Nonlinear Beam Loading and Dynamical Limiting Currents in a High-Power Microwave Gap

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We show that the intense space charge of a high-current beam may increase the gap capacitance by a factor of 2 to 3 over the vacuum value. The limiting current under an ac gap voltage is computed and compared with the steady-state value. The implications of these findings on operation efficiency and frequency stability of high-power microwave devices are discussed.

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Recent advances in relativistic klystron amplifiers[1,2] (RKA) led to electron acceleration to energies greater than 60 MeV over a distance of 1 m.[3,4] In the relativistic klystron configurations at the Naval Research Laboratory,[2,3] an intense electron beam was used, whose instantaneous current may reach the limiting value beyond which electrons begin to be reflected by a gap. The beam's intense space charge, while providing the unusual advantage of electrostatic insulation[5] against vacuum breakdown at the cavity gaps, could significantly load the gap, leading to (a) reduction in the operation efficiency, (b) long buildup time in the current modulation, and (c) detuning of the cavities. Some of these features were suggested in recent experiments and in particle simulation results.[6] As indicated below, how to account for the beam-loading capacitance in the presence of intense space charge was an open question.

Another basic question which has hardly been addressed concerns the limiting current which can be transported across a gap under dynamical conditions. The crucial interactions in an RKA take place when the highly modulated intense beam enters a gap, whose gap voltage has a dominant sinusoidal component. The usual notion of gap transit-time factor, for instance, needs to be reassessed when the limiting current is approached. In practice, the limiting current under ac conditions underlies the operation efficiency and the stability of RKA's. If the modulating gap voltage is too low, the beam power will not be effectively transferred to the load, and if the modulating gap voltage is too high, virtual cathodes are formed and electrons are reflected, destroying the amplifier stability. These issues of beam loading and ac limiting current are of such a general nature that they are likely to arise in many other high-power microwave devices, such as the relativistic magnetrons, vircators, etc.[7] They may have caused the lower efficiencies (compared with conventional sources) that were observed in experiments on those high-current devices.

In spite of a long history of studies of beam-gap interaction,[8] what should be taken as the appropriate gap capacitance, when an intense beam crosses the gap, is far from clear, even under the quasi-dc condition. For a number of reasons, the nonlinear capacitances introduced by Friedman and Serlin,[9] which were generalized from the ones by Bull,[10] are inadequate. For example, they are inconsistent with those established in conventional klystron literature when the limit of a nonrelativistic, low-current beam is taken. Nevertheless, Friedman and Serlin[9] made two important observations. First, the nonlinear dependence on the gap voltage of the stored electrostatic energy can lead to significant harmonic generation in an intense beam (and these harmonics may be utilized for pulse shaping). Second, the simple, one-dimensional parallel-plate model of the gap captures the essential features of the stored energy of the more complex coaxial cavity gaps that were used in the experiments. To isolate the roles of convection current, image charge, and displacement current, we consider the simple parallel-plate model henceforth.

The model consists of an electron beam impinging upon a gap formed by two parallel plates. The plate (K) located at $x=0$ is grounded and the plate (A) at $x=D$ is held at the gap voltage $V_g(t)$. The electrons may or may not carry a current modulation or velocity modulation when they enter the gap at plate K. In the limit of a nonrelativistic, weak beam, low transit time, and small signal gap voltage, the effective gap capacitance C is modified by the beam's dc space charge in the following way:[8,11]

$$C=C_0(1+\tfrac{1}{6}\,\omega_p^2D^2/v_0^2)\,. \tag{1}$$

Here $C_0=A\epsilon_0/D$ is the gap capacitance in vacuo (A is the area of the plate), v_0 is the streaming velocity of the electrons at the gap entrance, and $\omega_p=(e\rho_0/m_0\epsilon_0)^{1/2}$ is the electron plasma frequency corresponding to the beam's space-charge density ρ_0. Equation (1) may easily be derived from the beam-loading factor in classical klystron theory.[12]

Equation (1) already hints of the following effects of space charge on beam-gap interactions:[6] (a) They increase the gap capacitance, thereby providing additional shunts of the current to the load and resulting in a lower efficiency; (b) the rise time ("RC" time constant) is longer, when the gap is connected to a circuit of impedance R; (c) the space charge may lead to detuning of the natural frequency because of the additional capacitive element. It is clear that these effects are particularly pronounced as the limiting current is approached, and a first step toward understanding their magnitude is to provide a suitable, consistent, definition of gap capacitance which is valid up to the point of limiting current and then evaluate its magnitude.

