One-Jet Inclusive Cross Section at Order α_s^3: Quarks and Gluons

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The next-to-leading-order inclusive jet cross section is calculated including both quarks and gluons. The general structure of the dependence of the cross section on the explicit jet definition and on the choice of the renormalization scale μ^2 is found to be very similar to the case of gluons only, calculated earlier. The resulting cross section is shown to involve a much reduced theoretical uncertainty compared to the lowest-order result. First comparisons with experimental data are also exhibited.

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A characteristic feature of many of the most interesting events observed in the recent data[1] from hadron colliders is the obvious appearance of "sprays" of hadrons or jets.[2] Since these jets are thought to arise from the large-angle scattering of elementary constituents of the hadrons, the quarks and gluons, they offer an important quantitative test of our understanding of the strong-interaction theory, QCD. This is particularly true in the search[3] for a breakdown of the standard model due, for example, to the possible composite structure of quarks and gluons. For this purpose it is essential to analyze the scattering of these elementary partons at the largest-p_T scale possible. The experimental signal for hard parton-parton scattering is jet production, and the most straightforward jet cross section is that for the inclusive production of a jet.

Unfortunately, there exist important ambiguities which limit our ability to perform detailed quantitative studies with the observed jet cross sections. An important source of theoretical ambiguity is the uncertainty in the parton distribution functions[4] and the corresponding value of $\Lambda_{\overline{\rm MS}}$ used in the calculation. ($\overline{\rm MS}$ denotes the modified minimal-subtraction scheme.) Another important source of ambiguity, both theoretically and experimentally, arises from the fact that jets are *not* intrinsically precisely defined. Thus different experiments are free to define jets in somewhat different fashions, resulting in somewhat different results. On the theory side, at the Born level in perturbation theory, one looks at a cross section for parton scattering and assumes that each outgoing parton materializes into a narrow jet of particles. However, for reasons of color conservation, energy-momentum conservation, and quantum-mechanical interference, a jet of hadrons *cannot* be the residue of a *single* parton. This issue first arises directly in the perturbative calculation at one order beyond the Born level. At this level a jet can consist of more than a single parton and a careful definition of a jet is required in analogy to the experimental situation. The higher-order perturbative calculation discussed in this Letter addresses this issue plus the related theoretical issues of the choice of the renormalization-factorization[5] scale μ^2 and of the general magnitude of higher-order contributions (often referred to as the "K factor"). Such a calculation at one order beyond the Born approximation (i.e., at order α_s^3) can be expected to substantially reduce the theoretical uncertainty.

In earlier theoretical studies,[6] only incomplete QCD matrix elements at order α_s^3 were available. Recently, the full order-α_s^3 matrix elements in $4-2\epsilon$ dimensions have been calculated by Ellis and Sexton.[7] In previous papers[8] we described a calculation of the inclusive jet cross section using these full matrix elements but applied to the simplified case of gluons only. Here we present a brief summary of the results from a complete calculation involving both quarks and gluons for the process $p\bar{p} \to {\rm jet}+X$. The basic structure of the calculation is identical to the gluon-only case. Details will be presented elsewhere.[9] The primary differences are the greatly increased "bookkeeping" required to account for all possible parton participants and subprocesses and, more importantly, the possibility to perform serious comparisons to data.

An analysis based on the Ellis-Sexton matrix elements and focused primarily on single-particle inclusive production has also been given by Aversa *et al.*[10] These authors have also calculated a jet cross section, but only in the limit of small jet-cone size.

