New Class of Inhomogeneous Cosmological Perfect-Fluid Solutions without Big-Bang Singularity

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A new class of exact solutions to Einstein's field equations with a perfect-fluid source is presented. The solutions describe spatially inhomogeneous cosmological models and have a realistic equation of state $p=\rho/3$. The properties of the solutions are discussed. The most remarkable feature is the absence of an initial singularity, the curvature and matter invariants being regular and smooth everywhere. We also present an alternative interpretation of the solution as a globally regular cylindrically symmetric space-time.

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The fact that our Universe is not exactly spatially homogeneous and the lack of reasons to believe that it was so at early times, together with the desire to avoid postulating very special initial conditions, leads to the study of exact spatially inhomogeneous cosmological solutions. It is widely believed, however, and the singularity theorems[1] seem to indicate so, that inhomogeneous models originate in an initial big-bang singularity just as homogeneous (Bianchi) or standard Friedman-Robertson-Walker models do.

The history of inhomogeneous cosmological solutions has reinforced this belief. The first class of solutions of this type with a realistic equation of state $p=\gamma\rho$ $(0<\gamma<1)$ was discovered by Wainwright and Goode[2] and has an initial big-bang singularity in the finite past. Feinstein and Senovilla[3] recently found a new solution with equation of state $p=\rho/3$ and, again, a big-bang singularity in the past. Hitherto, the solutions in Refs. 2 and 3 were the only known solutions of this type with realistic equations of state.

In this Letter, we present a new class of spatially inhomogeneous cosmological models which generalizes the solution of Ref. 3. However, by choosing the parameters adequately, the solutions do not present any big-bang singularity either in the past or in the future. Before discussing this surprising property, we proceed to show the solutions.

The existence of two spacelike commuting Killing vectors $\partial/\partial y$ and $\partial/\partial z$, which are hypersurface orthogonal and orthogonal to each other, is assumed. The line element then takes the form

$$ds^2=e^{2f}(-dt^2+dx^2)+G(q\,dy^2+q^{-1}dz^2)\,, \quad (1)$$

where G, q, and f depend on t and x. Explicitly, we have found the solution given by

$$\begin{aligned} e^f&=[AC(at)+BS(at)]^2C(3ax)\,,\\ G&=[AC(at)+BS(at)]S(3ax)C^{-2/3}(3ax)\,, \quad (2)\\ q&=[AC(at)+BS(at)]^3S(3ax)\,, \end{aligned}$$

where

$$S(u)\equiv\sinh(u),\quad C(u)\equiv\cosh(u)\,, \quad (3)$$

and a, A, and B are arbitrary constants. Feinstein and Senovilla's solution[3] is the particular case $A=0$ of the previous metric.

The velocity of the fluid is

$$u=-e^f dt \quad (4)$$

and the pressure and energy density have the following expressions:

$$\chi p=5a^2[AC(at)+BS(at)]^{-4}C^{-4}(3ax),\quad \rho=3p\,, \quad (5)$$

where χ is the gravitational constant. From (5) we learn that the equation of state is realistic for hot radiation-dominated epochs. We also see that both pressure and density are positive everywhere; therefore, the energy conditions are satisfied. But most important is the fact that ρ and p *are finite and well behaved for any possible value the coordinates can take* as long as

$$A^2>B^2. \quad (6)$$

Then, we take the range of the coordinates to be

$$-\infty<t,x,y,z<\infty\,. \quad (7)$$

It is obvious that making the transformation $t\to t+\text{const}$, we can always set $B=0$ whenever the condition (6) is satisfied. On the other hand, if (6) is not verified (that is, $B^2\geq A^2$), it is possible to set $A=0$ by means of a similar transformation. Thus, there are only two essentially different solutions in our family, that of Feinstein and Senovilla[3] $(A=0)$ and the case $B=0$. From now on we assume

$$B=0 \quad (8)$$

and, by rescaling, we set

$$A=1\,, \quad (9)$$

so that the metric functions (2) and the density and pressure (5) should be rewritten appropriately using (8) and (9).

Keeping this in mind, in order to see if the solution has any curvature singularity, we first calculate the components of the Weyl tensor in the natural null tetrad of the metric (see Ref. 3). These are

$$\Psi_0 = 3a^2[C(2at)C(3ax)+S(2at)S(3ax)] \times C^{-6}(at)C^{-3}(3ax)\,,$$
$$\Psi_2 = a^2[C(2at)-S^2(3ax)]C^{-6}(at)C^{-4}(3ax)\,, \quad (10)$$
$$\Psi_4 = 3a^2[C(2at)C(3ax)-S(2at)S(3ax)] \times C^{-6}(at)C^{-3}(3ax)\,,$$

so that it is easily seen that the Weyl tensor is of Petrov type I. Obviously, the Ψ's are regular everywhere. Taking into account that the Weyl and Ricci invariants are expressed in terms of the Ψ's and p and ρ (see Ref. 2), one can further show that *all the curvature invariants are regular over the whole space-time.*

The hypersurface $x=0$ is special in the sense that the Weyl tensor is of Petrov type D there. The vanishing of the determinant of the metric at this hypersurface corresponds merely to a coordinate singularity, since all the invariants are finite at $x=0$. The metric exhibits also the discrete symmetry $x\rightarrow -x$.

