Mechanism for Rough-to-Rough Transitions in Surface Growth

Amar and Family[1] presented interesting numerical results for a restricted solid-on-solid (RSOS) growth model in 2+1 dimensions. In this model the height $h_t(\mathbf{x})$ of the surface above a substrate site $\mathbf{x}$ at time t is increased ($h_t \to h_t + 1$) with probability $\min(e^{-\kappa \Delta E}, 1)$, where ΔE is the change in the number of broken bonds, *provided* the RSOS condition on the nearest-neighbor height differences, $|\Delta h_t| \leq 1$, is preserved by the deposition event. As κ increases from zero, the authors observe a sudden drop in the surface-roughness exponents at a critical value $\kappa_c \cong 0.62$, with logarithmic roughness in the transition region, followed by a subsequent rise which indicates another rough phase for $\kappa > \kappa_c$.

In this Comment we show how such transitions can be understood in the framework of the Kardar-Parisi-Zhang (KPZ) theory of kinetic roughening.[2] KPZ proposed the following generic continuum equation to describe a growing surface in a moving frame:

$$(\partial/\partial t) h_t = \nu \nabla^2 h_t + (\lambda/2)(\nabla h_t)^2 + \eta_t , \qquad (1)$$

where $\nu > 0$ and $\eta_t(\mathbf{x})$ is white noise in space and time. The crucial term in (1) is the nonlinearity $(\nabla h_t)^2$. In 1+1 dimensions its presence changes the roughness exponent β (which describes the increase to the surface width with time, $W \propto t^{\beta}$) from $\beta = \frac{1}{4}$ to $\beta = \frac{1}{3}$. In 2+1 dimensions the effect is even more drastic, as the linear ($\lambda = 0$) theory predicts logarithmic roughening [$W \propto (\ln t)^{1/2}$] while the most recent simulation results (see Ref. 1) indicate that $\beta \cong \frac{1}{4}$ for $\lambda \neq 0$. It is therefore important to understand the origin of the nonlinearity.[3] It arises from a small-gradient expansion of the macroscopic inclination-dependent growth velocity $v(\nabla h)$.[3,4] The coefficient λ is thus given by

$$\lambda = (1/d) \nabla^2 v(\mathbf{0}) \qquad (2)$$

in d substrate dimensions. For growth rules which contain a tunable parameter, $v(\nabla h)$ will vary smoothly as the parameter is changed. One must then be prepared for λ to vanish at isolated parameter values. We have found such behavior for a class of (1+1)-dimensional models for which $v(\nabla h)$ can be computed exactly.[4] In the following we argue that it is precisely this mechanism which operates in the case of the RSOS model.[1]

For $\kappa = 0$, growth can occur at any site which is not located next to a step. Tilting the surface increases the number of steps and thereby reduces the density of growth sites. Hence the growth velocity is *reduced* and we conclude from (2) that $\lambda(\kappa = 0) < 0$. On the other hand, for large κ, growth occurs almost exclusively at existing steps. The growth mechanism becomes nucleation dominated. Now tilting the surface *increases* the growth velocity and $\lambda > 0$. It follows that at some intermediate point κ_c, $\lambda(\kappa_c) = 0$. To check this picture we have numerically determined $v(\nabla h)$ for the (1+1)-dimensional RSOS model (Fig. 1). As expected we find a change from concave ($\lambda < 0$) to convex ($\lambda > 0$) behavior at $\kappa \cong 1$.

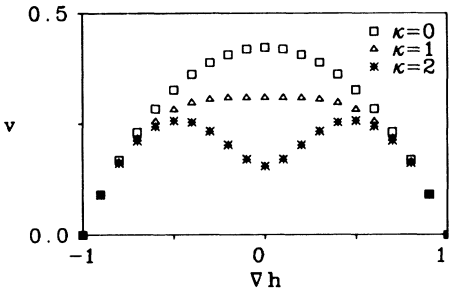

FIG. 1. Inclination-dependent growth velocity for the (1+1)-dimensional RSOS model. Each data point is an average over 5×10^6 attempted depositions on a lattice of length 1000.

This explains in a natural way the findings of Amar and Family:[1] At κ_c, the nonlinearity in (1) is absent and one observes the logarithmic roughness predicted by the linear theory, while for $\kappa < \kappa_c$ *and* $\kappa > \kappa_c$ one expects asymptotically to see the usual KPZ exponents.

It remains to clarify why in Ref. 1 the exponents for $\kappa > \kappa_c$ appear to be smaller than for $\kappa < \kappa_c$. We attribute this to finite-size effects which are much more severe for $\kappa > \kappa_c$. For large κ, the density of surface steps is small and hence the effective lattice constant, which is the average distance between surface steps (i.e., the typical island size), may become comparable to the system size. This effect is quite apparent in Fig. 5 of Ref. 1.

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