Now consider how a jet is defined.[11] We imagine detecting the jet with a segmented calorimeter consisting of cells i distributed in pseudorapidity η $[=-\ln\tan(\theta/2)]$ and azimuth ϕ, in which the transverse energies $E_{T,i}$ (the energy in cell i times $\sin\theta_i$) are measured. We define cells in a jet cone of radius R in η-ϕ space, centered on a cone axis (η_c,ϕ_c), by

$$(\eta_i-\eta_c)^2+(\phi_i-\phi_c)^2<R^2. \tag{1}$$

The transverse energy E_T of the jet is then

$$E_{T,J}=\sum_{i\text{ in cone}} E_{T,i}. \tag{2}$$

The jet axis is defined by the following weighted averages:

$$\eta_J=\frac{1}{E_{T,J}}\sum_{i\text{ in cone}} E_{T,i}\eta_i,\qquad \phi_J=\frac{1}{E_{T,J}}\sum_{i\text{ in cone}} E_{T,i}\phi_i. \tag{3}$$

This process is iterated until the cone axis (η_c,ϕ_c) agrees with the jet axis (η_J,ϕ_J) determined by Eq. (3). Note that it is exactly this process of counting all of the energy in a finite cone that ensures that the jet cross section is finite to all orders in perturbation theory, in analogy to what occurs for similar quantities in e^+e^- physics.[12]

In our calculation, there are at most three partons in the final state. A single isolated parton with parameters (E_T,η,ϕ) is "reconstructed" as a jet with these same parameters. Two partons with parameters $(E_{T,1},\eta_1,\phi_1)$ and $(E_{T,2},\eta_2,\phi_2)$ may be combined into a single jet, using Eqs. (1)-(3) above. Note that it can occur that two partons qualify both as two individual jets and a combined jet. We count only the combined jet in this case.

The specific quantity calculated is the inclusive jet cross section $d\sigma/dE_T\,d\eta$ for the production of a jet with pseudorapidity η and transverse energy E_T plus anything. As a first illustration of this cross section at order α_s^3 we exhibit the dependence on the cone size R. This is illustrated in Fig. 1, where the inclusive jet cross section is plotted versus R for $\eta=0$, $E_T=100$ GeV, and $\sqrt{s}=1800$ GeV. For comparison the *R-independent* Born cross section (with the one-loop form of the running coupling) is also plotted. The order-α_s^3 result is well described by the form $A+B\ln R+CR^2$, where we recognize the second term as the remnant of the cancellation between a (negative) infinity in the virtual correction to the $2\rightarrow 2$ process and a (positive) collinear singularity in the $2\rightarrow 3$ process. Clearly the corrections to the Born result become arbitrarily large (and negative) as $R\rightarrow 0$, indicating that fixed-order perturbation theory is inadequate in this limit. Physically, the higher-order "showering" effects play an ever more important role as $R\rightarrow 0$. This dependence of the cross section on R is an intrinsic feature of QCD. It will be important to test it experimentally.

We now turn to the dependence of the jet cross section on the scale μ. This scale appears as the argument of the running coupling constant $\alpha_s(\mu)$ and as the evolution parameter in the parton structure functions $G(x,\mu)$. The scale μ arises as an artifact of the truncation of the perturbation series. Any residual dependence on it is a reminder of the missing higher-order contributions and is not physical. In Fig. 2, the inclusive jet cross sections at both lowest order and order α_s^3 are plotted versus μ, again for $\eta=0$, $E_T=100$ GeV, and $\sqrt{s}=1800$ GeV, but now with $R=0.6$. Clearly the Born cross section is a monotonic function of μ while the higher-order result is quite stable in the range around $\mu/E_T\sim 0.5$. In the range $0.5\le\mu/E_T\le 1$ the Born cross section varies by approximately 40% while the higher-order result varies