As is well known, the rotation of the fluid in this type of model vanishes identically. The only nonzero components of the acceleration, expansion, and shear of the fluid for the solution are

$$a_1 = -3aS(3ax)C^{-2}(at)C^{-2}(3ax)\,,$$
$$\theta = 3aS(at)C^{-3}(at)C^{-1}(3ax)\,, \quad (11)$$
$$\sigma_{33} = -2\sigma_{11} = -2\sigma_{22} = 2aS(at)C^{-3}(at)C^{-1}(3ax)\,,$$

where all the components have been computed in the natural orthonormal tetrad of the metric (see Ref. 3). As we can see, all the kinematical quantities are regular everywhere. The three slices orthogonal to the fluid flow are not conformally flat, and their intrinsic properties are just the same as those of the solution in Ref. 3.

From expressions (5), (10), and (11), it follows that as $t\rightarrow -\infty$ the solution tends to a nearly flat space-time, with all the kinematical quantities, as well as the Weyl tensor and the density and pressure, going to zero. From there, as t increases, the fluid starts to contract and, of course, the density increases. In this period, there is a positive shear in the x and y directions, while the shear in the z direction is negative. This occurs until $t=0$, where the expansion and shear vanish and the density reaches its maximum value for each x. However, at this instant, as it happens over the whole history of the solution, the fluid is accelerated in the x direction, the acceleration being positive for $x<0$, negative for $x>0$, and vanishing at the hypersurface $x=0$. From then on, as t continues to increase, the solution undergoes an expansion epoch, and the density decreases. The shear properties are now reversed with respect to the contracting epoch. Finally, as $t\rightarrow\infty$, the solution approaches a similar nearly flat space-time as that of $t\rightarrow -\infty$. We thus complete a cycle of contraction and expansion which, of course, could be made periodic. It is also obvious that we could reverse the history of the solution by simply changing the sign of t.

The most intriguing fact of the solution is the complete absence of a spacelike singularity from which the Universe originated (i.e., a big-bang singularity). This property does not, in principle, contradict the powerful singularity theorems (see Ref. 1), since they only establish that some timelike or null geodesic is incomplete. Whether or not our solution has this property is not clear, since, for example, the region $x=0$ could have some pathological behavior. But, in any case, this would only mean that there exist some isolated singularities which are of no importance for the evolution of our model, especially because the fluid congruence itself does not find any singularity at all. Our model, therefore, having a realistic equation of state and satisfying the energy conditions, seems to indicate that the singularity theorems do not rule out completely the possibility of universes without a beginning (without a big-bang singularity). Thus, we see that the introduction of inhomogeneities in the Universe should somehow change our current views on the subject. The solution presented here is a very simple model in the sense that it has only a one-dimensional inhomogeneity. It is necessary to study more general inhomogeneous solutions to reach a clear answer.

Finally, we give an alternative interpretation of our solution. The properties of the $x=0$ hypersurface give rise to the idea of changing the topology of the coordinates so that x becomes a cylindrical radial coordinate. We can certainly do so and by renaming the coordinates as $x=r$ and $y=\phi$, and choosing the constants appropriately, the line element becomes

$$ds^2 = C^4(at)C^2(3ar)(-dt^2+dr^2) + (1/9a^2)C^4(at)S^2(3ar)C^{-2/3}(3ar)d\phi^2 + C^{-2}(at)C^{-2/3}(3ar)dz^2\,. \quad (12)$$

With this choice of the constants, the regularity condition on the axis $r=0$ is automatically satisfied and the 2π periodicity of the coordinate ϕ is well defined. We can thus take the range of the coordinates to be

$$-\infty < t,z < \infty,\quad 0<r<\infty,\quad 0<\phi<2\pi\,.$$

As is apparent, (12) is a globally regular cylindrically symmetric space-time, thus reinforcing the arguments of the previous paragraph. The question of how our solutions fit in with the general conclusions of the singularity theorems is under current consideration, and we hope to report on whether or not they are geodesically complete (and why) shortly.

All the quantities appearing in the Letter have been calculated by using the algebraic computing programs CLASSI and REDUCE.

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