Production of Soft Dileptons in the Quark-Gluon Plasma

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The production rate for soft dileptons in a hot quark-gluon plasma is computed to leading order in the QCD coupling constant through a systematic resummation of perturbation theory. The partial rate from the annihilation and decay of soft quarks and antiquarks exhibits unexpected structure—sharp peaks due to Van Hove singularities. These peaks are overwhelmed by the partial rate from processes involving hard quarks and gluons. The total rate for soft dileptons is larger than the naive one-loop prediction by orders of magnitude.

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Quantum chromodynamics predicts that at high temperature hadronic matter undergoes a phase transition to a quark-gluon plasma, a prediction that may be testable in relativistic heavy-ion collisions. Lepton pairs are one of the most promising probes of hot hadronic matter, because once produced they tend to escape without further interactions. Dilepton pairs with large total energy are created predominantly by quark-antiquark annihilation.[1]

In this Letter we study the production of low-energy dileptons using perturbation theory. We assume that the quark-gluon phase of hadronic matter can be approximated by a system in equilibrium at a temperature T. For the temperatures accessible in heavy-ion collisions, the QCD coupling constant g is not likely to be small. Nevertheless, perturbative calculations are still of value: They provide a self-consistent picture of the quark-gluon phase that may apply qualitatively even at strong coupling.

Our calculations are an application of the resummation of perturbation theory developed to solve the long-standing problem of the gauge dependence of the gluon damping rate.[2] In applying perturbation theory to field theories at high temperature, it is necessary to differentiate between hard momenta, on the order of T, and soft momenta, on the order of gT. If all lines are hard, ordinary perturbation theory applies and loop corrections are suppressed by powers of g^2. If a momentum P is soft, however, there are loop corrections that are of the same order in g as the tree amplitudes. In order to include all effects of leading order in g, these "hard-thermal-loop" corrections must be resummed.

It was pointed out by Kapusta[3] that the production rate for soft dileptons could differ by orders of magnitude from the naive prediction from quark-antiquark annihilation. Here we apply the resummation techniques of Ref. 2 to give a systematic calculation of the production rate. The differential rate for producing pairs of massless leptons with total energy E and total momentum $\mathbf{p}$ is related to the discontinuity in the photon self-energy $\Pi^{\mu\nu}(P)$ as

$$\frac{dW}{dE\,d^3p}=\frac{\alpha}{12\pi^3}\frac{1}{P^2}\frac{1}{e^{E/T}-1}\frac{1}{2\pi i}\,\mathrm{Disc}\Pi^{\mu\mu}(P)\,. \quad (1)$$

In the imaginary-time formalism, the momentum of the virtual photon is Euclidean: $P^\mu=(p^0,\mathbf{p})$, with $p^0=2n\pi T$. The discontinuity in (1) is obtained by analytically continuing p^0 to a continuous Minkowski energy $E=ip^0$. We work only to lowest order in the electromagnetic coupling, assuming that photons do not thermalize.

The production of hard dileptons with total energy E of order T can be computed by standard perturbative techniques. The leading-order contribution to $\Pi^{\mu\nu}$ is from the one-loop diagram with a quark loop. The rate for the production of a lepton pair with total momentum $\mathbf{p}=0$ is particularly simple:

$$\frac{dW}{dE\,d^3p}(\mathbf{p}=0)=\frac{5\alpha^2}{36\pi^4}\tilde{n}\left(\frac{E}{2}\right)^2, \quad (2)$$

where $\tilde{n}(\omega)=1/[\exp(\omega/T)+1]$ is the Fermi-Dirac statistical distribution function. We include only the contribution from up and down quarks, which we take to be massless. Perturbative corrections arise from higher-loop diagrams. At two-loop order, the cancellation of