In the one-dimensional model, the total current (convection current and displacement current)

$$I_A(t)=I(x,t)-A\epsilon_0\partial E(x,t)/\partial t \qquad (2)$$

is independent of the position x within the gap.[8,11] It is also the current which is delivered to the load. Equation (2) is exact, since it is obtained by taking the divergence of the Maxwell equation, $\nabla\times\mathbf{H}=-\rho\mathbf{v}+\epsilon_0\partial\mathbf{E}/\partial t$. In Eq. (2), $I(x,t)$ represents the convection current ($A\rho v$) due to the space-charge flow within the gap. It also includes the reflected currents within the gap, if any. (We assign $\rho>0$; and $I>0$ if v is positive.)

Since Eq. (2) is independent of the position x, we may evaluate I_A anywhere within the gap. The most convenient place to evaluate I_A is right in front of the plate K at $x=0^+$, in which case $I(0^+,t)$ is simply the input current $I_i(t)$ entering the gap (if it is below the limiting value[13]) and this incident current is presumably given as an input parameter. Thus, we treat the input current $I(0^+,t)$ as an ideal current source. The remaining quantity $-A\epsilon_0\partial E(0^+,t)/\partial t$ in Eq. (2) is then the current that is shunted by the (nonlinear) gap capacitance. Note that this way of viewing beam loading is just borrowed from the conventional klystron theory. It is also important to note that, as $I_A(t)$ is independent of x, we have in effect eliminated the ambiguity stressed in Refs. 9 and 10 regarding whether the beam-loaded capacitance should be evaluated at plate K or plate A. By Gauss's law $E(0^+,t)=\sigma_k/\epsilon_0$, where σ_k is the total surface charge density on plate K.

To determine the degree of beam loading when the limiting current is approached, let us now pretend that the intense beam enters the gap without any initial current modulation or velocity modulation. If the gap voltage $V_g(t)$ is quasistatic over the gap transit time, we may write

$$\epsilon_0\partial E(0^+,t)/\partial t=\partial\sigma_k(t)/\partial t=(\partial\sigma_k/\partial V_g)\partial V_g/\partial t$$

and identify the quasi-dc, nonlinear gap capacitance as [cf. Eq. (2)]

$$C=-A\frac{\partial\sigma_k}{\partial V_g}=C_0-A\frac{\partial\sigma_{ki}}{\partial V_g}. \qquad (3)$$

In the second form of Eq. (3), we have expressed the total surface charge as $A\sigma_k=-C_0V_g+A\sigma_{ki}$, the sum of the surface charge ($-C_0V_g$) due to the vacuum field, and the (remaining) image charges $A\sigma_{ki}$ on plate K due to the space charge of the beam within the gap.

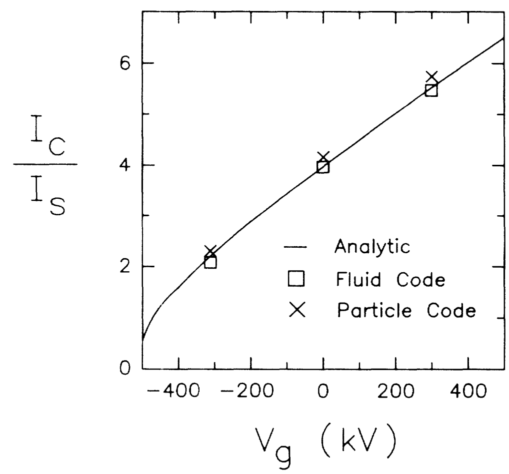

FIG. 1. The dc limiting current I_c as a function of the dc gap voltage V_g. Here, the incident kinetic energy of the electrons is 511 keV.

