Time-Dependent Thermal Fluctuations and the Giant Dipole Resonance in Hot, Rotating Nuclei

W. E. Ormand,(1) P. F. Bortignon,(1,2) R. A. Broglia,(1,3,4) T. Døssing,(4) and B. Lauritzen(5)
(1)*Istituto Nazionale di Fisica Nucleare sezione di Milano, Via Celoria 16, 20133 Milano, Italy*
(2)*Istituto di Ingegneria Nucleare, via Ponzio 34-3, 20133 Milano, Italy*
(3)*Dipartimento di Fisica, Università di Milano, Via Celoria 16, 20133 Milano, Italy*
(4)*The Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*
(5)*NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*
(Received 30 October 1989)

A model accounting for the time dependence of thermal shape fluctuations in hot nuclei and their effects on the giant dipole resonance (GDR) is presented. Experimental data for Er and Sn isotopes are compared with theoretical calculations, and evidence for the motional narrowing of the GDR is investigated.

PACS numbers: 24.30.Cz, 24.60.Dr, 24.60.Ky

Heavy-ion collisions permit the study of the giant dipole resonance (GDR) built upon states with relatively high excitation energy. This has prompted an extensive experimental investigation of the properties of the GDR in hot nuclei.[1-5] The GDR acts as a probe of the nuclear shape because of its coupling with the quadrupole shape deformation, which can be characterized by the parameters β and γ, defining an ellipsoid with radii $R_k = R_0\exp[-\sqrt{5/4\pi}\beta\cos(\gamma+2\pi k/3)]$, where $k=1$, 2, and 3 denotes the x, y, and z axes. The coupling between the dipole and the quadrupole shapes then leads to three fundamental vibrations with frequency[6]

$$\omega_k = \omega_0\exp[-\sqrt{5/4\pi}\beta\cos(\gamma+2\pi k/3)]\,, \tag{1}$$

where ω_0 is the frequency of the spherical shape. In addition to the equilibrium shape, thermal fluctuations allow the nucleus to sample a variety of shapes and orientations. If the time scale of these fluctuations is slow compared to the changes in the frequencies induced by the fluctuations, then the frequency changes may be viewed as adiabatic, and the photoabsorption cross section for the μth spherical $(\pm 1,0)$ component in a nucleus with temperature T and spin J may be written as

$$\sigma_\mu(E) = Z^{-1}\int d\tau\exp[-F(\beta,\gamma,\Omega;J,T)/T] \times\sigma_\mu(\beta,\gamma,\Omega;E,J)\,, \tag{2}$$

where E is the photon energy, Ω represents the Euler angles (ϕ,θ,ψ), F is the free energy, $d\tau$ is the volume element,[7] and $Z=\int d\tau\exp(-F/T)$ is the partition function. This procedure for including thermal shape fluctuations was followed by Gallardo *et al.*,[8,9] Pacheco, Yannouleas, and Broglia,[9,10] and Alhassid, Bush, and Levit.[11] A schematic estimate of the spread in dipole frequencies implied by this procedure can be obtained by using liquid-drop parameters for the free energy. One finds[12]

$$\delta\omega_k \sim 1.3\sqrt{T}\ \text{MeV}\,. \tag{3}$$

It is expected, however, that the time available for the nucleus to experience each deformation and orientation decreases with increasing excitation energy.[12,13] In this case, the nucleus spends, on the average, a time τ at a given deformation and orientation before "hopping" to another. When hopping is considered, the simple idea of superimposing the line shapes at each deformation and orientation is inadequate. In particular, when $\Gamma=1/\tau$ is much larger than the spread in frequencies, $\delta\omega$, caused by the ensemble of shapes and orientations, a narrowing of the line shape is expected. This phenomenon, known as motional narrowing, occurs whenever a periodic resonant process undergoes a time-dependent perturbation with a rate of frequency shifts that exceeds $\delta\omega$ (i.e., $\delta\omega/\Gamma \ll 1$).