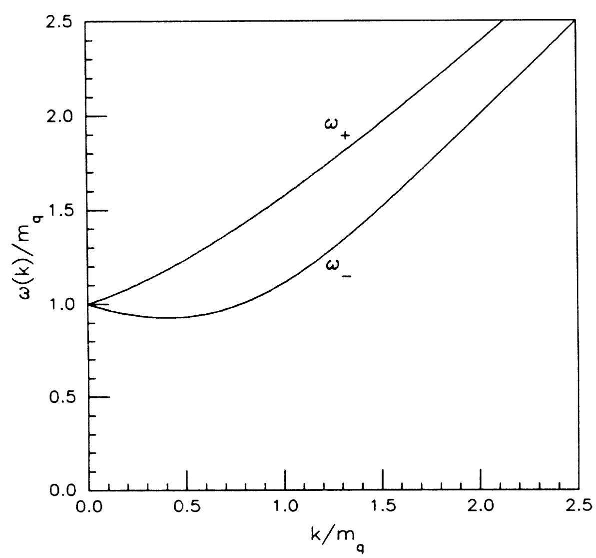

FIG. 1. The quark dispersion relations $\omega_+(k)$ and $\omega_-(k)$.

mass singularities has been verified,[4] and the complete result to order g^2 has been computed.[5]

When the energy E is soft, of order gT, the one-loop calculation is not complete. There are hard-thermal-loop corrections that are as large as the result in (2). These are resummed into a diagrammatic expansion using the methods developed in Ref. 2. For lines carrying soft momentum, bare propagators are replaced by effective propagators, which resum the self-energy corrections from hard thermal loops. If all external lines entering a vertex are soft, an effective vertex, which includes a vertex correction from a hard thermal loop, is required. The only diagram that contributes at leading order to the discontinuity in $^*\Pi^{\mu\mu}$ at soft P is the one-loop diagram constructed out of effective quark propagators $^*\Delta_f$ and effective quark-photon vertices $^*\tilde{\Gamma}^\mu$:

$$^*\Pi^{\mu\mu}(P)=\tfrac{5}{3}e^2\mathrm{Tr\,tr}[^*\Delta_f(K)\,^*\tilde{\Gamma}^\mu(K-P,-K;P)\,^*\Delta_f(K-P)\,^*\tilde{\Gamma}^\mu(K,P-K;-P)]\,, \tag{3}$$

where Tr is the integral over the loop momentum K, $\mathrm{Tr}=T\sum_{k^0}\int d^3k/(2\pi)^3$, and tr is the trace over Dirac indices. There is a second diagram that contributes to $^*\Pi^{\mu\nu}$, which is constructed out of an effective vertex $^*\tilde{\Gamma}^{\mu\nu}$ that couples a quark pair to two photons.[2] It is essential for verifying the Ward identity $P^\mu\,^*\Pi^{\mu\nu}=0$, but it does not contribute to (3) because $^*\tilde{\Gamma}^{\mu\mu}=0$.

The effective quark propagator $^*\Delta_f(K)$ includes the hard thermal loop in the quark self-energy computed first by Klimov and Weldon.[6] Explicitly,

$$^*\Delta_f(K)=\frac{1}{D_+(K)}\,\frac{\gamma^0+i\hat{\mathbf{k}}\cdot\boldsymbol{\gamma}}{2}+\frac{1}{D_-(K)}\,\frac{\gamma^0-i\hat{\mathbf{k}}\cdot\boldsymbol{\gamma}}{2}\,, \tag{4}$$

where

$$D_\pm(K)=-ik_0\pm k+\frac{m_q^2}{k}\left[Q_0\left(\frac{ik_0}{k}\right)\mp Q_1\left(\frac{ik_0}{k}\right)\right], \tag{5}$$

and $m_q=gT/\sqrt{6}$ is the quark "mass" induced by the thermal medium. Q_n are Legendre functions of the second kind. At zero momentum $\mathbf{k}$, the poles in $\omega=ik^0$ occur at $\omega=\pm m_q$. Notice that $^*\Delta_f(K)$ anticommutes with γ_5, so the quark mass is chirally symmetric.

For explicit calculations, it is convenient to introduce a spectral representation for the effective quark propagator. The spectral densities are defined by Disc$[1/D_\pm(K)]=2\pi i\rho_\pm(\omega,k)$, where $\omega=ik^0$:

$$\rho_\pm(\omega,k)=\frac{\omega^2-k^2}{2m_q^2}[\delta(\omega-\omega_\pm(k))+\delta(\omega+\omega_\mp(k))]+\beta_\pm(\omega,k)\theta(k^2-\omega^2)\,. \tag{6}$$

