} L$, where R_{dc} and L_{dc} are the dc-SQUID junction resistance and loop inductance, γ is a numerical factor of order unity, a is the inductive coupling strength, and T is the ambient temperature. Thus, for a dc SQUID with loop inductance L_s and junction resistance R_s on the order of the rf-SQUID parameters L and R, the equivalent noise temperature for the broadband flux noise linked into the rf SQUID is on the order of the ambient temperature reduced by the inductive coupling strength a^2. This is an easily satisfied constraint on the coupling strength.

Thus, the dc SQUID without junction shunts is a good candidate for a polarizer for the rf SQUID. The extremely hysteretic dc SQUID is operated as a flux-activated variable threshold switch. This switch produces a negligible back-action on the rf SQUID for one of the two flux states of the rf SQUID. However, when activated, the switch will produce a significant back-action transient if the rf SQUID is in the other flux state. This back-action is not dissipative, and, thus although one of the states of the rf SQUID may be significantly altered after the action of the switch, the coherent oscillations of the rf SQUID are not destroyed.

The complete measurement circuit is shown in Fig. 1. The rf SQUID [Fig. 1(a)] is inductively coupled to a pair of highly hysteretic dc-SQUID switches [1(b)] which are the analog of a pair of polarizers. The nonhysteretic dc-SQUID magnetometers [1(c)] are weakly coupled to the switches, and thus extremely weakly coupled to the rf SQUID. The magnetometers act as "particle detectors" which not only read out the final state of the switches, but also destroy the coherent oscillations in the rf SQUID. This process occurs over a time interval which is long compared to the times at which the switches are activated.

The dynamics of the dc-SQUID switch is described by a potential surface analogous to the double-well potential for the rf SQUID.[10] The two coordinates of the surface are the sum and difference of the phase drops across the Josephson junctions. The potential contains an infinite series of alternating potential wells which form a chain extending down a single deep valley. The shape and location of these wells are determined by the dc bias current I_b and externally applied magnetic flux ϕ_s. The switch is biased so that the system is initially in a shallow potential well (No. 1) separated by large barriers from much deeper adjacent wells (No. 2). The height of the barriers are determined by the total applied flux, including the signal flux from the rf SQUID, and by I_b. Thus, by choosing I_b appropriately, the barrier can be made very large during the free oscillation of the rf SQUID.

The rf SQUID oscillates symmetrically if the flux bias point of the rf SQUID is $\phi = \phi_0/2$. Deviations from this bias point change the coherent oscillation period and increase the damping rate.[3] The coherent oscillations are significantly altered when the effective tunneling frequency is on the order of the difference in energy between the two potential wells. For the rf-SQUID parameters given above, this occurs at a shift in the flux bias of $5\times10^{-8}\phi_0$. This is a stringent requirement on the stability of the flux bias which can easily be violated by the back-action of the dc-SQUID switch. If the coupling is reduced so that the transient flux is considerably less than the flux stability requirement, then the effect of the back-action flux during the coherent oscillation of the rf SQUID is negligible. As an example, a dc-SQUID switch with $L_s = 23.5$ pH, $I_s = 44$ μA, and $C_s = 100$ pF has good switching characteristics for $I_b = 13.67$ μA and externally applied flux $\phi_a = 0.4198\phi_0$. If the mutual inductance to the rf SQUID is $M = 0.023$ pH, the back-action flux is $5\times10^{-8}\phi_0$.

The state of the rf SQUID is registered in the dc-SQUID switch during some interval δt at time t_1, where t_1 is on the order of the coherent oscillation period, and $\delta t \ll t_1$. At t_1, the bias current of the dc-SQUID switch is increased until one of the barriers between wells 1 and 2 almost vanishes. At this point, the flux applied by the rf SQUID in the $-$ state increases the barrier to a value which is large enough to prevent transitions into well 2, while the flux applied by the rf SQUID in the $+$ state reduces the barrier until the dc SQUID tunnels or thermally activates into well 2 during δt. At $t_1+\delta t$, the bias current is restored to the initial value. As a result, if the rf SQUID is in the $-$ state, a negligible transient in the dc-SQUID screening current is generated during δt, and the dc-SQUID switch remains in well 1. Using expressions for tunneling in an rf SQUID as a guide, the tunneling rate in the dc SQUID is exponential in the junction capacitance C_s. Thus for the example given above, the difference in the tunneling rates between the $+$ and $-$ states can be as large as 13 orders of magnitude for C_s on the order of 100 pF. Furthermore, the back-action for the $-$ state during δt can be reduced to any desired level by decreasing the current bias step. This leads to a decrease in the tunneling barrier, which can be compensated for by an increase in C_s.