The first discussion of the time dependence in the thermal fluctuations of the GDR was that of Lauritzen *et al.*[12] This model, based on the formalism for strength-function phenomena (cf. Appendix 2D of Ref. 14), although having the correct general behavior for a one-dimensional resonant process, did not allow for either fluctuations in the orientation or a coupling between resonant lines, features that are essential for the proper description of the GDR in excited nuclei. In this Letter, we propose a model making use of the Kubo-Anderson process[15] to describe the giant dipole in a hot, rapidly rotating nucleus that is free from these limitations. Comparisons with experimental data for Er and Sn isotopes are made, and a discussion of the current theoretical problems is given, along with suggestions for future experiments that may isolate the phenomenon of motional narrowing in nuclei. We note that, recently, Alhassid and Bush[16] addressed this same problem using a different technique based on the Langevin equation.

The collective vibration of the GDR is simulated by a rotating, three-dimensional classical harmonic oscillator, whose equations of motion in the rotating frame are[17]

$$\ddot{d}_k + \Gamma_k^i\dot{d}_k + \omega_k^2 d_k = -[\boldsymbol{\omega}_R\times(\boldsymbol{\omega}_R\times\mathbf{d})]_k - 2(\boldsymbol{\omega}_R\times\dot{\mathbf{d}})_k - (\dot{\boldsymbol{\omega}}_R\times\mathbf{d})_k\,, \tag{4}$$

where Γ_k^i is the intrinsic dipole width, ω_k is given by Eq.

(1), and $\boldsymbol{\omega}_R$ is the angular velocity of the rotating nucleus, which was obtained by integrating the equations of motion for a rotating rigid body with spin $\mathbf{J}=J\hat{\mathbf{z}}$ in the fixed laboratory frame.

The intrinsic width Γ_k^i is composed of two parts. The first, which we refer to as Γ_0, is due to the coupling between the GDR and low-lying 2p-2h states, and finite-temperature calculations have found it to be roughly independent of temperature.[18] The second is due to nucleon-nucleon collisions, and is estimated to increase with temperature according to $\Gamma_{\text{coll}}=0.35T^{1.6}$ MeV.[19] This calculation, however, did not include surface effects, and being conservative, we estimate the uncertainty in Γ_{coll} to be approximately a factor of 2. In keeping with this information, and making use of the parametrization of Ref. 20, we take the total intrinsic width as

$$\Gamma_k^i=(\Gamma_0+\Gamma_{\text{coll}})(\omega_k/\omega_0)^{\delta}, \tag{5}$$

where Γ_0, ω_0, and δ are parameters determined from experimental data on the GDR built on the ground state.

To account for time-dependent thermal fluctuations, a mechanism allowing for jumps between the various deformations and orientations must be introduced. The simplest model for such a jumping is what is known as a Kubo-Anderson process (KAP).[15,21] In this model, jumps are made between the various deformations and orientations independent of the initial starting point. The conditional probability $P(\boldsymbol{\alpha},t\,|\,\boldsymbol{\alpha}_0,t_0)$ of having the deformation $\boldsymbol{\alpha}=(\beta,\gamma,\Omega)$ at time t after having been at the point $\boldsymbol{\alpha}_0$ at time t_0 is simply

$$P(\boldsymbol{\alpha},t\,|\,\boldsymbol{\alpha}_0,t_0)=e^{-\Gamma(t-t_0)}\delta(\boldsymbol{\alpha}-\boldsymbol{\alpha}_0)+(1-e^{-\Gamma(t-t_0)})p(\boldsymbol{\alpha}), \tag{6}$$

where Γ is the mean jumping rate and $p(\boldsymbol{\alpha})$ is the stationary probability distribution, $Z^{-1}e^{-F/T}$.