The δ functions come from the poles in $1/D_\pm$ and correspond to the propagation of quasiparticles. At positive energy, $\omega>0$, the pole in the effective quark propagator has two branches. The first branch, $\omega=\omega_+(k)$, represents the propagation of ordinary quarks with a thermal mass. We denote this quasiparticle by q_+, since the ratio of its chirality to its helicity is +1. The second branch, $\omega=\omega_-(k)$, represents a collective mode with no analog at zero temperature. We denote this quasiparticle by q_-, since the ratio of its chirality and helicity is -1. The dispersion relations for q_+ and q_- are shown in Fig. 1. Notice that q_+ and q_- have the same mass: $\omega_+(0)=\omega_-(0)=m_q$. Away from zero momentum, the q_+ branch is monotonically increasing, while that for q_- has a shallow minimum at $k=0.408m_q$, where $\omega_-=0.928m_q$. At high momentum, both branches approach the light cone from above.

An equally important feature of the effective quark propagator $^*\Delta_f$ is the existence of a branch cut below the light cone, for $|\omega|\le k$. The spectral density along this cut is

$$\beta_\pm(\omega,k)=\frac{(m_q^2/2k)(1\mp\omega/k)}{\{\omega\mp k-(m_q^2/k)[Q_0(\omega/k)\mp Q_1(\omega/k)]\}^2+[(\pi m_q^2/2k)(1\mp\omega/k)]^2}\,. \tag{7}$$

The physical origin of this cut is Landau damping,[2] the absorption of a spacelike quark by a hard gluon or hard antiquark.

The effective vertex $^*\tilde{\Gamma}^\mu$ in (3) is the sum of the bare vertex γ^μ and a vertex correction from a hard thermal loop given in Ref. 2. For $\mathbf{p}=0$, it reduces to

$$^*\tilde{\Gamma}^0(K-P,K;P)=\left[1-\frac{m_q^2}{ip^0k}\delta Q_0\right]\gamma^0-\frac{m_q^2}{ip^0k}\delta Q_1 i\hat{\mathbf{k}}\cdot\boldsymbol{\gamma}\,, \tag{8}$$

$$^*\tilde{\Gamma}^i(K-P,K;P)=\left[1+\frac{m_q^2}{ip^0k}(\delta Q_0-\tfrac{1}{3}\delta Q_2)\right]\gamma^i-\frac{m_q^2}{ip^0k}\delta Q_1 i\hat{k}^i\gamma^0+\frac{m_q^2}{ip^0k}\delta Q_2\hat{k}^i\hat{\mathbf{k}}\cdot\boldsymbol{\gamma}\,, \tag{9}$$

where we have introduced the shorthand $\delta Q_n=Q_n(ik^0/k)-Q_n(i(k^0-p^0)/k)$. Note that the effective vertex anticommutes with γ_5, and so respects chiral symmetry.

Inserting (4), (8), and (9) into (3), $^*\Pi^{\mu\mu}$ reduces to a sum of terms of the form $f_1(k^0)f_2(p^0-k^0)$. The discontinuity required in (1) can only be computed after evaluating the sum over k^0. We use the identity

$$\mathrm{Disc}\,T\sum_{k^0}f_1(k^0)f_2(p^0-k^0)=2\pi i(1-e^{E/T})\int_{-\infty}^{+\infty}d\omega\int_{-\infty}^{+\infty}d\omega'\,\tilde{n}(\omega)\tilde{n}(\omega')\delta(E-\omega-\omega')\rho_1(\omega)\rho_2(\omega')\,, \tag{10}$$

where ρ_1 and ρ_2 are the spectral densities for f_1 and f_2. All the spectral densities that are required can be expressed simply in terms of $\rho_\pm$ in (6). Our final expression for the dilepton rate is

