Dimensional Crossover in the Hopping Regime Induced by an Electric Field

Dan Shahar and Zvi Ovadyahu
Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel
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The current-voltage characteristics of indium-oxide films in the activationless-hopping regime exhibit an inflection point at a thickness-dependent electric field. It is argued that this electric field reflects a 2D-3D dimensionality crossover expected of a variable-range-hopping system.

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Variable range hopping (VRH) is the general form of charge transport at low temperatures for Anderson (disorder-induced) insulators. The concept of "variable range" refers to the spatial extent of the most probable hop r which is usually a temperature-dependent quantity.[1] The hopping length is believed to be the spatial cutoff for quantum-interference effects such as conductance fluctuations that have recently received renewed attention.[2] It also determines the effective dimensionality of the hopping process: Given a film of thickness d, it is expected that the VRH process be two dimensional (2D) or three dimensional (3D) when $r > d$ or $r < d$, respectively. In theory, the effective dimensionality of the VHR may be inferred from the temperature dependence of the resistance that takes the following form:[1]

$$\ln R(T) \sim (T_*/T)^{1/3} \text{ (in 2D)}, \qquad k_B T_* \sim 1/N(0)\xi^2 d, \tag{1a}$$

or

$$\ln R(T) \sim (T_0/T)^{1/4} \text{ (in 3D)}, \qquad k_B T_0 \sim 1/N(0)\xi^3, \tag{1b}$$

where $N(0)$ is the density of states at the Fermi level (assumed to be independent of the energy), ξ is the localization length, and k_B is Boltzmann's constant. For a film of finite thickness, a crossover from 2D to 3D behavior [which is described by Eqs. (1a) and (1b), respectively] should result when $r(T)$ becomes smaller than d. The difference between the functional dependence of (1a) vs (1b) is rather subtle. An unambiguous assignment of either dimensionality to experimental data just on the basis of this functional difference is an uncertain procedure unless a very extended temperature range is covered. That is perhaps the main reason why a dimensional crossover in a single sample as a function of temperature has not yet been convincingly observed.

In this Letter we report on the observation of such a crossover in a VRH system using the electric field F rather than the temperature to control r. As is well known,[3] for a sufficiently strong F such that $F > k_B T/e\xi$, r and the resistance itself are temperature independent. In this so-called activationless-hopping regime, the I-V characteristics of the VRH system take the following form:

$$\ln I \sim -(F_*/F)^{1/3} \text{ (in 2D)}, \qquad F_* \sim k_B T_*/e\xi, \tag{2a}$$

and

$$\ln I \sim -(F_0/F)^{1/4} \text{ (in 3D)}, \qquad F_0 \sim k_B T_0/e\xi, \tag{2b}$$

which reflects the fact that the hopping length is controlled by the electric field and follows:

$$r \sim \xi (F_*/F)^{1/3} \text{ (in 2D)}, \tag{3a}$$

$$r \sim \xi (F_0/F)^{1/4} \text{ (in 3D)}. \tag{3b}$$

The functional difference between (2a) and (2b) is not greater than that of (1a) and (1b). Nevertheless, the crossover as a function of field should be easier to observe experimentally due to two reasons: First, it is relatively simple to cover a wide range of fields with high precision. More importantly, it is not necessary to have tight control over the temperature in the field experiment since the resistance at high fields is temperature independent, while in a temperature-induced crossover it would have been mandatory to control F at each temperature such that "Ohmic" conditions are maintained throughout.

We have observed the field-induced crossover on insulating polycrystalline In_2O_{3-x} films prepared as described elsewhere.[4] The films in the present study had thicknesses ranging from 130 to 350 Å and exhibited (low-field) $R(T)$ consistent with a 2D behavior below 2.17 K (except for the 350-Å film that was only marginally 2D at 1.3 K). The various ξ's were calculated using Eq. (1a) and taking $N(0) = 10^{32}$ erg^{-1}cm^{-3} as in Ref. 5. Assigning a 2D behavior to our samples below 2 K is consistent with the fact that the calculated $r(T = 2\text{ K})$ is generally larger than the film thickness, as can be seen in Table I. It is noted that some of our samples are not too deep in the 2D regime, a fact that may have introduced some uncertainty in calculating ξ and $r(T)$ from the "pure" 2D expression for $R(T)$. This error, however, is

TABLE I. Relevant parameters for the studied films.

