Forces upon Vortices in Anisotropic Superconductors

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Forces on two parallel vortices in anisotropic materials are generally not directed along the line connecting the vortices. The interaction results not only in a repulsion but, in addition, in a torque. On a macroscopic scale, if the magnetic induction B or the anisotropy parameters m_{ik} change within a crystal, the flux-line lattice is subject to forces associated with spatial variation of the torque originated by anisotropy. This force is additional to and different from the Lorentz force, and both forces are, in general, comparable in magnitude.

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The forces between vortices in type-II superconductors can be evaluated by extracting the interaction part from the total energy.[1] When the cores are not overlapped, the repulsive force exerted by a vortex A upon a parallel vortex B can be written as the Lorentz force $\mathbf{j}_A\times\phi_0\hat{\mathbf{z}}/c$, where $\mathbf{j}_A$ is the current density of the vortex A at the location of B, $\hat{\mathbf{z}}$ is the direction of the vortex axes, and ϕ_0 is the flux quantum. The lines of $\mathbf{j}_A$ are circles centered at A, resulting in the Lorentz force along the radius; i.e., the interaction force is directed along AB. In the anisotropic case, the current lines are no longer circles (unless the vortices are directed along the c axis of a uniaxial material); see Fig. 1. Then the direction of $\mathbf{j}_A\times\hat{\mathbf{z}}$ deviates from AB, and the question arises whether or not the force is still given by the common Lorentz expression.

To show that this is the case, one can start with the London free energy in the standard notation (see, e.g., Ref. 2):

$$F=\int(\mathbf{h}^2+\lambda^2 m_{ik}\,\mathrm{curl}_i\mathbf{h}\,\mathrm{curl}_k\mathbf{h})\,dx\,dy/8\pi . \qquad (1)$$

Here m_{ik} is the dimensionless "mass tensor" and λ is the (geometric) average penetration depth. Minimization of F with respect to $\mathbf{h}(\boldsymbol{\rho})$ $(\boldsymbol{\rho}=\{x,y\})$ yields the London equations.[3] Although a sufficiently simple analytic solution $\mathbf{h}(\boldsymbol{\rho})$ for a single vortex is not available,[4] the Fourier transform $\mathbf{h}(\mathbf{k})$ is readily obtained for a vortex oriented arbitrarily within a uniaxial crystal:[2,3]

$$\begin{aligned}&h_x(\mathbf{k})=-h_y(\mathbf{k})k_x/k_y=\phi_0\lambda^2 m_{xz}k_y^2/d\,,\\&h_z(\mathbf{k})=\phi_0(1+\lambda^2 m_{zz}k^2)/d\,,\\&d=(1+\lambda^2 m_{zz}k_x^2+\lambda^2 m_c k_y^2)(1+\lambda^2 m_a k^2)\,.\end{aligned}\qquad (2)$$

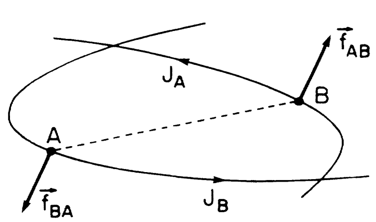

FIG. 1. The interaction forces upon two parallel vortices, $\mathbf{f}_{AB}$ and $\mathbf{f}_{BA}$, are not parallel to AB.

Here $m_{zz}=m_a\sin^2\theta+m_c\cos^2\theta$ and $m_{xz}=(m_a-m_c)\sin\theta\times\cos\theta$; $m_{a,c}$ are eigenvalues of m_{ik} along the $\hat{\mathbf{a}},\hat{\mathbf{c}}$ axes of the uniaxial crystal; θ is the angle between the $\hat{\mathbf{c}}$ axis and the vortex $\hat{\mathbf{z}}$ axis at the origin $\boldsymbol{\rho}=0$; we choose $\hat{\mathbf{y}}=\hat{\mathbf{c}}\times\hat{\mathbf{z}}$. For another vortex at $\boldsymbol{\rho}=\mathbf{R}$, the equation for $h_z(\boldsymbol{\rho})$ contains the term $\phi_0\delta(\boldsymbol{\rho}-\mathbf{R})$ [instead of $\phi_0\delta(\boldsymbol{\rho})$ for the vortex at the origin]; therefore, the Fourier components of $\mathbf{h}(\boldsymbol{\rho}-\mathbf{R})$ are obtained from Eqs. (2) just by replacement of ϕ_0 with $\phi_0 e^{i\mathbf{k}\cdot\mathbf{R}}$.