$$\frac{dW}{dE\,d^3p}(\mathbf{p}=0)=\frac{10\alpha^2}{9\pi^4}\frac{1}{E^2}\int_0^\infty k^2dk\int_{-\infty}^{+\infty}d\omega\int_{-\infty}^{+\infty}d\omega'\tilde{n}(\omega)\tilde{n}(\omega')\delta(E-\omega-\omega')$$
$$\times\Bigg\{4\left(1-\frac{\omega^2-\omega'^2}{2kE}\right)^2\rho_+(\omega,k)\rho_-(\omega',k)$$
$$+\left(1+\frac{\omega^2+\omega'^2-2k^2-2m^2}{2kE}\right)^2\rho_+(\omega,k)\rho_+(\omega',k)$$
$$+\left(1-\frac{\omega^2+\omega'^2-2k^2-2m^2}{2kE}\right)^2\rho_-(\omega,k)\rho_-(\omega',k)$$
$$+\theta(k^2-\omega^2)\frac{m_q^2}{4kE^2}\left(1-\frac{\omega^2}{k^2}\right)$$
$$\times\left[\left(1+\frac{\omega}{k}\right)\rho_+(\omega',k)+\left(1-\frac{\omega}{k}\right)\rho_-(\omega',k)\right]\Bigg\}. \qquad (11)$$

The ranges of integration for ω and ω' are actually finite, because the spectral densities only have support for $|\omega| \leq \omega_\pm(k)$. Note that ρ_+ and ρ_- are positive, and so (11) is manifestly positive. Strictly speaking, the Fermi distribution functions $\tilde{n}(\omega)\simeq\frac{1}{2}-\omega/4T$ in (11) should be set equal to $\frac{1}{2}$ when the energy is soft, because then ω/T is of order g and represents a higher-order perturbative correction.

Each of the terms in (11) has a simple physical interpretation. The spectral densities $\rho_\pm$ contain contributions from the q_+ and q_- poles and from the cuts. The terms with two powers of $\rho_\pm$ give three types of contributions: pole-pole, pole-cut, and cut-cut. The pole-pole terms represent processes involving two soft quasiparticles q_+ or q_-. The partial rate from these terms is shown as the lower curve in Fig. 2, and exhibits a remarkable structure—sharp peaks at $E=0.470m_q$ and $1.856m_q$. We discuss each of the contributions to this partial rate in detail.

$q_+\to q_-\gamma^*$.—A q_+ decays into a q_- with the same momentum k, emitting a virtual photon with energy $E=\omega_+(k)-\omega_-(k)$. This is a transition from the upper to the lower branch of the dispersion relation in Fig. 1. The spectrum from this process begins at $E=0$ and terminates at $E=0.470m_q$, the maximum value of $\omega_+(k)$

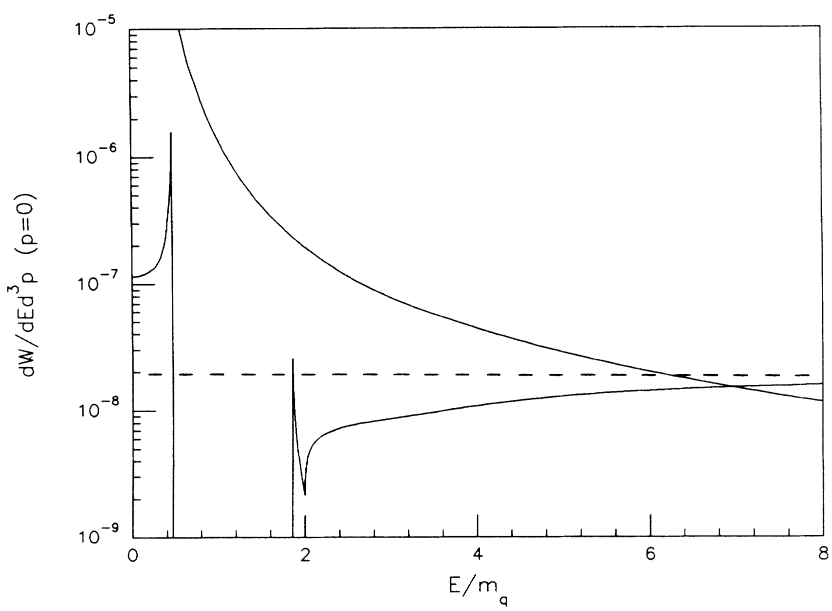

FIG. 2. The production rate for soft dileptons $dW/dE\,d^3p$ with total momentum $\mathbf{p}=0$ as a function of E/m_q. The dashed line is the naive one-loop prediction. The lower curve is the partial rate from pole-pole terms, while the upper curve is from terms involving cuts.