Sample No.	d (Å)	ξ (Å)	$r(2\ \mathrm{K})$ (Å)	F_0 (10^6 V/cm)	$r(F_{co})/d$
1	130	12	200	2.2	0.92
2	150	13.5	215	2.1	1.15
3	160	23	250	0.32	1.28
4	170	17	220	1.9	1.00
5	200	30	255	1.18	1.65
6	200	11	240	2.3	0.70
7	220	19	210	0.22	0.75
8	350	NA	NA	0.045	0.87

probably not significant as the values we obtained for ξ are close to those obtained elsewhere[2,5] for insulating In_2O_{3-x} films with similar bulk resistivities, but with either a pure 2D or 3D behavior. The films thicknesses were measured both *in situ*, during the deposition process by a quartz oscillator, and again after completion of the transport studies by a Tolansky interferometer (which involved a post deposition of a reflective layer). The two measurements agreed to within ±30 Å.

A typical I-V curve of one of the studied films is shown in Fig. 1. As a reference we show in Fig. 2 the I-V characteristic for a 1200-Å film similar to those studied by Faran and Ovadyahu[5] that exhibited 3D VRH down to 1.3 K. Above a certain (temperature-dependent) electric field, the I-V curves in both figures conform to the behavior of a 3D system [Eq. (2b)]. Such behavior has been seen in numerous disordered systems[6] and it is invariably characterized by a straight slope on a $\ln I$ vs $F^{-1/4}$ plot [or $F^{-1/2}$ in the case of a Coulomb-gap dominated VRH (Ref. 7)] at the "activationless" (high-field) regime, and a smoothly varying slope of the I-V curves at lower fields. The novel feature observed in the intermediate thickness films of the present study is the inflection point (cf. Fig. 1) that gives the I-V curve a characteristic "stretched-out" z shape. This inflection point results from the appearance of a steeper slope segment in the I-V curve beginning at a characteristic field F_{co} (cf. Fig. 1) followed by a concave region as one moves towards still weaker fields. The latter is, of course, the temperature-dependent regime common to all I-V curves regardless of thickness (cf. Figs. 1, 2, and 4). F_{co} is found to obey the following: (a) It depends strongly on the film thickness, as illustrated in Fig. 3, and (b) below the temperature where it can be resolved, F_{co} is temperature independent, as can be seen in Fig. 1 for the two lowest traces. We have checked this important point by cooling one of the samples down to 0.3 K in a dilution refrigerator and found that F_{co} is unchanged within our experimental error.

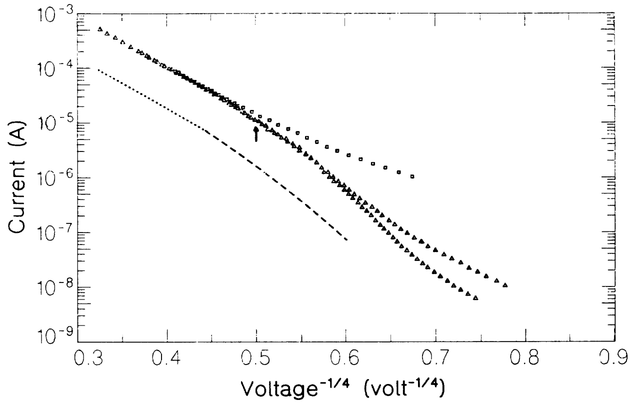

FIG. 1. Current-voltage characteristics for a 200-Å-thick In_2O_{3-x} film (No. 6 in Table I). The distance between the voltage probes was 1 mm. Data were taken at 4.11, 1.7, and 1.3 K for the top to bottom curves, respectively. Note that no 2D behavior (no inflection point) can be detected in the I-V characteristic at the higher temperature. The arrow marks F_{co} (see text). The dashed [dotted] curve is the theoretical simulation of the 2D [3D] I-V curves expected by Eq. (2a) [Eq. (2b)]. Note that the temperature-dependent part of the I-V is not included in the simulation and thus no inflection point emerges.