The free energy (1) can be rewritten in terms of $\mathbf{h}(\mathbf{k})$:

$$8\pi F=\int d^2\mathbf{k}[\,|\mathbf{h}|^2-\lambda^2 m_{ij}(\mathbf{k}\times\mathbf{h})_i(\mathbf{k}\times\mathbf{h})_j]/4\pi^2 . \qquad (3)$$

For *two* parallel vortices, A at $\boldsymbol{\rho}=0$ and B at $\boldsymbol{\rho}=\mathbf{R}$, the Fourier components of $\mathbf{h}$ are given in Eqs. (2) with ϕ_0 replaced by $\phi_0(1+e^{i\mathbf{k}\cdot\mathbf{R}})$; substituting them into Eq. (3) one obtains the interaction energy

$$\begin{aligned}F_{\rm int}=F-2F_0&=\frac{\phi_0^2}{4\pi}\int d^2\mathbf{k}\frac{1+\lambda^2 m_{zz}k^2}{4\pi^2 d}\cos(\mathbf{k}\cdot\mathbf{R})\\&=\frac{\phi_0}{4\pi}h_{Az}(\mathbf{R})\,.\end{aligned}\qquad (4)$$

Here F_0 is the line energy of a single vortex and $\mathbf{h}_A(\mathbf{R})$ is the field of vortex A at point $\mathbf{R}$, where vortex B is situated. The force upon the vortex at $\mathbf{R}$ is

$$f=-\frac{\partial F}{\partial\mathbf{R}}=-\frac{\phi_0}{4\pi}\frac{\partial}{\partial\mathbf{R}}h_{Az}(\mathbf{R})=\frac{\phi_0}{c}(\mathbf{j}_A\times\hat{\mathbf{z}})\,; \qquad (5)$$

i.e., the interaction in the anisotropic situation is indeed represented by the common Lorentz expression.

As a simple example, let us consider the vortices parallel to $\hat{\mathbf{a}}$ $(\hat{\mathbf{z}}=\hat{\mathbf{a}})$. In this case $m_{zz}=m_a$, $m_{xz}=0$, and $h_{Az}(\mathbf{k})=\phi_0/(1+\lambda^2 m_a k_x^2+\lambda^2 m_c k_y^2)$ for vortex A at the origin; $h_{Ax}=h_{Ay}=0$. $h_{Az}(\mathbf{R})$ can be obtained by inverting $h_{Az}(\mathbf{k})$ or, directly, by integrating the London equations in the x-y space:

$$h=\frac{\phi_0}{2\pi\lambda^2 m_a m_c}K_0\left(\frac{R_1}{\lambda}\right),\quad R_1^2=\frac{X^2}{m_a}+\frac{Y^2}{m_c}\,, \qquad (6)$$

where K_0 is the modified Bessel function, and $\{X,Y\}=\mathbf{R}$.

The force (5) now reads

$$f=\frac{\phi_0 K_1(R_1/\lambda)}{8\pi^2\lambda^3 m_a m_c R_1}\left(\frac{X}{m_a}\hat{\mathbf{x}}+\frac{Y}{m_c}\hat{\mathbf{y}}\right). \quad (7)$$

As was pointed out above, this force is not parallel to $\mathbf{R}=X\hat{\mathbf{x}}+Y\hat{\mathbf{y}}$, which connects the interacting vortices. Therefore, the system of two vortices is subject to a torque

$$\begin{aligned}\boldsymbol{\tau}&=\mathbf{R}\times\mathbf{f}=\phi_0\mathbf{R}\times[\mathbf{j}_A(\mathbf{R})\times\hat{\mathbf{z}}]/c\\&=-\phi_0[\mathbf{j}_A(\mathbf{R})\cdot\mathbf{R}]\hat{\mathbf{z}}/c\,.\end{aligned} \quad (8)$$

In the case considered ($\hat{\mathbf{x}}=\hat{\mathbf{c}}$, $\hat{\mathbf{y}}=\hat{\mathbf{b}}$, $\hat{\mathbf{z}}=\hat{\mathbf{a}}$),

$$\boldsymbol{\tau}=-\frac{\phi_0^2 K_1(R_1/\lambda)}{8\pi^2\lambda^3 m_c R_1}(m_c-m_a)XY\hat{\mathbf{z}}\,. \quad (9)$$

It is worth noting that $\boldsymbol{\tau}$ vanishes if both cores (A and B) are situated at either x or y axes. However, the position along the x axis (or along $\hat{\mathbf{c}}$ in this example) is stable with respect to rotations of the pair AB (with a fixed intervortex spacing R), unlike that along $\hat{\mathbf{y}}=\hat{\mathbf{b}}$. This result is related to the tendency of vortices to align in "chains" along $\hat{\mathbf{c}}$, observed in decoration experiments with a small field in the a-b plane of a YBaCuO single crystal:[5] One should, however, consider a system of many vortices, not a pair, to find out whether the bare fact of anisotropy suffices for chain formation.