Let us verify that the nonlinear capacitance C given by Eq. (3) is indeed consistent with the expression (1) in the classical, low-frequency limit. In the presence of a constant gap voltage V_g, a nonrelativistic, weak beam undergoes uniform acceleration $a=(e/m_0)V_g/D$. The velocity at position x is $v(x)=v_0+at=v_0+ax/v_0$, and the space-charge density $\rho(x)=\rho_0v_0/v(x)\simeq\rho_0(1-ax/v_0^2)$ when the gap voltage is weak. The potential distribution $\psi(x)$ between $x=0$ and $x=D$ may easily be solved from the Poisson equation $d^2\psi/dx^2=\rho_0(1-ax/v_0^2)/\epsilon_0$ subject to the boundary conditions $\psi(0)=0$ and $\psi(D)=V_g$. It is obvious that the solution $\psi(x)$ is a cubic polynomial in x, which consists of the vacuum potential V_gx/D, and of the remaining parts representing the self-fields of the beam. From this polynomial, we may readily calculate $\sigma_k=-\epsilon_0d\psi(x)/dx$, to be evaluated at $x=0$. We find $\sigma_k=-(C_0V_g/A)(1+\omega_p^2D^2/6v_0^2)+p$, where p is a quantity independent of V_g. Substituting this expression of σ_k into Eq. (3) yields Eq. (1).

The limiting current I_c depends on the gap voltage. Under dc conditions, it can be obtained analytically[14] and is shown by the solid curve in Fig. 1. (In the figures, the current is in units of the current scale $I_s\equiv C_0m_0c^2/e$, where c is the speed of light.) Also shown in Fig. 1 are the data obtained from a particle code and from a fluid code.[15] To specify the problem, we need to provide the input current $I_i(t)$ and velocity $v_i(t)$ of the electrons as they enter plate K, as well as the gap voltage $V_g=-\int_0^D dx\,E$. The two codes are validated against the analytic theory in Fig. 1.

The dependence of the nonlinear capacitance C on the quasi-dc gap voltage is shown in Fig. 2, at various levels

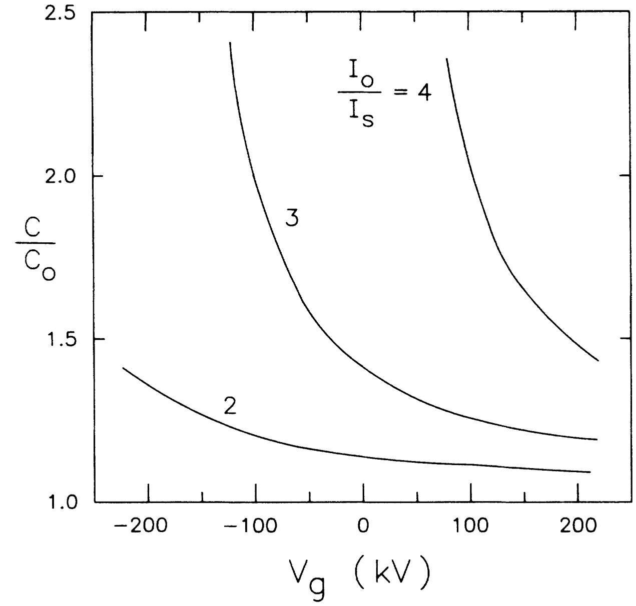

FIG. 2. The nonlinear dependence of the gap capacitance on the quasi-dc gap voltage, at various levels of steady current.

of injected dc beam current I_0. The data were obtained from the particle code, in which a continuous dc beam is injected into an empty gap at $t=0$, and the data are taken at a later time when the transients have died out and C is then obtained according to Eq. (3). From Fig. 2, one sees that the nonlinear capacitance can be a factor of 2 to 3 times higher than the vacuum capacitance, when the limiting current is approached. This ought to be taken as significant beam loading. Beam loading is most pronounced when the gap retards the beam ($V_g<0$), in which case the gap holds more space charge.