The mean time between jumps may be viewed as a relaxation time for the quadrupole degree of freedom. No clear knowledge of this quantity exists, and different estimates depend on the *Ansatz* made concerning the nature of the nuclear configurations (each having a definite deformation and orientation) participating in the compound-nucleus eigenstates. Assuming these configurations to be uncorrelated, multiquasiparticle states, one can relate Γ to the single-quasiparticle width, obtaining $\Gamma\approx 0.34E^*\approx 0.034AT^2$.[19] Making the *Ansatz* that the (unperturbed) nuclear configurations are coherent surface states, information on the relaxation time may be obtained from fission data, which lead to $\Gamma\leq 0.1$ MeV for $A>150$ ($T\approx 2$ MeV, $J\approx 40\hbar$).[22] Comparing with Eq. (3), it is seen that the first estimate implies motional narrowing already at $T\geq 1$ MeV, whereas the second leads to an adiabatic situation. Furthermore, it is entirely possible that the jumping rate for the deformation coordinates β and γ may be different from that associated with the Euler angles. This last quantity is likely to be connected with the damping width Γ_{rot} of rotational motion.[13] With these points in mind, it remains an open question as to the proper time scales for the variety of thermal fluctuations relevant to the study of the GDR observed in compound-nuclear reactions.

The photoabsorption cross section can be obtained by applying a harmonic, external force to Eq. (4). If the magnitude of the force is unity, then

$$\sigma_\mu(E)=\frac{4\pi^2\hbar}{mc}\frac{ZN}{A}\frac{2}{3}\langle P_\mu(E)\rangle, \tag{7}$$

where the external force is composed of spherical tensor (μ) components in the laboratory frame with frequency $\omega=E/\hbar$, and $\langle P_\mu(E)\rangle$ is the average power delivered by the force (cf. p. 50 of Ref. 17). In general, experiments from heavy-ion collisions do not measure the individual components of the GDR directly, but rather the total cross section, $\sigma_T-\sum_\mu\sigma_\mu$. In the case of the rotating nucleus, however, information regarding the individual components may be obtained from the angular distributions. In particular, the a_2 angular distribution coefficient, $a_2=[0.5\sigma_0-0.25(\sigma_1+\sigma_{-1})]/\sigma_T$.[23]

In this work, σ_μ was evaluated via Eq. (7) by first selecting an ensemble of 400 initial points, and then integrating the equations of motion for the time period $t_{\max}=200\Gamma$, i.e., approximately 200 jumps.[24] At each time-integration step, $t_i=t_{i-1}+\Delta t$, the probability of a jump to a new deformation and orientation $\boldsymbol{\alpha}_i=(\beta,\gamma,\Omega)_i$ is $\Delta t\Gamma$. If this probability is satisfied, then a new point is selected from the stationary probability distribution $p(\boldsymbol{\alpha})$ by applying the von Neumann rejection technique. After each jump, the angular velocity was adjusted so that the angular momentum of the rigid rotor was conserved. In addition, after each change in orientation of the intrinsic system, the dipole coordinates d_i and $\dot{d}_i$ were adjusted so that the corresponding quantities in the fixed laboratory frame were continuous.

The free energy for each nucleus was calculated using the standard Nilsson-Strutinksy procedure[25] with the liquid-drop parameters of Ref. 26 and the Nilsson-model parameters given in Ref. 27. At finite spin, the rotational part of the free energy was taken to be that of a rigid rotor. The general features for the two nuclei studied in this work are that for ^{166}Er the shell corrections lead to a prolate deformed nucleus at zero temperature with $\beta\approx 0.3$, disappearing gradually at $T\approx 2$ MeV. For ^{110}Sn, the shell corrections are essentially negligible and the free energy is governed by the liquid-drop part.