We now show that these features are consistent with the expected 3D-2D field-induced crossover. First it is noted that the change of slope at F_{co}, going from high to low values of F, is in the "right" direction (cf. the theoretical simulation in Fig. 1). Next, taking the crossover field F_{co} as the field where the I-V deviates from the 3D form by, arbitrarily, 10% and F_0 from the high-field logarithmic slopes, we calculated for each film the values of $r(F_{co})$ through Eq. (3b). These values are compared with the respective d in Table I for the studied films. As can be seen, $r(F_{co})/d$ turns out to be of the order of unity with unsystematic scatter of ±50%. This observation supports the conjecture that the crossover involves $r(F)$ rather than the correlation radius,[3] L_c. In the range of the parameters relevant to our measurements, L_c is typi-

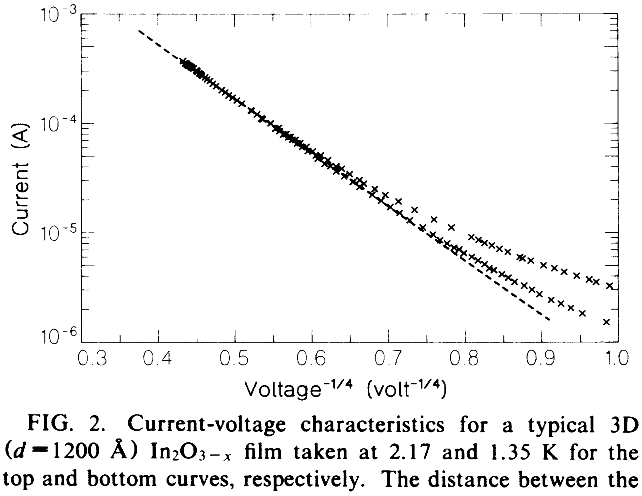

FIG. 2. Current-voltage characteristics for a typical 3D (d=1200 Å) In_2O_{3-x} film taken at 2.17 and 1.35 K for the top and bottom curves, respectively. The distance between the voltage probes was 2 mm. The dashed curve is the high-field behavior expected by Eq. (2b). Note the absence of an inflection point.

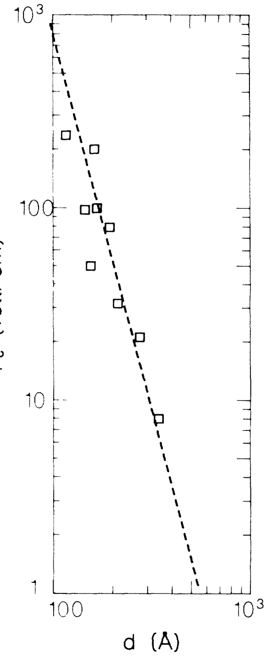

FIG. 3. The crossover field as a function of film thickness for various In_2O_{3-x} films. The dashed curve depicts the theoretical d^{-4} dependence.

cally an order of magnitude larger than the hopping length. If it were possible for L_c to become smaller than d, it would have meant that our estimates for r and ξ are off by an order of magnitude, which seems unlikely in view of previous studies.[2,5] At the same time, it also means that even in the high-field regime where a typical 3D behavior is observed, L_c is not smaller than d. It is a moot point whether a pure 3D regime should exist under such conditions.[8] The fact that a rather suggestive $F^{-1/4}$ I-V portion is observed may indicate that the percolation picture of VRH has to be modified under high-field conditions.