The above considerations pertain to quasi-dc conditions. We now use the particle code to determine the limiting current when the gap voltage $V_g(t)$, or the incident current $I_i(t)$ at plate K, is modulated.[13] Since a particle code is used, the nonlinear, transient effects on the limiting current due to beam loading and to gap transit-time factors are accounted for fully. We assume $I_i(t)=I_0+I_1\sin(\omega t)$ and $V_g(t)=V_{g0}\cos(\omega t+\phi)$, where ω is the angular frequency of modulation and I_0, I_1, V_{g0}, and ϕ are constants. Note that $\Omega\equiv\omega D/c$ is roughly the transit angle, in radians, for a relativistic electron. We assume that the beam has no velocity modulation when it enters the gap, i.e., $v_i(t)=v_0=0.866c$, as in Figs. 1 and 2. Shown in Fig. 3 is the limiting current as a function of Ω when $I_1=0$, $\phi=0$, and $V_{g0}=150$, 300, and 450 kV. This figure may be understood as follows. In the low-frequency limit ($\Omega\rightarrow 0$), the limiting current is determined by the retarding phase of the gap voltage. It therefore approaches the value of I_c(dc) given in Fig. 1 corresponding to a decelerating dc gap voltage $V_g=-|V_{g0}|$. At a high frequency ($\Omega\gg 1$), the electronic motion does not respond to the rapidly oscillating

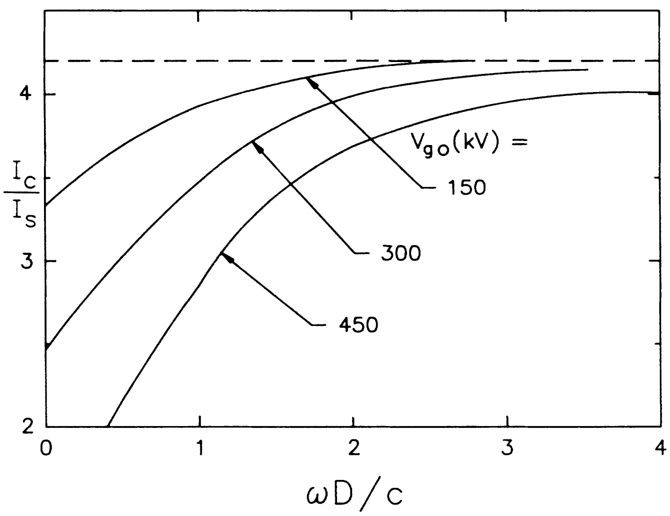

FIG. 3. The critical current of a dc beam when it is subject to a sinusoidal gap voltage of amplitude V_{g0} and normalized frequency $\Omega\equiv\omega D/c$.

gap voltage. Thus, the limiting current in that case approaches the dc value I_c(dc) corresponding to $V_g=0$ in Fig. 1. Note from this figure that for $\Omega\gtrsim 2$, the presence of an ac gap voltage has a diminishing influence on the limiting current.

We next compute the limiting current when the entering beam is fully modulated: $I_i(t)=I_0(1+\cos\omega t)$. We assume a zero gap voltage. Shown in Fig. 4 is the critical value of the peak current ($2I_{0c}$) as a function of Ω when $v_i(t)=0.866c$. Again, the critical current $2I_{0c}$ increases with the modulation frequency, from the dc value as $\Omega\rightarrow 0$ and to an increase by 40% for $\Omega\simeq 2$. In the limit $\Omega\rightarrow\infty$, it is projected[16] to reach twice the value corresponding to $\Omega=0$.

In summary, we have used a standard model to cut through a number of cloudy issues concerning intense

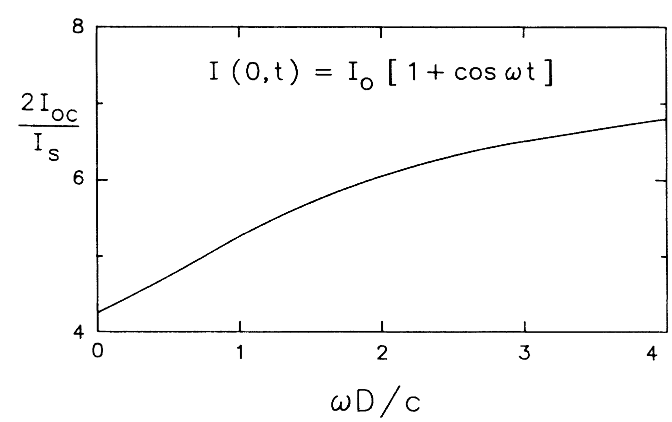

FIG. 4. Peak values ($2I_{0c}$) of the critical current of a fully modulated beam as a function of the normalized frequency $\omega D/c$. The beam enters a short-circuited gap.