If the rf SQUID is in the $+$ state during the interval δt, a large transient is generated in the dc-SQUID screening current as the dc SQUID begins to oscillate within the deep well No. 2. This transient produces a large back-action on the rf SQUID in the form of an oscillation in the rf-SQUID flux bias point. As a result, the development in time of this state of the rf SQUID is strongly affected. Note, however, that the rf SQUID is not prepared into the $-$ state by this process. It would be necessary to damp the transient response of the rf-SQUID–dc-SQUID switch system, which does not occur since neither of these systems has any appreciable dissipation. This is an important point. The action of the dc-SQUID switch on the $+$ state of the rf SQUID during the interval δt does not constitute a measurement of the state of the dc SQUID either. In principle, the dc SQUID could tunnel coherently back into the metastable state. In fact, the coupled system consisting of the rf SQUID and the dc-SQUID switch admits description by a three-dimensional potential with states analogous to those of the single-dimensional potential for the rf SQUID.[10] As a result, the heuristic discussion of the operation of the dc-SQUID switch given above can be replaced by a detailed quantum-mechanical model in which the system undergoes coherent oscillation between four distinct states. However, there is no significant coupling to dissipative elements in the combined system which would provide irreversibility in the form of a large number of degrees of freedom. Thus, even though information that that the combined system was in the $-$ state, No. 1 at time t_1 is registered by a lack of transition to well 2 during and after time t_1, no irreversible change in a dissipative macroscopic system has occurred. Thus, if the rf SQUID at some later time has been in the $-$ state at time t_1, the interaction between the dc-SQUID switch and the rf SQUID is noninvasive in the sense that the coherent oscillations in the rf SQUID are preserved unchanged.

The dc-SQUID switch generates a disruptive back-action only if the dc SQUID switches into well 2. Furthermore, the dc-SQUID magnetometer, which contains substantial dissipation, destroys the coherent oscillations of the rf SQUID over some much longer time interval T_a. Thus, it is possible to make a second recording of the state of the rf SQUID with a second dc-SQUID switch at time t_2, where $t_1, t_2 \ll T_a$. The second dc-SQUID switch is coupled to an independent dc-SQUID magnetometer, which also amplifies over T_a. At the end of the amplification interval, the states of both dc-SQUID switches have been measured, and thus the state of the rf SQUID at times t_1 and t_2 has been determined.

An ensemble of identically prepared rf-SQUID systems can now be used to determine joint probability densities $\rho(Q_1, Q_2)$, where $Q = +1$ for the + state, and $Q = -1$ for the − state. Following the scheme suggested by Leggett and Garg,[4] the resultant data are selected such that the Q_1 at t_1 is -1. Thus the joint probability density $\rho(-1, Q_2)$ has been determined as required through a noninvasive process at time t_1. Note that Q_2 is determined through an invasive measurement to be either -1 or $+1$. By reversing the bias points of the dc-SQUID switch, $\rho(+1, Q_2)$ can be measured on a separate ensemble of rf SQUID's.

This gedanken experiment has illustrated that it is possible, in principle, to determine the correlation functions K_{ij} for the rf SQUID. If the correlation functions are found to be in agreement with those derived from a quantum-mechanical treatment of the system, then the reduction of the dissipation in both the rf SQUID and the dc-SQUID switch will have allowed us to witness an essentially quantum-mechanical process in the two macroscopic systems, and thus to extend the experimentally determined domain of applicability of the quantum theory. On the other hand, if the correlations are in violation with the quantum-mechanical predictions even though the dissipation in the rf SQUID and dc-SQUID switch can be clearly identified and shown to be negligible, then the possibility that there is a limit to the applicability of the quantum theory must be seriously considered.

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[1] A. J. Leggett, Prog. Theor. Phys. Suppl. **69**, 80 (1980).

[2] S. Chakravarty and A. J. Leggett, Phys. Rev. Lett. **52**, 5 (1984).

[3] H. Grabert and U. Weiss, Phys. Rev. Lett. **53**, 1787 (1984).

[4] A. J. Leggett and Anupam Garg, Phys. Rev. Lett. **54**, 857 (1985).

[5] Asher Peres, Phys. Rev. Lett. **61**, 2019 (1988).

[6] L. E. Ballentine, Phys. Rev. Lett. **59**, 403 (1987).

[7] J. R. Kirtley, C. D. Tesche, W. J. Gallagher, A. W. Kleinsasser, R. L. Sandstrom, S. I. Raider, and M. P. A. Fisher, Phys. Rev. Lett. **61**, 2372 (1988).

[8] C. D. Tesche, in *New Techniques and Ideas in Quantum Measurement Theory* (New York Academy of Sciences, New York, 1986).

[9] C. M. Caves, K. S. Thorne, R. W. P. Drever, V. Sandberg, and M. Zimmerman, Rev. Mod. Phys. **52**, 341 (1980).

[10] C. D. Tesche, J. Low Temp. Phys. **44**, 119 (1979).