We can also account for the dependence of F_{co} on the film thickness. Using Eqs. (2) and (3), and the natural condition for a dimensional crossover [i.e., $r(2D) = r(3D) = d$], one gets that F_{co} should scale like d^{-4}. The experimental dependence (Fig. 3) compares favorably with this theoretical prediction.

Finally, Fig. 4 depicts the I-V characteristics of a 100-Å film that shows the 2D variation of the activationless current-voltage characteristics in the same temperature and electric-field ranges used in the present study. In this particular case and in agreement with a previous work,[5] only 2D behavior [i.e., Eq. (2a)] is observed in the range of fields and temperatures used. In particular, no inflection point is seen in the data. This is consistent with our interpretation since even for the largest field used (namely, $F = 500$ V/cm) the hopping length is still larger than $d = 100$ Å and a crossover to 3D is expected only for $F > 1000$ V/cm (cf. Fig. 3).

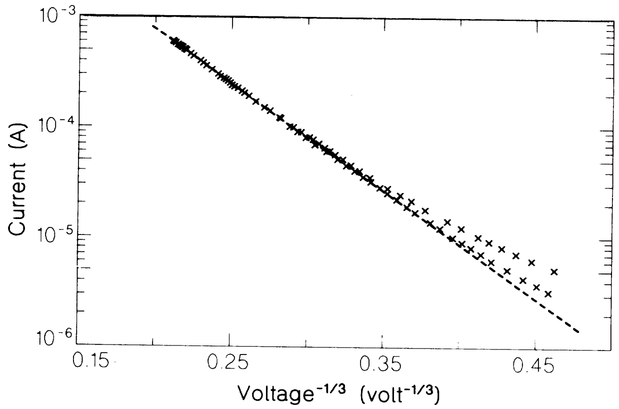

FIG. 4. Same as in Fig. 2, but for a 2D (100-Å) film. Note the different abscissa.

It is easy to understand why F_{co} and the inflection point are not observed above a certain temperature (Fig. 1): To observe a significant range (in terms of F) of the 2D I-V [Eq. (2a)] both conditions $F' > k_B T/e\xi$ and $r'(F') > d$ have to be satisfied simultaneously. This places a bound on the temperature below which F_{co} can be identified. We would like to point out that a substantial range of a pure 2D behavior was observed only for the thinnest films at the lowest temperature, and therefore we were prevented from using the 2D part of the I-V curve to get a less arbitrary experimental definition of F_{co} than the one we used. In other words, it is the breakaway from $r(F) < d$ that the present work can firmly establish. This reflects itself as a well-defined "kink" in the I-V curve at F_{co} even though no "pure" 3D regime nor a 2D one are necessarily resolved as explained above.

One may question that the validity of the assumption (implicitly used above) derived from $R(T)$ (low-field data) is the proper one to use for the analysis of the high-field results. In other words, to what degree is the localization length affected by the field? This is a rather complicated question for which we know of no definite answer. It seems unavoidable that the asymptotic tail of the wave function is modified under high-field conditions. Nevertheless, the structure of the wave function on a scale of the "original" ξ is unlikely to be significantly altered as long as $r(F) > \xi$. That is so because of the following reason: The initial decay of the wave function results from the intense multiple scattering due to the disordered potential of the neighboring sites within a distance ξ. This potential is only weakly perturbed from its low-field value as long as the field is such that the energy gained by the electron from it, over a distance ξ, is much smaller than the average energy separation within a localization volume, T_0. In other words, this perturbation will be negligible as long as $eF\xi \ll k_B T_0$ (or $F \ll F_0$) and according to Eq. (3b) this means $r(F) > \xi$. The highest fields used in our experiment were always smaller than 10^3 V/cm which was typically 2–3 orders of magnitude smaller than F_0 (cf. Table I). The experimental limit on

F was set by the necessity to limit Joule heating rather than anything else.

In summary, we have shown the feasibility of observing a dimensional crossover in a VRH system as a function of an electric field. This demonstrates, in a more direct way than hitherto achieved, the basic nature of the notion of "variable range" long believed to be an inherent feature of charge transport in Fermi glasses.

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