beam interaction with a gap. Quantitative measures are given for the nonlinear gap capacitance and the limiting currents under dynamical conditions which are otherwise difficult to isolate in a full-scale electromagnetic simulation of complex geometries. Not only is beam loading found to be an important factor in RKA's, it is likely to be just as important in all radiation sources (or accelerator gaps) whenever the space-charge potential of the intense beam is a significant fraction of the diode voltage. One may then wonder whether the limited efficiencies or the nonmonochromatism observed in relativistic magnetrons, vircators, etc.,[7] may in part be attributed to the nonlinear beam-gap interaction of the sort described in this paper. While our analysis has accurately modeled the important contributions from the displacement currents, the inductive effects have been excluded from the simple model.

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[1]M. A. Allen *et al.*, SLAC Publications No. 4801 and No. 5070 (1989).

[2]M. Friedman, J. Krall, Y. Y. Lau, and V. Serlin, J. Appl. Phys. **64**, 3353 (1988); Rev. Sci. Instrum. **61**, 171 (1990).

[3]M. Friedman, V. Serlin, Y. Y. Lau, and J. Krall, Phys. Rev. Lett. **63**, 2468 (1989).

[4]M. A. Allen *et al.*, Phys. Rev. Lett. **63**, 2472 (1989).

[5]M. Friedman and V. Serlin, IEEE Trans. Electr. Insul. **23**, 51 (1988).

[6]M. Friedman, V. Serlin, D. G. Colombant, J. Krall, and Y. Y. Lau, in Proceedings of the SPIE Meeting, Los Angeles, January 1990 (to be published).

[7]A discussion of these microwave sources may be found in *High Power Microwave Sources* edited by V. L. Granatstein and I. Alexeff (Artech House, Inc., Norwood, MA, 1987).

[8]See, e.g., C. K. Birdsall and W. B. Bridges, *Electron Dynamics of Diode Regions* (Academic, New York, 1966), Chap. 1; also, R. B. Miller, *An Introduction to the Physics of Intense Charged Particle Beams* (Plenum, New York, 1982), Chap. 3.

[9]M. Friedman and V. Serlin, Phys. Rev. Lett. **54**, 2018 (1985); J. Appl. Phys. **58**, 1460 (1985).

[10]C. S. Bull, Proc. Inst. Electr. Eng. Part B **104**, 374 (1957).

[11]M. V. Chodorow and C. Susskind, *Fundamentals of Microwave Electronics* (McGraw-Hill, New York, 1964), Chap. 3.

[12]According to Eq. (1), beam loading contributes an additional capacitance $C' \equiv C_0 \omega_p^2 D^2 / 6 v_0^2$. The additional admittance $j\omega C'$ is simply the low-frequency limit ($\omega \to 0$) of the admittance Y_B given in Eqs. (66)-(68) on p. 80 of Ref. 11.

[13]In this paper, the input current I_i is said to reach the limiting value when reflected electrons begin to be detected at $x=0$ in the particle code.

[14]Y. Y. Lau, J. Krall, M. Friedman, and V. Serlin, Photo-Opt. Instrum. Eng. **1061**, 48 (1989). See also the second paper cited in Ref. 2.

[15]The fluid code solves $E(x,t)$, $\rho(x,t)$, and $v(x,t)$ from the equations $(\partial/\partial t + v\,\partial/\partial x)\gamma v = -(eE/m_0)$, $\partial\rho/\partial t + \partial(\rho v)/\partial x = 0$, and $\partial E/\partial x = -\rho/\epsilon_0$. The particle code solves these equations in Lagrangian form to account for charge overtaking and particle reflection. See Birdsall and Bridges (Ref. 8). The more recent works on these equations may be found in A. Kadish, R. J. Faehl, and C. M. Snell, Phys. Fluids **29**, 4192 (1986), and in E. A. Coutsias and D. J. Sullivan, Phys. Rev. A **27**, 1535 (1983).

[16]P. Sprangle (private communication).