For comparisons with experimental data, we note that in compound-nuclear reactions, GDR γ rays are emitted from all angular momenta up to the maximum value, and from all daughter nuclei of a light-particle cascade. Here, the summations were replaced by inserting properly averaged angular momentum and temperature.[19]

Given the uncertainties in the estimates of Γ mentioned above, rather than make comparisons with experimental data at each temperature, we focus on two sets of experimental data to see if evidence for motional narrowing in the GDR exists. In Fig. 1, a comparison between experimental data and our model for the compound-nuclear systems ^{110}Sn at $T \approx 1.3$ MeV (Ref. 1) and $T \approx 2.8$ MeV,[5] and ^{166}Er at $T \approx 1.2$ MeV (Ref. 2) is shown. The calculations were performed with $\omega_0 = 15.5$ MeV, $\Gamma_0 = 4.5$ MeV, and $\delta = 1.6$ for ^{110}Sn and $\omega_0 = 14.3$ MeV, $\Gamma_0 = 3.97$ MeV, and $\delta = 1.8$ for ^{166}Er.[28] For comparison, three values of Γ were used: $\Gamma = 0$ MeV (adiabatic), $\Gamma = \delta\omega$, and $\Gamma = 6\delta\omega$ (motionally narrowed). Although experimental information is not available, we have also performed the calculations for the ^{166}Er compound system with $T = 2.8$ MeV and $\langle J \rangle = 40\hbar$ [Fig. 1(d)].

At $T \approx 1$ MeV the Sn data seem to indicate the presence of motional narrowing. The experimental centroid, $E_0 = 15.5$ MeV, and the full width at half maximum, $\Gamma_{\rm FWHM} = 6.0$ MeV, are inconsistent with the adiabatic calculation, $E_0 = 14$ MeV and $\Gamma_{\rm FWHM} = 7.8$ MeV, but are in good agreement with the motionally narrowed results. We point out that the narrowing of the line shape in the motionally narrowed limit in our calculations is primarily caused by rapid fluctuations of the orientation variables. This effect is easily understood by noting that jumps in the orientation effectively transfer a vibration from one principal axis to another. If the time scale of these transfers is very short, then the system cannot distinguish the different frequencies along each axis, and at zero spin the response to an external force is with a frequency that is the average of the fundmental modes (cf. Ref. 21, pp. 138–145). If, on the other hand, the jumps in the orientation variables are very slow, while those in the β and γ variables are fast, the individual peaks corresponding to vibrations along the intrinsic axes may be motionally narrowed but in practice lead to a total cross section σ_T that is essentially the same as the adiabatic result because of the large intrinsic dipole width. The a_2 coefficient, on the other hand, would be enhanced somewhat over the adiabatic result.

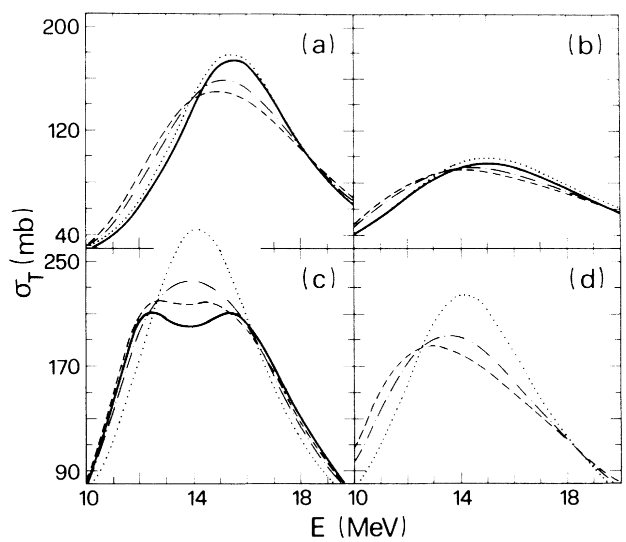

FIG. 1. Comparison between experimental data (solid lines representing the results of cascade calculations) and our model for the compound systems ^{110}Sn at (a) $T = 1.3$ MeV, $\langle J \rangle = 21\hbar$ and (b) $T = 2.8$ MeV, $\langle J \rangle = 40\hbar$, and ^{166}Er at (c) $T = 1.2$ MeV, $\langle J \rangle = 15\hbar$ (with $\Gamma_{\rm coll} = 0.9T^{1.6}$) and (d) $T = 2.8$ MeV, $\langle J \rangle = 40\hbar$. Results for three values of Γ are shown: $\Gamma = 0$ (dashed line), $\Gamma = \delta\omega$ (dash-dotted lines), and $\Gamma = 6\delta\omega$ (dotted lines). Typical experimental errors (not shown) amount to $\approx 10\%$ of the corresponding cross section. Experimentally, the total GDR strength is rather poorly determined, and in the figure the data were normalized to 100% of the classical sum rule.