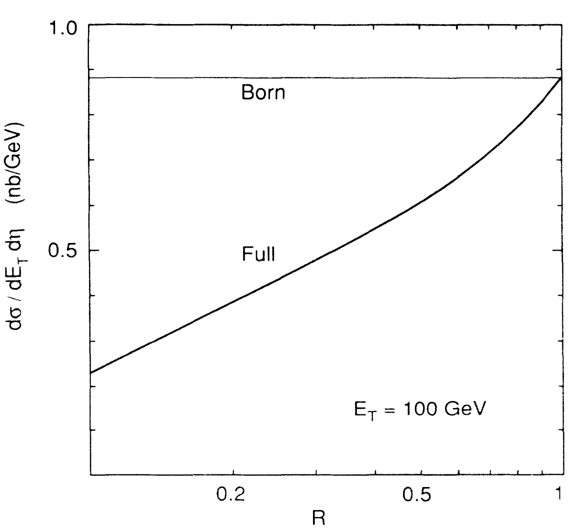

FIG. 1. Inclusive jet cross section $d\sigma/dE_T\,d\eta$ vs the cone size R for $\sqrt{s}=1800$ GeV, $E_T=100$ GeV, $\eta=0$, and $\mu=0.5\times E_T$.

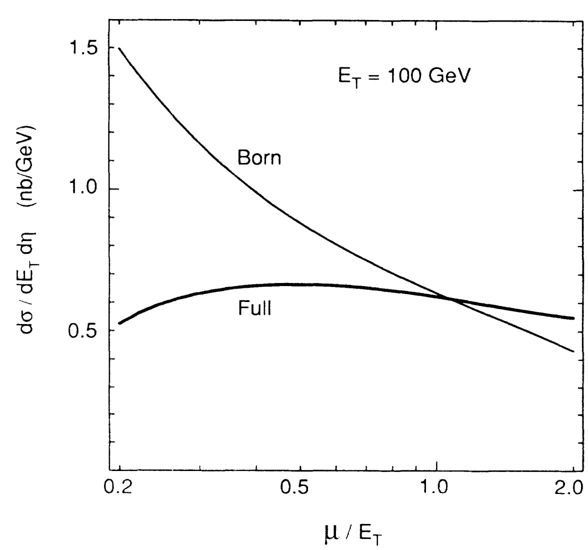

FIG. 2. Inclusive jet cross section $d\sigma/dE_T\,d\eta$ vs the ratio μ/E_T for $\sqrt{s}=1800$ GeV, $E_T=100$ GeV, $\eta=0$, and $R=0.6$.

only by about 5%. In this sense the inclusion of the higher contributions has greatly reduced the theoretical uncertainty due the arbitrary choice of μ.

Finally, we compare the calculated inclusive jet cross section to data. Figure 3 exhibits the data from the Collider Detector at Fermilab[1] (CDF) for $R=0.6$ and $\sqrt{s}=1800$ GeV vs E_T. The data are averaged over the angular range $0.1 \leq |\eta| \leq 0.7$. In the original CDF publication,[1] the experimental values for E_l were adjusted in an attempt to account for the part of the original parton's energy that fell outside the cone and for "background" energy (corresponding to the general energy level in the calorimeter) that fell inside the cone. It is important to recognize that the status of such corrections is different when comparing to the higher-order cross section. The order-α_s^3 perturbative contribution to both corrections is, in fact, correctly *included* in the calculation. Thus we have attempted to remove these corrections from the data plotted in Fig. 3 by subtracting 1 GeV from each quoted jet E_T. There are, of course, residual corrections. We include these in our estimate of the theoretical uncertainty described below.

The theoretical curve in Fig. 3 corresponds to $R=0.6$ and $\sqrt{s}=1800$ GeV, and is averaged over η in the same way as the data. To estimate the overall theoretical uncertainty we include three contributions. We first note that, in the range around $E_T \sim 100$ GeV, the cross section behaves as $\sim E_T^{-6}$. Thus an uncertainty in the jet transverse energy $\Delta E_T \sim 1$ GeV due to nonperturbative sources will yield a relative uncertainty in the cross section of the form

$$\frac{\Delta\sigma_{\rm NP}}{\sigma} \equiv \epsilon_{\rm NP} \sim 6\frac{\Delta E_T}{E_T} \sim \frac{6\ \rm GeV}{E_T}, \tag{4}$$

which is less than 6% for $E_T \gtrsim 100$ GeV. There will also be a perturbative contribution to the theoretical uncertainty of the form

$$\frac{\Delta\sigma_{\rm P}}{\sigma} \equiv \epsilon_{\rm P} \sim c\alpha_s^2 . \tag{5}$$

One would normally expect the uncalculated higher-order perturbative coefficient c to be of order 1, but in this process, with its steeply falling cross section, a larger value may be anticipated. We estimate that $c \sim 5$. With $\alpha_s^2 \sim 0.015$, this implies a residual perturbative uncertainty $< 10\%$, which is consistent with the variation of the calculated cross section with μ.