$-\omega_-(k)$. The sharp peak at the end point in Fig. 2 is a Van Hove singularity. The density of states that contributes to this process is inversely proportional to $d(\omega_+ - \omega_-)/dk$, which vanishes at $k = 1.179 m_q$, producing a square-root singularity. Physically, the singularity is smoothed out by higher-order effects, such as damping. In Fig. 2, it has been cut off when the peak becomes narrower than the pen width.

$q_- \bar{q}_- \rightarrow \gamma^*$.—A q_- pair annihilates into a virtual photon. The process opens up with a square-root singularity at $E = 1.856 m_q$, twice the minimum value of $\omega_-(k)$. This is again a Van Hove singularity: The density of states that satisfies the energy-conservation condition $E = 2\omega_-(k)$ diverges at $k = 0.408 m_q$. The contribution from this process falls off rapidly for large E, as $\exp(-E^2/m_q^2)$.

$q_+ \bar{q}_+ \rightarrow \gamma^*$.—This is the normal annihilation process between a quark and an antiquark. It turns on slowly at the threshold $E = 2m_q$ like $(E - 2m_q)^2$. For sufficiently large E, this contribution dominates, approaching the naive prediction in (2).

$q_+ \bar{q}_- \rightarrow \gamma^*$.—This process also opens up at $E = 2m_q$, with the square-root behavior $(E - 2m_q)^{1/2}$. As E increases, the rate quickly peaks, and then falls off like $\exp(-E^2/2m_q^2)$.

In addition to these pole-pole contributions, there are also terms from the cuts in the effective quark propagator. Because the spectral density along the cut is a smooth function, these contributions do not produce any dramatic structure. Their sum is shown as the upper curve in Fig. 2, and is surprisingly large. It falls below the $q_+ \bar{q}_+$ annihilation term only for energies greater than $7m_q$. At low energies, the pole-cut term grows like $1/E^2$ and the cut-cut term grows like $1/E^4$, and they completely overwhelm the structure due to the pole-pole terms. For comparison, the naive prediction (2) is shown as a dashed line in Fig. 2. For $E \simeq m_q$, the total rate exceeds the naive prediction by orders of magnitude.

The physical origin of the pole-cut and cut-cut contributions is Landau damping, interactions of a soft quark with hard quarks and gluons. The physical processes which give the pole-cut contributions involve the absorption of a soft quasiparticle q_+ or q_- by a hard particle, with a soft virtual photon radiated from the $q_\pm$ or from a hard quark or antiquark. Examples are $q_\pm G \rightarrow Q\gamma^*$ and $q_\pm \bar{Q} \rightarrow G\gamma^*$, where Q represents a hard quark and G a hard gluon. Similarly, the cut-cut terms come from processes in which two hard particles scatter by exchange of a soft quark, while radiating a soft virtual photon; for example, $QG \rightarrow GQ\gamma^*$ or $Q\bar{Q} \rightarrow GG\gamma^*$. It is remarkable that the combined effect of all these diverse processes can be summed up into the compact expression (11).

These calculations provide a physical picture of the quark-gluon plasma in the weak-coupling regime that is much richer than naive expectation. The elementary excitations are not massless quarks, but quasiparticles with thermal "masses" of order gT. It is sensible to speak of quasiparticle states in the weak-coupling regime, for the damping rates are of order g^2/T, and so are much smaller than the masses.[2] The thermal masses produce threshold behavior in partial rates. Even more dramatic are the Van Hove singularities due to the nontrivial momentum dependence of $\omega_\pm(k)$. Such singularities may also appear in dilepton production from the hadronic phase, if the pion dispersion relation in nuclear matter has a minimum at nonzero k like $\omega_-(k)$.[7]

Unfortunately, at weak coupling the presence of dramatic structure in the partial rates due to the quasiparticles is moot. At soft momenta, the effects of hard particles overwhelm the structure due to soft quasiparticles. What is lost in structure, however, is gained in the overall rate. For dilepton production at energies of the order of the thermal quark mass m_q, the total rate is larger by orders of magnitude than the naive expectation.

We conclude with the caution that these results are derived in the limit of weak coupling. At temperatures of experimental interest, the coupling is not weak. Nevertheless, we expect that the qualitative features of our calculation persist, including the existence of both quasiparticle peaks and a continuum due to multiparticle states. Whether in strong coupling the continuum overwhelms the quasiparticle peaks, as at weak coupling, is a quantitative question which may have to await an experimental answer.

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