Finally, with regard to ^{110}Sn at $T = 1$ MeV, the adiabatic cross section could be brought into agreement with the experimental data only in the event that ^{110}Sn were to have very strong spherical shell corrections at $T \approx 1.3$ MeV. This is not very likely, however, because, although the protons fill a complete major shell ($Z = 50$), the neutrons have an open-shell configuration, leading to a near cancellation in the proton and neutron shell corrections.

At higher temperatures, we note that all calculations yield a $\Gamma_{\rm FWHM}$ that is somewhat smaller than the experimental value for ^{110}Sn. However, the collisional contribution to the intrinsic width is somewhat uncertain, and we find that if $\Gamma_{\rm coll}$ is increased by approximately a factor of 2, the motionally narrowed calculation is in overall agreement with the experimental data.

For ^{166}Er, the data seem to favor the adiabatic picture, which is not inconsistent with the relaxation times obtained from fission data in this region. In the motionally narrowed limit, the strength function tends towards one peak, apparently destroying the deformation effects. This is again due to rapid fluctuations in the orientation of the system.

In Fig. 2, we display the coefficients a_2 for the four systems shown in Fig. 1. We note that the differences in the two regimes are more apparent in the high-spin and high-temperature limits. In particular, we observe a clear repolarization in the case of ^{110}Sn with increasing temperature [see Fig. 2(b)]. We note that even though the average spin is increased by ≈ 20 units, the adiabatic prediction for the a_2 coefficient is essentially unchanged from the low-temperature result. However, if motional narrowing occurs, then there should be a clear enhancement in the a_2 coefficient with increasing temperature, even though there is no clear difference in the total cross section, as is evident in Fig. 1(b). The experimental measurement of a_2 is quite difficult, and given the current quality of the data, the differences shown in Fig. 2 are not yet observable. However, with the development and deployment of large solid-angle γ-ray detectors, it may be possible to improve the experimental precision to the point that such a comparison is possible.

We conclude by noting that the observation of the giant dipole resonance in hot nuclei is affected in an essential way by large-amplitude thermal fluctuations. In this work, a model simulating the GDR in hot, rotating nu-

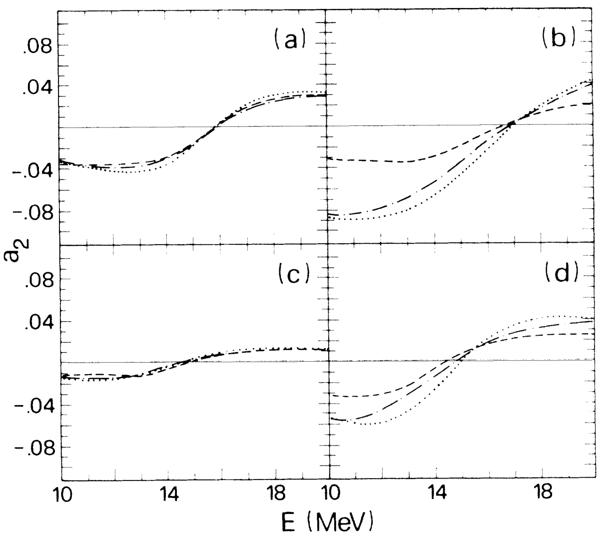

FIG. 2. The a_2-angular distribution coefficients for the four systems shown in Fig. 1, with $\Gamma=0$ (dashed lines), $\Gamma=\delta\omega$ (dash-dotted lines), and $\Gamma=6\delta\omega$ (dotted lines).