Let us now consider in what way the anisotropy affects the macroscopic forces upon flux-line lattices (FLL). The forces f_i (per unit volume) experienced by any macroscopic physical system are described by the stress tensor σ_{ik}: $f_i=\partial\sigma_{ik}/\partial x_k$. As a direct consequence of the angular momentum conservation, this tensor is symmetric, $\sigma_{ik}=\sigma_{ki}$, for a system in the *isotropic* space.[6] An anisotropic superconductor constitutes an "anisotropic space" with respect to the system of vortices it hosts. Therefore, σ_{ik} for FLL's may have an antisymmetric part, $\sigma^a_{ik}=-\sigma^a_{ki}$. This part is directly related to the torque density $\boldsymbol{\tau}$ acting upon the system. The tensor $\tau_{ik}=e_{ikn}\tau_n$, dual to the vector τ_n, is given by $\tau_{ik}=\sigma_{ki}-\sigma_{ik}=-2\sigma^a_{ik}$.[7] Hence,

$$\sigma^a_{ik}=-e_{ikn}\tau_n/2\,. \quad (10)$$

The torque density in high-T_c superconductors (which are uniaxial or almost uniaxial) has been measured in fields that were constant over the sample (and large with respect to the lower critical field H_{c1}).[8,9] The stresses (10) in these experiments were uniform ($\boldsymbol{\tau}=\mathbf{M}\times\mathbf{B}$; both the magnetization $\mathbf{M}$ and $\mathbf{B}$ were constants) and no force was generated. If, however, the torque $\boldsymbol{\tau}$ changes from point to point ($\mathbf{B}\neq$const, the anisotropy m_{ik} is not uniform, etc.), an extra force acts on the FLL:

$$f^a_i=\partial\sigma^a_{ik}/\partial x_k=-\mathrm{curl}_i\boldsymbol{\tau}/2\,. \quad (11)$$

This force and the corresponding stresses can also be written in the form

$$\mathbf{f}^a=\mathrm{curl}(\mathbf{B}\times\mathbf{H})/8\pi,\quad \sigma^a_{ik}=(B_iH_k-B_kH_i)/8\pi\,, \quad (12)$$

which shows explicitly that they vanish if $\mathbf{B}$ is parallel to $\mathbf{H}$ (in a small vicinity of the point where $\mathbf{f}^a$ is evaluated).[10] In general, $\mathbf{f}^a$ is comparable in magnitude to the Lorentz force $(\mathbf{j}\times\mathbf{B})/c=\mathrm{curl}\mathbf{H}\times\mathbf{B}/4\pi$. Therefore, it should be incorporated in the macroscopic conditions for the FLL to be in rest:

$$\mathbf{j}\times\mathbf{B}/c-\mathrm{curl}\boldsymbol{\tau}/2+\mathbf{P}=0\,, \quad (13)$$

where $\mathbf{P}$ stands for the pinning force density (possible corrections to the force balance due to spatial variations of the FLL structure are disregarded).

To demonstrate qualitatively what the extra force (11) does, let us consider the case of intermediate fields, $H_{c1}\ll B\ll H_{c2}$, for which the *equilibrium* free-energy density F is known; see, e.g., Ref. 11, where F is expressed in terms of components of $\mathbf{B}$ and of m_{ik} in a special coordinate system (with $\hat{\mathbf{z}}=\mathbf{B}/B$). One can write F in any Cartesian coordinates in terms of the invariant $B^*=(m_{ik}B_iB_k)^{1/2}$ (B^* turns into B in the isotropic case and into $Bm_{zz}^{1/2}$ for the uniaxial situation in notation of Ref. 11):

$$F=\frac{B^2}{8\pi}+\frac{\phi_0}{32\pi^2\lambda^2}B^*\ln\left(\frac{\phi_0\beta}{2\pi\xi^2B^*}\right). \quad (14)$$

Here λ and ξ are the geometric averages for the penetration depth and the coherence length, and β is a constant of order unity. The torque density $\boldsymbol{\tau}=\mathbf{M}\times\mathbf{B}=\mathbf{B}\times(\partial F/\partial\mathbf{B})$ is readily found:

$$\begin{aligned}\tau_i&=e_{ikn}m_{nm}B_kB_m\Phi(B^*)\,,\\ \Phi&=(\phi_0/32\pi^2\lambda^2B^*)\ln(\phi_0\beta/2\pi\xi^2eB^*)\end{aligned} \quad (15)$$

($e=2.718\ldots$). This is, in fact, the result of Ref. 12, written in arbitrary Cartesian coordinates. According to Eq. (10), the antisymmetric part of the stress tensor is

$$\sigma^a_{ik}=(m_{ij}B_k-m_{kj}B_i)B_j\Phi(B^*)/2\,. \quad (16)$$

Thus, the force $f^a_i=\partial\sigma^a_{ik}/\partial x_k$ can be obtained as long as the induction $\mathbf{B}(\mathbf{r})$ and $m_{ik}(\mathbf{r})$ are known.

