Comment on "Phase Transition in a Restricted Solid-on-Solid Surface-Growth Model in 2+1 Dimensions"

In a recent Letter,[1] Amar and Family (AF) have done extensive numerical simulations of surface growth in $d=2+1$ with a finite-temperature restricted solid-on-solid (RSOS) growth model.[2] They find an unusual transition as a function of their parameter κ which plays the role of an inverse temperature. In this Comment, we propose an explanation of their results based on a continuum growth model. We argue that the observed "transition" is not due to a nonequilibrium analog of a roughening transition, but to the vanishing of the coefficient λ of the nonlinear term in the Kardar-Parisi-Zhang (KPZ) equation of a growing interface.[3]

We assume that the interface is always rough at long length scales,[4] so that in the continuum limit the growth subject to a height restriction may be described by

$$\partial h/\partial t = \mu e^{-a(\nabla h)^2}[1+b\nabla^2 h + c(\nabla h)^2 + \cdots],$$

where $h(\mathbf{x},t)$ is the height of the interface on a $(d-1)$-dimensional substrate, a, b, and c are positive constants, μ is the random deposition rate with $\langle\mu\rangle=\mu_0$, and

$$\langle[\mu(\mathbf{x},t)-\mu_0][\mu(\mathbf{x}',t')-\mu_0]\rangle = 2D\delta(\mathbf{x}-\mathbf{x}')\delta(t-t').$$

The exponential term represents a height restriction with strength a. Expanding this term and neglecting irrelevant higher-order terms one recovers the KPZ equation[3] $\partial h/\partial t = \nu\nabla^2 h + \lambda(\nabla h)^2 + \mu$, with $\nu=\mu b$ and $\lambda=\mu\times(c-a)$. For the RSOS model with $\kappa=0$, λ is negative[5] since some step sites are unavailable for growth. However, increasing κ will increase b since deposition at a step edge becomes more favorable than on a flat surface which, because of irreversibility, causes c to increase. A "peak" for which $\nabla^2 h<0$ can neither evaporate nor spread out, and c must increase to cancel the effect of the negative $\nabla^2 h$ term. Since a remains constant there exists a κ_c for which the effective λ is zero. This case of an ideal interface can be solved exactly with exponents[6] $\beta=\alpha=0$ in $d=2+1$. Thus, a logarithmic growth is expected at κ_c, as observed by AF. Moreover, since the sign of λ is irrelevant, the growth exponent should remain intact for $\kappa>\kappa_c$. The same conclusion can be obtained from a more microscopic argument. For $\kappa>0$ the growth on step edges becomes more favorable than on the flat areas, and thus λ must be positive for large κ.

We can estimate the value of κ_c in $d=2+1$ with a simple argument. Namely, the growth rate of a step edge is proportional to $e^{-2\kappa}$, while that of the flat region is $e^{-4\kappa}$. Simulations give the result[5-7] that about one-third of the sites are active (for $\kappa=0$). Equating $\frac{1}{3}e^{-2\kappa}$ to $e^{-4\kappa}$ gives $\kappa_c>(\ln 3)/2=0.55$. In higher dimensions, kinks in steps must be taken into account which leads to a lower value of κ_c. This argument also implies that the size of a flat region will be $\sim e^{2\kappa}$, so that finite-size effects will be very severe for large κ.

As a consequence of our arguments, in $d=1+1$ there should be a change in the exponent β from the value of $\frac{1}{3}$ to $\frac{1}{4}$ at κ_c, while $\alpha=\frac{1}{2}$ remains unchanged. We have done simulations of the AF model with $\rho=0$ and find that β indeed decreases from its $\kappa=0$ value to the ideal surface value of $\frac{1}{4}$ at κ_c. We estimate that $0.8<\kappa_c<1.2$. For $\kappa=2.0$, the effective value of β increases from 0.27 ± 0.01 ($L=2000$) up to 0.30 ± 0.01 ($L=4000$), and for $\kappa=3.0$ from 0.28 ± 0.01 ($L=1200$) to 0.31 ± 0.01 ($L=4000$). For smaller systems ($L=400$) it was not possible to obtain meaningful exponents for $\kappa>2$. On the other hand, we get $\alpha=0.5\pm0.02$ at both $\kappa=1$ and 2, as expected. All these numerical results are consistent with the idea that growth using the AF algorithm can be explained by the KPZ equation with $\lambda=0$ at κ_c. The effective exponents observed by AF for $\kappa>\kappa_c$ are due to the large effective microscopic length scale compounded by a very slow crossover from the ideal interface behavior in $d=2+1$.

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