Finally, there is the uncertainty due to the choice of parton structure functions. We have used the set B structure functions of Martin, Roberts, and Stirling[13] (MRS) with $\Lambda_{\overline{\rm MS}}^{5\ \rm flavors}=200$ MeV to generate the results in Figs. 1-3. We have compared the Born-level jet cross section calculated with these structure functions (MRS set B) to the Born-level cross section obtained with three other sets of structure functions (MRSE,[13] MRS3,[4] and EHLQ1[14]). In the kinematic range of interest we find the differences between the resulting cross sections to be of order $\pm 20\%$.[15] Thus we estimate

$$\epsilon_{\rm SF} \sim 0.20 . \tag{6}$$

We have added the explicit forms in Eqs. (4)-(6) in quadrature to determine the width of the theoretical curve indicated in Fig. 3.

It should be noted that, although our jet definition is close to that used by the UA1 and CDF groups,[1] there is a residual uncertainty associated with detailed differences in the way the jet definition is applied to the data and to the theoretical calculation (e.g., whether one adds the E_T from each calorimeter cell or first adds the E from each cell and then projects the sum onto the transverse plane). This issue has not yet been studied in detail but we do not expect the associated effects to be larger than the uncertainties discussed above.

In conclusion, we have calculated the inclusive jet cross section at order α_s^3, including the contributions of all types of partons. The resulting cross section exhibits reasonable dependence on the jet-cone size, with which it will be important to compare experimental data. The theoretical uncertainty due to perturbative effects has been considerably reduced from the situation at lowest order. The first comparison to data for the cross section indicates very good agreement, which raises the possibility of precision tests of QCD in the context of jet physics and the possibility of using jet measurements to determine the form of the gluon structure function in the proton.

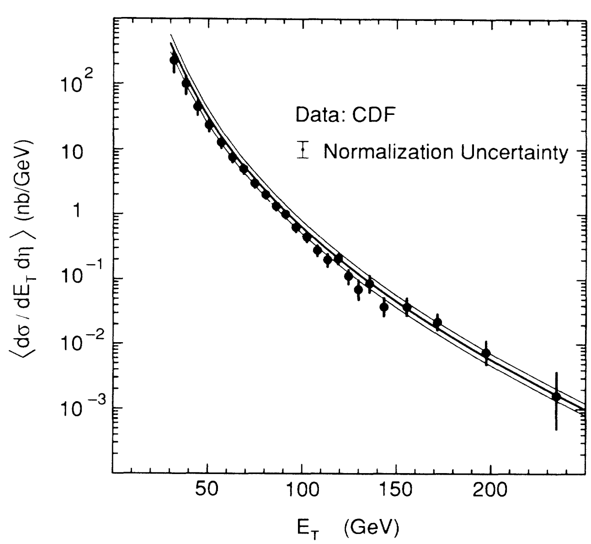

FIG. 3. Inclusive jet cross section $d\sigma/dE_T\,d\eta$ averaged over $0.1 < |\eta| < 0.7$ vs E_T at $\sqrt{s}=1800$ GeV for $R=0.6$ and $\mu=0.5E_T$. The data are from the CDF Collaboration (Ref. 1). The experimental error bars include both the energy-dependent systematic errors and the statistical errors as given in Ref. 1. The magnitude of the energy-independent systematic uncertainty is indicated by the single, isolated error bar.

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