Strictly speaking, Eqs. (14)–(16) hold only in a uniform field. If, however, the changes in $\mathbf{B}$ occur on distances L large with respect to the cell size $(\phi_0/B)^{1/2}$, one can consider the FLL as being in equilibrium *locally*. This condition is satisfied easily in large fields: The macroscopic current density J is of order

$$cB/4\pi L\ll cB^{3/2}/4\pi\phi_0^{1/2}\sim 2\times10^3B^{3/2}\ \mathrm{A/cm^2}$$

(B in gauss); for $B\simeq10^4$ G, this yields $J\ll10^9$ A/cm^2.

Consider, for example, the situation shown in Fig. 2(a), where the $\mathbf{B}$ lines are in the x-z plane ($B_y=0$); assume that B_x is odd while B_z is even in z. The crys-

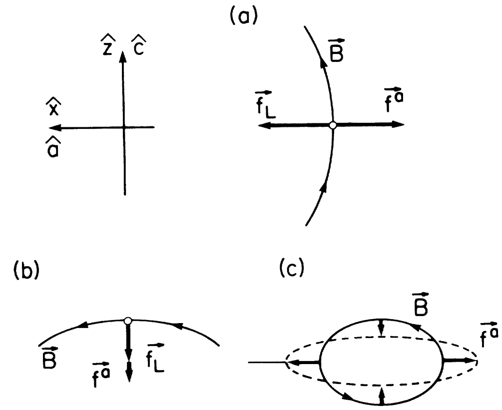

FIG. 2. Forces upon bent flux-line lattices. The crystal axes $\hat{\mathbf{c}}$ and $\hat{\mathbf{a}}$ are the same and $m_c > m_a$ for all cases shown. The anisotropy-originated force $\mathbf{f}^a$ is opposite to the Lorentz force $\mathbf{f}_L$ in (a), whereas they are of the same direction in (b). If the vortex lines (i.e., the lines of **B**) are closed as in (c), forces $\mathbf{f}^a$ tend to deform vortices as to make them go mostly along the "easy" direction (in the a-b plane), where their line energy is minimum. The Lorentz force, which is not shown, acts inward everywhere along the vortex loop. Therefore, under the Lorentz force the loop would contract, while $\mathbf{f}^a$ tends to make it "flat."

tal axes $\hat{\mathbf{a}},\hat{\mathbf{b}},\hat{\mathbf{c}}$ coincide with $\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}}$, respectively. Then, the only nonzero components of σ^a_{ik} [Eq. (16)] are $\sigma^a_{xz} = -\sigma^a_{zx} = (m_a - m_c)B_xB_z\Phi(B^*)/2$ and $B^{*2} = m_aB_x^2 + m_cB_z^2$. At $z=0$, $B_x=0$, and one obtains $f_x^a = (m_a - m_b)B\Phi(B^*)\partial B_x/\partial z$, $B^* = Bm_c^{1/2}$. For $m_c > m_a$, this force is positive because, as is seen from Fig. 2(a), $\partial B_x/\partial z < 0$. Note that the Lorentz force $\mathbf{f}_L = (\mathbf{j}\times\mathbf{B}/c)_x < 0$ in this particular situation. Therefore, the anisotropy-originated force $\mathbf{f}^a$ reduces the effect of the Lorentz force in the configuration of Fig. 2(a).

A situation similar to that of Fig. 2(a) may occur in long thin-film strips (with the $\hat{\mathbf{c}}$ axis perpendicular to the film plane) close to the strip edges. The field and the current density are usually large near the edges, which therefore constitute "weak spots" where the flux flow under the Lorentz force can occur first. One is tempted to speculate that the reduction of $\mathbf{f}_L$ by the force $\mathbf{f}^a$ at the edges helps in achieving the high critical currents observed in c-axis-oriented films of high-T_c superconductors.[13]

Similar arguments show that in the configuration of Fig. 2(b), $\mathbf{f}^a$ and $\mathbf{f}_L$ act in the same direction. The flux flow in this situation, however, may be prevented by the "intrinsic" pinning of vortices oriented along the layered structure or by large energy barriers associated with shear deformations of the FLL;[14] the reenforcement of $\mathbf{f}_L$ by $\mathbf{f}^a$ near the top and bottom surfaces of c-axis-oriented films does not make the above speculation on the cause for high J_c's less appealing.

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Note added.—It has recently been shown that for a vortex inclined with respect to the c axis, the field $h_z(x,y)$ may change sign at distances on the order λ [A. M. Grishin, A. Yu. Martynovich, and S. V. Yampolskii (private communication)]. The interaction given in Eq. (4) becomes attractive for parallel (to z) vortices in the c-z plane or close to it.

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