clei taking into account the time dependence of these fluctuations was presented. Further, experimental data on the Sn system support the hypothesis that the time scale of these thermal fluctuations may be such that the GDR is motionally narrowed even at relatively low excitation energies, $T\approx 1$ MeV. On the other hand, data at comparable temperatures for the Er system do not support this conclusion. However, the *Ansatz* of a single relaxation time for all the quadrupole degrees of freedom is not likely to be valid in the case of deformed nuclei. In fact, the Er data could still be compatible with motional narrowing provided that the relaxation width associated with the orientation Γ_Ω is small compared to $\delta\omega$, while that for the β and γ degrees of freedom is larger than $\delta\omega$. In order to clarify this question, experimental data at higher excitation energies for the ^{166}Er compound system are needed, as are higher-resolution data for the angular distributions, and data on the fission relaxation time in Sn nuclei.

[1]J. J. Gaardhøje *et al.*, Phys. Rev. Lett. **53**, 148 (1984).

[2]C. A. Gosset *et al.*, Phys. Rev. Lett. **54**, 1486 (1985).

[3]J. J. Gaardhøje *et al.*, Phys. Rev. Lett. **56**, 1783 (1986).

[4]D. R. Chakrabarty *et al.*, Phys. Rev. C **36**, 1886 (1987).

[5]A. Bracco *et al.*, Phys. Rev. Lett. **62**, 2080 (1989).

[6]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, 1975), Vol. II, p. 445; Hill-Wheeler coordinates are used here.

[7]In this work, $d\tau=\beta^4 d\beta \sin(3\gamma) d\gamma d\Omega$ was used.

[8]M. Gallardo *et al.*, Nucl Phys. **A443**, 415 (1985).

[9]In these works, $d\tau=\beta d\beta d\gamma$ was used.

[10]J. M. Pacheco, C. Yannouleas, and R. Broglia, Phys. Rev. Lett. **61**, 294 (1988).

[11]Y. Alhassid, B. Bush, and S. Levit, Phys. Rev. Lett. **61**, 1926 (1988).

[12]B. Lauritzen *et al.*, Phys. Lett. **B207**, 238 (1988).

[13]B. Lauritzen *et al.*, Nucl. Phys. **A457**, 61 (1986); R. A. Broglia *et al.*, Phys. Rev. Lett. **58**, 326 (1987).

[14]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. 1.

[15]R. Kubo, J. Phys. Soc. Jpn. **9**, 935 (1954); P. W. Anderson, J. Phys. Soc. Jpn. **9**, 316 (1954).

[16]Y. Alhassid and B. Bush, Phys. Rev. Lett. **63**,. 2452 (1989).

[17]K. R. Symon, *Mechanics* (Addision Wesley, Reading, 1971), p. 278.

[18]P. F. Bortignon *et al.*, Nucl. Phys. **A460**, 149 (1986).

[19]P. F. Bortignon *et al.*, Nucl. Phys. **A495**, 155c (1989).

[20]P. Carlos *et al.*, Nucl. Phys. **A219**, 61 (1974).

[21]S. Dattagupta, *Relaxation Phenomena in Condensed Matter Physics* (Academic, Orlando, 1987), p. 156.

[22]D. J. Hinde *et al.*, Phys. Rev. C **39**, 2268 (1989), and references contained therein.

[23]This formula is appropriate for angular distributions in which the angle is measured between the emitted photon and the incident beam direction.

[24]The results could be obtained from only one point after a sufficient number of jumps. However, it is more efficient to solve the equations of motion for a large ensemble in parallel.

[25]M. Brack and P. Quentin, Nucl. Phys. **A361**, 35 (1981).

[26]C. Guet, E. Strumberger, and M. Brack, Phys. Lett. B **205**, 427 (1988).

[27]S. G. Nilsson *et al.*, Nucl. Phys. **A131**, 1 (1969).

[28]The parameters were obtained from the equilibrium shape at zero temperature.